

Efficiency and equity in a socially-embedded economy*

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Abstract

A model that only focuses on economic relations, and in which efficiency and equity are defined in terms of resource allocation may miss an important part of the picture. We propose a canonical extension of the standard general equilibrium model that embeds economic activities in a larger game of social interactions. Such a model combines general equilibrium effects with social multiplier effects and considerably enriches the analysis of efficiency and equity. Efficiency involves coordination between economic and social interactions, may depend on social norms, and may strongly interact with the distribution of resources. Equity can be defined in a comprehensive, socioeconomic way, and a decomposition into an economic and a social component is possible.

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1 Introduction

This paper aims at making three contributions. First, it presents a general model of society that embeds the economic market equilibrium in other non-market social interactions, and encapsulates key tenets of the main models of social interactions. Second, it examines the conditions for achieving efficiency and equity in such a broad “socioeconomic model” and compares these with the conditions for efficiency and equity in the standard general equilibrium model, focused on the economy alone. Third, the paper explores implications for some of the most significant ideas to have emerged from the discipline of economics, such as the workings of the Invisible Hand (as elucidated by the two fundamental theorems of welfare economics), or the relation between efficiency and equity. These implications constitute an important motivation for our socio-economic model.

Our model relates straightforwardly to the existing economic thinking on economic and social interactions. The core models of economic theory focus on markets, contracts and trade and tend to ignore other forms of social interactions. Since Adam Smith’s *Theory of Moral Sentiments*, Veblen’s *Theory of the Leisure Class* (Veblen 1934) and Becker’s “Theory of social interactions” (Becker 1974), however, many economists have been aware that there is an important social side to people’s lives that cannot be ignored because it influences the economic world and is affected by it in return. A very large economic literature on social interactions has blossomed in the last decades.

This literature can be divided into three broad categories. One branch studies social influences with game-theoretic and related tools and focuses on the specific patterns that emerge in social interactions, as distinct from market transactions. It highlights specific phenomena that naturally appear in such interactions, such as conformism and social multiplier effects, as well as multiple equilibria, and it generally focuses on one variable of choice, such as a particular cultural, moral or consumption behavior (e.g., Bernheim 1994, Akerlof 1997, Brennan and Pettit 2004, Brock and Durlauf 2001, Durlauf 2001, Durlauf and Ioannides 2010, Gintis 2017, Gui and Sugden 2005, Hoff and Stiglitz 2016, Manski 2000). In some approaches there is a direct desire to conform to the others’ behavior, whereas in others there is external policing through approval and blame. Diffusion through networks has recently been extensively studied (Jackson 2008, Bramoullé et al. 2016).

The other important branch of the literature studies the interdependence between social mechanisms and economic activity. It has adopted the sociological notion of social capital (Bourdieu 1979, Coleman 1994, Putnam 2002) and examined how social capital, competition for social status, social identities and social norms or even mere narratives influence economic decisions, in particular human capital investment, savings and consumption behavior, as well as how, conversely, economic decisions and economic transformations shape social structures such as class hierarchies and segregation (Akerlof and Kranton 2000, Cole, Mailath and Postlewaite 1992, 1998, Becker and Murphy 2000, Benabou 1993, 1996, Frank 1985, Hoff and Stiglitz 2016, Kolm 2005, Loury 1977, Coate and Loury 1993, Mailath and Postlewaite 2003, 2006, Snower and Bosworth 2016, Shiller 2019).

A third branch of the literature explores the microfoundations of social influences. Taking inspiration from psychology (e.g., Fiske 2005, Kahneman et al. 1999) and direct experimental evidence, it reveals how sensitive people are to social comparisons and fairness evaluations, how influenced they are by the

social context, and how much they care about others (Rabin 1993, Fehr and Schmidt 1999, Fehr et al. 2002, Cherchia et al. 2017, Layard 2005). Brennan and Pettit (2004) propose to consider the desire to be esteemed as a key social enforcement lever beside the fear of punishment and economic incentives, and this can be enlarged to other desires such as genuine caring, a desire for belonging, and so on.

As this broad literature provides many illuminating analyses of the way in which real-life economies, embedded in social settings, differ considerably from the textbook economic model, the time may be ripe for rethinking the basic general equilibrium model and revisit the standard economic concepts of efficiency and equity that have been developed for this model. This is the purpose of our socio-economic model.

The paper introduces a canonical model depicting basic channels of interdependence between economic and social interactions. This model is a standard economic Arrow-Debreu model embedded in a social game, resembling the integrated models of the economy and the environment (e.g., Nordhaus and Boyer 2000). The social game is a simple interactive game in which every individual directs some action to every individual (including oneself), and social success depends on the level of support or reinforcement received from others. The Walras-Nash equilibrium of this model contains the competitive equilibrium in the economy in the Nash equilibrium of the larger social game.

This model is general enough to encompass the conformism mechanism of social interaction models, the enforcement of norms by social pressure, patterns of grouping and segregation, power relations as well as competition for status and power. It displays general equilibrium effects alongside social multiplier effects, which enriches the analysis of social and economic transformations induced by exogenous parametric changes. The model is static and has a fixed population, and therefore leaves out a key set of dynamic social phenomena, in particular intergenerational transmission and the dynamics of interpersonal interactions. But, as is well-known, static analysis is able to uncover key structural insights of steady-state dynamic equilibria.

This model distinguishes two channels of interdependence between the economic module and the larger social game. One channel operates through people's preferences, or more generally character formation, which may involve influence of the social context on economic preferences. The other channel lies in the determination of social success, i.e., in the rules of the social game, in which economic activities may influence social outcomes, and therefore may be determined by social strategic considerations. These are the two senses in which the economic model of Walrasian equilibrium is embedded in the social model of Nash equilibrium. Each channel brings in externality effects and thereby important sources of inefficiency.

As far as equity is concerned, this model makes it possible to examine the relationship between economic inequality and social inequality, and raises the question of defining equity in a way that encompasses the economic and the social dimensions of people's lives simultaneously. As can be expected, economic inequality is likely to be correlated with social inequality when the social game makes social status partly depend on economic assets, but it is insufficient as a metric for general inequality. General socioeconomic inequality can be measured by extending money-metric utilities, which are already familiar in the literature on well-being (Fleurbaey and Blanchet 2013), to include social dimensions. This provides a convenient generalization of the measurement of economic inequality and offers the

possibility to disentangle the contribution of economic inequality from other social factors in the measure of general socioeconomic inequality.

Three limitations of this model must be mentioned: it assumes perfect competition and the absence of market failures on the economic side; it ignores the fact that some social interactions occur within market transactions, which is likely to generate market power as well; it excludes social impacts on the available production technology. These issues have been briefly examined in Fleurbaey et al. (2021), from which this paper originates, and will be further explored in follow-up papers.

The paper is structured as follows. Section 2 presents an overview of the underlying ideas and the main insights. Section 3 introduces the basic socioeconomic model and three special cases of this model that disentangle the various types of interactions between the economic sphere and the broader social sphere. Sections 4 and 5 then examine what happens to the study of efficiency and equity, respectively, in this model. Section 6 concludes.

2 Insights from this analysis

Our analysis offers several important insights and suggests open questions that we would like to highlight here, before diving into the details of the model. The ideas emerging from this paper are not all new, and many have been suggested by various existing models of social interactions. Our socio-economy model however provides a general framework for understanding and exploring these various dimensions, which together have far-reaching implications for our understanding of how the market economy interacts with society and how economic policy affects this interaction.

The first set of insights from this work is methodological, offering a critical perspective on conventional economic models, but also the positive outlook of a convenient modeling framework.

- **Conventional economic models make unrealistic implicit assumptions about society:** Conventional economic analysis focuses on a special component of a more general socioeconomic theory. But by ignoring the social component, it implicitly assumes that society is well-functioning and that there is perfect coordination between the social and economic spheres (see section 4.1). Economic theories of market imperfections (such as theories of asymmetric information and market power) and most environmental economic theories recognize the existence of externalities and other market failures, but still assume a well-functioning society and ignore the issue of the coordination between the spheres. A less charitable interpretation of conventional economic models is that they assume away social interactions altogether, even if a world in which people interact economically but not socially is totally foreign to any reasonable conception of humanity.

- **A simple and tractable extension of the canonical general equilibrium model can serve as a general template for the study of systemic interactions between the economic and social spheres:** Adjoining a social game to the general equilibrium Arrow-Debreu model provides a convenient framework (see section 3). The standard analytical tools for the investigation of equilibrium existence, efficiency and equity can be extended to this richer model. This makes it possible to achieve a deeper understanding of the tenets of efficiency and equity in complex societies, to simultaneously

track distinct general equilibrium effects and social multiplier effects in one and the same model, and to enlarge the array of interactions between efficiency and equity covered in the analysis.

A second set of insights is that efficiency in the economy is much less relevant than claimed in economics, and that conditions for efficiency shed light on the importance of the interaction between the social and economic spheres:

- **The fundamental theorems of welfare economics are undermined:** In the presence of the social sphere, the two fundamental theorems of welfare economics no longer hold. In particular, the perfectly competitive general equilibrium no longer leads to a Pareto efficient allocation of resources, even under complete markets, symmetric information and no economic externalities (see section 4). Since competitive markets do not lead to efficiency, the conventional theoretical basis for the Invisible Hand is undermined. Lump-sum transfers combined with free markets would not suffice to simultaneously achieve efficiency and equity.

- **Efficiency requires appropriate coordination between the economic and social spheres:** More precisely, Pareto efficiency requires (i) efficient economic markets, (ii) efficient social interactions, and (iii) efficient coordination between the economic and social spheres. Sufficient conditions for efficiency, encapsulating the three conditions, can be expressed in terms of net zero value of social and economic externalities, given the distribution of resources and the induced willingness to pay for externalities among the population. Coordination between the economic and social spheres is both important and difficult to fulfill, since the commodification of many social relations is impossible, while the typical coordination mechanisms of the social sphere (social norms, tacit reciprocity, or centralized coordination) are usually not designed for such a task.

A third set of insights is about inequalities, which now have an economic and a social dimension.

- **Economic inequality interacts with social inequality:** Economic inequalities correlate with social inequalities under mild conditions about the nature of the social competition, even if there is not necessarily a perfect correlation between economic inequality and social inequality (see section 5.1). The impact of the distribution of economic wealth on social inequality can be decomposed into a direct effect (wealth raises status), general equilibrium effects through prices and wages, and a social multiplier effect through the strategies in the social game.

- **Heterogenous preferences over social status question a narrow focus on economic equity:** A focus on economic equity misses important issues and clashes with the Pareto principle, in a new form of efficiency-equity trade-off. Indeed, we show that for a social welfare criterion respecting individual preferences, it is logically impossible, when individuals have heterogeneous preferences over the relative importance of economic and social dimensions, to always treat reducing inequalities in resources as socially desirable, even when such reduction is implemented among individuals with identical social standing.

- **Socioeconomic inequality can be measured comprehensively:** Measuring well-being in a comprehensive way covering economic and social dimensions is possible and offers ways to analyze the contribution of each sphere to well-being inequalities (see section 5.2). This paper develops a way of conceptualizing and measuring socio-economic inequality, building on the money-metric approach

which can be extended beyond the narrow measurement of living standards. It also sheds light on the limited benefits for social optimality of economic transactions that transfer resources from the rich to the poor and reduce inequalities in consumption but widen the gap in their social standing, e.g., when they enter master-servant, or employer-employee relations.

- **Complementarity between efficiency and equity:** A huge literature has devoted attention to the correlation between economic inequalities and economic productivity and efficiency. Here we find that the interdependence between social relations and economic inequalities provides a natural channel by which reducing economic inequality can enhance general socio-economic efficiency (as distinct from economic efficiency). In particular, when economic inequality is sufficiently high and sufficient status-seeking activities sufficiently intense, economic redistribution can create more commonality among people and thereby enables a wider range of mutually beneficial social relations. Under suitable circumstances, redistribution is good for everyone (see sections 4.3, 5.1).

The insights above call into question the conventional view of Adam Smith's Invisible Hand and bring us closer to Arrow's (1973) views about the complementarity between economic interests and moral norms. The history of capitalism may be understood in terms of the gradual extension of economic markets into domains that used to be governed by social relations. In his magnum opus, *The Great Transformation*, Karl Polanyi describes how economies that were based on reciprocity and communal relationships became transformed into competitive markets, accompanied by industrialization and rising influence of the state. It was this transformation that enabled us to conceive of human behavior in terms of the satisfaction of consumption wants through voluntary exchange. In the process, economics lost sight of Polanyi's insight that "man's economy, as a rule, is submerged in his social relationships." (1944, p. 48) This paper is a tentative attempt to right the balance.

The insights above are also a first step towards a balanced appreciation of two major themes, emerging from two different disciplines: The first is "the magic of markets" theme of economics, explaining why people trading freely with one another under competitive conditions can make each other better off. The second is "the magic of societies" theme of anthropology, explaining how much of the evolutionary success of humanity can be explained by our capacities for social cooperation (see, for example, Christakis (2019), Henrich (2016), Novak (2012) and Turchin (2015)). The first may be understood as the economist's Invisible Hand; the second as the anthropologist's Invisible Hand. We need an appreciation of both themes within a single conceptual framework in order to gain a more balanced understanding of how economies, in interaction with the societies in which they are embedded, help people coordinate their actions in order to address the challenges they face. This paper is meant as a step towards such a more balanced understanding.

3 An integrated Walras-Nash model

The basic model starts with a standard Arrow-Debreu general equilibrium model and embeds it in a social game, in similar fashion as integrated climate-economy models feature a standard Solow growth model connected to a climate module, but with the key difference that the social sphere is still the locus of human strategic action. Even though this is only a first step toward the integration of the

economic and the non-economic, it already contains several channels of interactions, and therefore we will introduce specific subcases of this basic model in which the specific channels are isolated. This section introduces the general basic model and its relevant three special cases.

3.1 General framework

To keep things simple, and take the most favorable outlook for the economic sphere of the model as a starting point, the economy part of the model is an Arrow-Debreu economy with constant returns to scale in production, perfect competition and no externalities. Market imperfections will be introduced in sections 6-7.

3.1.1 Individual behavior

There is a finite number of individuals $i \in N = \{1, \dots, n\}$. Each individual's situation is described by a pair (x_i, y_i) , where $x_i \in X_i \subset \mathbb{R}_+^\ell$ is a vector of $\ell > 1$ commodities consumed by i , and y_i a vector of personal and collective outcomes in the social sphere that are relevant for individual i . Total production takes the form of a transformation of commodities $q \in Q \subset \{0\} \cup (\mathbb{R}^\ell \setminus \mathbb{R}_+^\ell)$, where a positive component of q is an output and a negative component an input, and Q is a cone. We assume constant returns to scale (i.e., Q is a cone) in order to avoid having to track the distribution of profit among individuals. When $Q = \{0\}$, there is no production and the model describes a pure exchange economy.

Individual preferences are represented by a utility function $u_i(x_i, y_i)$. This utility function can actually capture much more than standard preferences. Since social influences really shape individuals' mindset and determine their personal development, this function can capture deep and formative impacts of social interactions on the individual. In that sense, this model allows for dramatic departures from the standard economic model in which preferences over x_i are stable. What we keep from the standard approach, though, is that individual behavior is assumed to rationally strive for personal well-being according to the true function u_i . This means that in our analysis, individuals choose social relations that make them grow and they systematically shun social interactions that would have a nefarious influence on their personal development. It is obvious that this assumption is not realistic. But we retain it here because our focus is on the fact that, in spite of such a demanding rationality assumption, there are serious obstacles to achieving a social optimum in this model. It is obvious that things are much harder if individuals adopt self-destructive strategies.

Each individual faces two constraints, the economic and the social one. The economic constraint is a typical budget, and it is assumed in sections 1-6 that individuals are price-takers:

$$x \in X_i \text{ and } px_i \leq p\omega_i,$$

where ω_i is i 's endowment. Commodities can include labor services.

The other constraint brings the social game into the picture. It says that y_i is obtained through a

game form of the type

$$y_i = F_i(x, s),$$

where $x = (x_1, \dots, x_n)$ is the economic allocation and $s = (s_1, \dots, s_n)$ is the profile of social strategies $s_i \in S_i$ in the population. The function F_i encapsulates how i 's social outcome depends on social strategies but also on the economic allocation. Market prices do not appear explicitly as an argument of F_i but the market value of x (reflecting i 's wealth, a potentially relevant variable for social stratification) can be retrieved when a price vector supports every bundle x_i (i.e., delineates a hyperplane that is tangent to the upper contour set of the individual at x_i). Thus, this model makes room for wealth comparisons in the social game.

In this game, the interdependence between commodities and social interactions can go both ways. Some social patterns may require certain economic distributions, but conversely, it may be impossible for an individual to adopt an economic lifestyle without the realization of certain social strategies. To keep the analysis simple, we will assume that the set of strategies available to i is a fixed S_i , and that all interdependent feasibility constraints between x and s , or between s_i and s_{-i} , are embodied in the function F_i .

Many examples of social games can be drawn from the literature on social interactions cited in the introduction, and many examples are provided in this paper. For concreteness, think of s_i as a strategy which can consist in a combination of the following actions:

- extend, accept or decline invitations to and from other people (leading to family formation, friendships, joint activities including economic activities);
- directly engage in interactions and collective activities (e.g., cheering, booing, fighting, joining demonstrations);
- pass or withhold (mis)information, advice and admonitions, expressive symbolic messages that may influence others;
- behave in a way that enhances or undermines one's reputation (e.g., being truthful, faithful, courageous, empathetic, or the opposite);
- seek, keep or change one's or someone else's position in a network or a hierarchy of power or influence.

The game produces social outcomes, and the relevant outcomes for i are contained in the vector y_i . This vector can for instance contain a list of pairs (j, r) meaning that i is in relation r with individual j (e.g., i has authority over j regarding road use), or it can describe how i is viewed by other people in terms of identity, esteem, prestige, what type of social role, power and status i enjoys or endures, and so on. It can include features about the whole social situation and need not be limited to strictly personal outcomes, since the individual may care about who is related to whom, who has such or such position or social status and so on.

Moreover, since $y_i = F_i(x, s)$, the vector of social outcomes y_i can include many things about the social situation that the individual may care about: the distribution of resources x , the strategies played by

everyone s . This means that our model is able to capture preferences about procedures and not just about final outcomes. For instance, individuals may resent being forced to do certain things (e.g., follow a dress code), even though these things are not by themselves particularly unpleasant. This can be depicted in this model by listing two components in y_i , one representing the actual outcome, and another recording how it came about through specific strategies.

3.1.2 Fleshing out the social game

As there is a strikingly similar structure in most social interactions, it is possible and useful to provide a more precise description of the social game, at a similar level of abstraction as the Arrow-Debreu economic model. The common structure in social games comes from their distributed reinforcing nature. That is, every individual directs a more or less supportive action at every individual (including oneself), and the outcome for an individual is an increasing function of the level of support received by this individual. The support itself may be aimed at two different types of outcomes: a collective achievement (joint activity, common beliefs), or a personal outcome (status, power). Hybrid outcomes are also commonplace, when group gatherings and actions play a role in social competition for status or power.

Formally, let $s_i = (s_{ij})_{j=1,\dots,n}$, where each $s_{ij} \in \mathbb{R}^m$ is a level of support in dimensions $d = 1, \dots, m$. The individual outcome $y_i \in \mathbb{R}^m$ includes the same dimensions $d = 1, \dots, m$, some of which may simply be traces of x and s , if the individual cares about the distribution of consumption or the strategies deployed by the others. The function $F_{id}(x, s)$ in dimension d is assumed to be a non-decreasing function of the vector $(s_{jid})_{j=1,\dots,n}$. The number of dimensions m can be large because it can include the existence of a link between any pair of individuals in a network, and there may be several different networks for different joint activities.

Different types of functions $F_{id}(x, s)$ embody different social norms for various outcomes. Here are salient examples, with illustrations for collective achievements and personal outcomes, when relevant:

- the veto function $y_{id} = \min_{j \in N_{id}} s_{ji}$ enables every j in a certain subset N_{id} to limit the outcome y_{id} :
 - collective: whether i befriends l can be vetoed by i, l and perhaps a few others among their relatives;
 - personal: i 's credibility may be determined by the individual who trusts him the least, if such opinions are common knowledge;
- the claim function $y_{id} = \max_{j \in N_{id}} s_{ji}$ enables every j in a certain subset to up the outcome y_{id} :
 - collective: how often the Smiths and the Joneses have dinner together may depend on the one who extends the most invitations;
 - personal: in order to get a position, one acceptance may suffice;
- the additive function $y_{id} = \sum_{j \in N_{id}} s_{ji}$ enables every j in a certain subset to add up to the outcome :

- collective: community life depends on multiple contributions (as for a public good);
- personal: people heap praise or blame on i ;
- the rank function $y_{id} = r$ if $\sum_j s_{ji}$ has rank r in the distribution of $\sum_j s_{jl}, l = 1, \dots, n$:
 - personal: status or power depends on the relative support received, e.g. getting a plurality of votes in an election;
- the affordability function $y_{id} = \min \{x_{ik}, s_{ijd}\}$:
 - personal or social: a certain social strategy (e.g., initiating a relation) is possible only with the required wherewithal in resources;
- the gatekeeper function $y_{id} = x_{ik} \text{sgn} (s_{ijd} - s_{ijd}^*)$;
 - personal: the individual suffers a social penalty for consuming x_{ik} unless she “conforms” by having $s_{ijd} > s_{ijd}^*$; this can serve to police social behavior through market access discrimination;
- the contest function $y_{id} = f(x_{jk}, j = 1, \dots, n)$ where f is increasing in x_{ik} and decreasing in x_{jk} for $j \neq i$, and k is a commodity (or a subset of commodities) representing expenses in the contest:
 - personal: status may be obtained by ostentatious consumption.
- the solo function $y_{id} = s_{iid}$ marks a purely individual decision:
 - personal: i joins an institution (e.g., a religious denomination), makes a public statement (e.g., an outing), or adopts any observable behavior that may not directly affect others’ personal outcomes but will potentially shift their strategies in equilibrium;

The next to last example only involves economic actions, and one can interpret buying a quantity of commodity k as supporting one’s own status, while the others do the same on their own count. No support for others is considered in this example, but one could introduce it by allowing for gifts.

More examples will be given in the paper, but it should be clear that this model encompasses most of the social interaction games of the literature, with the exception of dynamic and intergenerational models.

Formal institutions other than the market economy can be depicted in this model in two ways. First, the functions F_i can embody different social roles. For instance, one individual occupying a position of authority may be entitled to promote or demote another individual, whereas the latter has no such reciprocal power. An alternative way to have institutions in the model may derive from the social game having multiple equilibria, each of which represents a particular institution. For instance, once an individual asserts authority, it may be a best response for another to accept it rather than undertake a costly rebellion, and this may enshrine the hierarchy among them. Informal institutions such as social norms are naturally incorporated in the model through preferences for certain behaviors (possibly out of sheer conformism) and willingness to punish deviations.

3.1.3 Walras-Nash equilibrium

In summary, individual i selects his economic and social behavior by solving the following program:

$$\begin{aligned} & \max_{x_i \in X_i, s_i \in S_i} u_i(x_i, y_i) \\ \text{such that } & \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i((x_i, x_{-i}), (s_i, s_{-i})). \end{cases} \end{aligned}$$

This program assumes that the individual takes prices and other individuals' strategies as given, corresponding to competitive economic behavior and Nash-type strategic behavior.

A Walras-Nash equilibrium of this model is a pair (x, y) such that, for a price vector p and a strategy profile s :

WN-i) every i solves the above program;

WN-ii) the markets clear: $\sum_i x_i = \sum_i \omega_i + q$;

WN-iii) q maximizes pq for $q \in Q$.

Observe that the market clearance condition is separate from individual maximization, whereas the feasibility constraints on social strategies and outcomes are included in the functions F_i .

A Walras-Nash equilibrium includes two component subequilibria in the two spheres. The Walras subequilibrium is defined for a given strategy profile s as follows. The economic allocation is a Walras subequilibrium if there is a price vector p such that:

W-i) every i solves the following program:

$$\max_{x_i \in X_i} u_i(x_i, y_i) \text{ such that } \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i((x_i, x_{-i}), s). \end{cases}$$

W-ii) the markets clear (same as WN-ii);

W-iii) production choice (same as WN-iii).

In the Walras subequilibrium, the individual takes account of the influence that the choice of x_i has on the social outcome y_i .

The Nash subequilibrium is defined as follows, for a given economic allocation x . The social outcome y is a Nash subequilibrium if there is a strategy profile s such that:

N-i) every i solves the following program:

$$\max_{s_i \in S_i} u_i(x_i, y_i) \text{ such that } y_i = F_i(x, (s_i, s_{-i})).$$

It is worth noting an apparent asymmetry between the Walras and the Nash subequilibria. In the former, the individual takes account of the social consequences y_i of economic decisions x_i , via the

function $F_i((x_i, x_{-i}), s)$, whereas in the latter, social strategies appear not to have any economic consequences. This may seem odd, since the function $F_i((x_i, x_{-i}), s)$ does encapsulate the possibility that certain economic actions are possible only in conjunction with certain social strategies (e.g., access to a particular market may be barred unless a social relation is established with a gatekeeper). But social outcomes and economic outcomes are inherently different. Once economic decisions are fixed, there is nothing that can be done about it through social actions, because physical transactions are given: in the case of private goods, strategies and outcomes are the same thing since there is no difference between buying a good and obtaining it. In contrast, an economic decision may have the power to alter social outcomes, which have symbolic dimensions, even if social strategies s are also physically given and unalterable.

Proposition 1 *For a pair (x, y) to be a Walras-Nash equilibrium, associated to a price vector p and a strategy profile s , it is necessary that x be a Walras subequilibrium for the given s (and associated to the same vector p) and y be a Nash subequilibrium for the given x (and associated to the same s). This is also sufficient if, for every i , there is an increasing transform φ such that the function $\Phi_i(x, s) = \varphi \circ u_i(x_i, F_i(x, s))$ is concave and continuously differentiable in (x_i, s_i) .*

Conditions for the existence of a Walras-Nash equilibrium can be derived from adapting Ghosal and Polemarchakis (1997, Prop. 1).

Proposition 2 *A Walras-Nash equilibrium exists under the following assumptions:*

- *The function $U_i(x, s) := u_i(x_i, F_i(x, s))$ is continuous in (x, s) and non-satiable in x_i ;¹*
- *The set X_i is closed and convex;*
- *The set S_i is compact and convex;*
- *The individual endowment $\omega_i \gg 0$;*
- *The cone Q is closed;*
- *For every p and (x_{-i}, s_{-i}) , the set of (x_i, s_i) maximizing $U_i(x, s)$ such that $px_i \leq p\omega_i$ is convex.*

Proof. See Appendix A. ■

These sufficient conditions for existence of an equilibrium are rather standard and in this light may not appear very demanding, except the last one regarding s_i and convexity of S_i . In many real-life social games, strategies are discrete (tell the truth or lie, accept or decline...), and in such cases, existence of equilibria in pure strategies is much harder to guarantee.

Overall, this model is meant to be as simple as possible while allowing for an interesting range of social phenomena. Many extensions and refinements of this model deserve to be considered. As already mentioned, the dynamics of social interactions, including through intergenerational transmission, would

¹“Non-satiable in x_i ” means that for every (x, s) , every neighborhood $N \subset X_i$ of x_i , there is $x'_i \in N$ such that $U_i(x'_i, x_{-i}, s) > U_i(x, s)$.

bring a new range of interesting issues, and relatedly, risk, uncertainty and imperfect information would bring about many important phenomena which have been the subject of much research in game theory and information economics. Other valuable extensions would allow further social interference in the economy through social games around public good contributions, voluntary and forced transfers of resources (gifts, theft and extortion), and social relations happening in market transactions themselves. Finally, behavioral issues appear even more relevant for social interactions than for economic transactions, and should also be a priority for possible refinements. While each of these directions appears highly relevant and promising, we hope that the simple model proposed in this paper can serve as a useful cornerstone on which such extensions can be built.

3.2 Disentangling interdependence between economy and society

The reason why the model is not simply written in terms of two variables (x_i, s_i) with a general other-regarding utility function $u_i(x, s)$, but also includes the “social” variable y_i is that it makes it possible to more concretely depict interactions between x and s in determining social outcomes, and thus, enables us to distinguish the interactions between the social and the economic that take place in the individuals’ preferences from those that come through feasibility constraints. In particular, it is illuminating to disentangle the various interactions by looking at specific variants of the model in which the interaction between the economic and the social is severed in specific ways. Four restrictions are worth considering, two on preferences and two on the social game (the restrictions listed here are meant to apply to all individuals):

- Separable preferences: $u_i(x_i, y_i) = v_i(f_i(x_i), g_i(y_i))$, where v_i is increasing in $f_i(x_i) \in \mathbb{R}$, the subutility on commodities, and $g_i(y_i) \in \mathbb{R}$, the subutility on social outcomes.
- Pure economic preferences: $u_i(x_i, y_i) = f_i(x_i)$, i.e., the individual does not care about social outcomes.
- Separable social function: $F_i(x, s) = G_i(h_i(x), k_i(s)) \in \mathbb{R}$, where G_i is increasing in $h_i(x) \in \mathbb{R}$, the economic outcome, and $k_i(s) \in \mathbb{R}$, the purely social outcome. A key restriction here is that F_i is assumed to be one-dimensional.
- Pure social game: $F_i(x, s) = F_i(s)$, i.e., the social outcome that matters to i is independent of the economic allocation (but may be multidimensional).

Table 1 briefly depicts what happens to the Walras and Nash subequilibria when various restrictions are combined. A subequilibrium is “separate” when it does not depend on the other sphere.

The contents of the cells numbered (1)-(4) of Table 1 are explained in detail in the next subsections, as they represent interesting variants of the main model. The conventional economic analysis, as in the classical Arrow-Debreu model, is featured in cell (0), where individuals are only interested in their own private consumption and do not care at all about the social game, which moreover exerts no constraint on the economy. This table shows how restrictive this conventional framework is. The other cells are not studied in more detail in this paper, as they provide more straightforward results or less valuable insights.

	$y_i =$	$F_i(x, s)$	$G_i(h_i(x), k_i(s))$ (separable social function)	$F_i(s)$ (pure social game)
$u_i =$				
$u_i(x_i, y_i)$	Full model	(4)	Separate Nash subequilibrium	(3) x_{-i} has no influence on i
$v_i(f_i(x_i), g_i(y_i))$ (separable preferences)	(2) Maximize $g_i(y_i)$ in Nash subequil.			(1) Separate Walras and Nash subequilibria
$f_i(x_i)$ (pure economic preferences)		s_i only used to lift constraints on trade		(0) Social game is irrelevant, no externalities at all

Table 1: Restricted interactions between the economic and the social spheres

3.2.1 The park model (social interactions independent of economic interactions)

The cell numbered (1) can be intuitively described by reference to the typical American “park”, where social interactions do occur, but in a break from the economic part of life since everyone comes in casual outfit, so that it is hard to notice economic inequalities. Conversely, what happens in the park has no influence on the economy.

This model, with separable preferences $u_i(x_i, y_i) = v_i(f_i(x_i), g_i(y_i))$ and a pure social game $y_i = F_i(s)$, completely separates the economic subequilibrium from the social subequilibrium. The economic subequilibrium in this model is an allocation x such that, for a price vector p :

- i) every i chooses $x_i \in \mathbb{R}_+^\ell$ so as to maximize $f_i(x_i)$ such that $px_i \leq p\omega_i$;
- ii) the markets clear: $\sum_i x_i = \sum_i \omega_i + q$;
- iii) q maximizes pq for $q \in Q$.

The social subequilibrium is a social situation y such that, for a strategy profile s :

- iv) every i chooses $s_i \in S_i$ so as to maximize $g_i(y_i)$ such that $y_i = F_i(s)$.

As one can see, (i)-(iii) form a standard Walrasian equilibrium, while (iv) forms a standard Nash equilibrium. This model is particularly interesting because, even though the two subequilibria are separate, it retains some interdependence in individual preferences over f_i, g_i .

3.2.2 The backyard model (economically supported social interactions)

The model in cell numbered (2) keeps preferences separable, $u_i(x_i, y_i) = v_i(f_i(x_i), g_i(y_i))$, but reintroduces economic affairs into social interactions, $y_i = F_i(x, s)$. This variant can again be illustrated by a typical American institution, the “backyard,” where social interactions happen only when one can

afford to invite people and offer them drinks and food, although it entails little interaction between economic and social aspects in preferences.

In this case, the Nash subequilibrium is simplified because it is now based on the individual program:

$$\max_{s_i \in S_i} g_i(y_i) \text{ such that } y_i = F_i(x, (s_i, s_{-i})),$$

where the influence of x is confined to the feasibility of social outcomes, and no longer bears on preferences. In contrast, in the Walras subequilibrium, the individual still has to take account of the influence of x_i over y_i and cannot simply maximize $f_i(x_i)$:

$$\max_{x_i \in \mathbb{R}_+^\ell} v_i(f_i(x_i), g_i(y_i)) \text{ such that } \begin{cases} px_i \leq p\omega_i, \\ y_i = F_i((x_i, x_{-i}), s). \end{cases}$$

For instance, being better dressed may make the individual able to invite a friend to a restaurant, and have a better chance that the friend will be interested in spending time with him. To sum up, separability in preferences does not produce separation of any of the subequilibria, because the interaction of x and s in function F_i plays a key role.

3.2.3 The community model (economic interactions with social contagion)

Cell number (3) contains a variant that does the opposite of the backyard model. It drops separability in preferences but shuts down any interaction between economic and social affairs on the feasibility side: $y_i = F_i(s)$. To make this configuration intuitive, one may think of contagion effects coming from being in relation with other people in a non-economic community.

In this case, it is now the Walras subequilibrium that is simplified, since the individual program boils down to:

$$\max_{x_i} u_i(x_i, y_i) \text{ such that } px_i \leq p\omega_i.$$

This does not mean, however, that when solving the full program, the individual neglects the social side when making the economic decision about x_i . This choice will alter the optimal strategy s_i through the preference interaction. This can occur, for instance, when certain social interactions make one seek certain commodities (e.g., gifts of a specific kind), which, conversely, makes these social relations more attractive when one has access to these commodities. It also happens when certain commodities can be substitutes for certain social relations (e.g., buying a TV set may reduce the need or time for chats with neighbors, having access to private insurance may reduce the need for solidarity arrangements with relatives and friends).

The distinction between the backyard and the community model is formally clear but, in substance, not very strong, because separable preferences in the backyard model do not imply much separability in the overall utility function

$$v_i(f_i(x_i), g_i(F_i((x_i, x_{-i}), s))),$$

and the distinction between interaction between x_i and s that appears in preferences proper (in the community model) or in the outcome function F_i (in the backyard model) is shallow, since F_i is meant to represent the outcomes of the social game that matter to i .

3.2.4 The separate-spheres model (separate economic and social spheres)

Cell number (4) involves separability of the F_i function, which is then assumed to be one-dimensional. The clearcut distinction between an economic outcome and a purely social outcome, both contributing to i 's social outcome y_i , evokes a society with separate economic and social rankings, as in (stereotypical) representations of some traditional societies in which hereditary privilege, clergy membership or literacy provide a special status that is disconnected from economic positions—as a matter of fact, disconnection is very rare, as mentioned by Weber (1947) in his discussion of class and status.

This variant has the interesting feature of rendering strategy choices in the social game independent of x , but it does not preserve x from being influenced by s even when preferences are separable in x_i and y_i . For instance, assume

$$v_i(f, g) = fg$$

$$G_i(h, k) = \sqrt{h + k},$$

with $g_i(y_i) = y_i$. Then the marginal rate of substitution (MRS) between goods 1 and 2 is equal to

$$\frac{\frac{\partial f_i}{\partial x_{i1}}(h_i + k_i) + 0.5f_i \frac{\partial h_i}{\partial x_{i1}}}{\frac{\partial f_i}{\partial x_{i2}}(h_i + k_i) + 0.5f_i \frac{\partial h_i}{\partial x_{i2}}},$$

which depends on k_i in general, in a systematic way: The greater k_i (in positive values), the more the MRS depends on consumption preferences (f_i) rather than social considerations (h_i).

In conclusion of this section, we can highlight the fact that the common assumption of an irrelevant social game in traditional economic analysis is a very special and restrictive case, and that separability assumptions generally appear to leave substantial channels of influence between the economic and the social spheres. The next two sections examine how this mutual interdependence alters the analysis of efficiency and equity.

4 Efficiency

Efficiency can be defined for the whole situation, or separately for the economic and the social situations.

- An allocation (x, y) is efficient if there is no other feasible allocation (x', y') such that $u_i(x_i, y_i) \leq u_i(x'_i, y'_i)$ for all i and $u_i(x_i, y_i) < u_i(x'_i, y'_i)$ for some i .
- The economic allocation x is efficient given the strategy profile s if there is no other feasible allocation x' such that, letting $y'_i = F_i(p, x', s)$, one has $u_i(x_i, y_i) \leq u_i(x'_i, y'_i)$ for all i and $u_i(x_i, y_i) < u_i(x'_i, y'_i)$ for some i .

- The social situation y is efficient given the economic allocation x if there is no other feasible strategy profile s' such that, letting $y'_i = F_i(p, x, s')$, one has $u_i(x_i, y_i) \leq u_i(x'_i, y'_i)$ for all i and $u_i(x_i, y_i) < u_i(x'_i, y'_i)$ for some i .

General conditions for the efficiency of the economic allocation are well-known, and in particular involve the absence of externalities from economic decisions. Obviously, if x_{-i} influences i 's utility via $F_i(x, s)$, externalities are likely to arise, and therefore the park model and the community model, where the technology $F_i(s)$ prevails, contain more favorable circumstances for economic efficiency.

Efficiency in the social game can be analyzed by distinguishing three cases, which may jointly appear in various dimensions of the outcome vectors. These cases correspond to different technologies introduced in section 2:

1. The joint activity case where the veto or the claim technology prevails is likely to produce efficient equilibria, because at least one individual obtains her preferred option.
2. The case of a public good that is collectively produced, where private optimization tends to lead to underproduction, is generally plagued with inefficiency.
3. The case of competition for status or power, via social and/or economic strategies, also generally leads to inefficiency because of excessive exertion in the competition.

Clearly, the introduction of a social game next to the economic equilibrium introduces several potential causes of inefficiency: externalities via social outcomes influenced by economic actions, public good effects and expensive competition for status or power. In this section, we further study the channels by which inefficiency may appear, as well as the conditions permitting efficiency to be obtained.

4.1 First-order analysis in the general model

Under standard smoothness and interiority assumptions, first-order conditions are necessary. Under additional convexity assumptions, they are also sufficient. They provide a very convenient heuristic tool, which we will employ in this first subsection.

To be able to write down first-order conditions, assume that, for some a homogeneous function T , one has $Q = \{q \in \{0\} \cup (\mathbb{R}^\ell \setminus \mathbb{R}_+^\ell) \mid T(q) \leq 0\}$. Let $y_i = (y_{il})_{l=1, \dots, L}$ be the vector of individual i 's social outcomes, and to simplify notations, let us flatten the dimensions of the social strategies into a single one for every individual: $s_i = (s_{ih})_{h=1, \dots, H}$.

The first-order conditions of efficiency of an allocation can be formulated as follows: there exist $\alpha \in \mathbb{R}_+^n, \lambda \in \mathbb{R}_+^\ell, \mu \in \mathbb{R}_{++}$ such that:

- The sum of marginal private and social impacts of consumption x_{ik} equal the Lagrange multiplier for the resource constraint on commodity k :

$$\forall i \in N, k \in \{1, \dots, \ell\}, \alpha_i \left[\frac{\partial u_i}{\partial x_{ik}} + \sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial x_{ik}} \right] + \sum_{j \neq i} \alpha_j \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = \lambda_k, \quad (1)$$

- The marginal private and social impacts of strategy s_{ih} add up to zero:

$$\forall i \in N, h \in \{1, \dots, H\}, \alpha_i \sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial s_{ih}} + \sum_{j \neq i} \alpha_j \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0, \quad (2)$$

- Technology choice equalizes the impact of q_k on T with the Lagrange multiplier for commodity k ;

$$\forall k \in \{1, \dots, \ell\}, \lambda_k = \mu \frac{\partial T}{\partial q_k}, \quad (3)$$

- The resource constraint is satisfied:

$$\sum_i x_i = \sum_i \omega_i + q. \quad (4)$$

Under what assumptions can a Walras-Nash equilibrium be efficient? The following proposition provides an array of sufficient conditions that exclude externalities in various ways—its proof also contains a necessary and sufficient condition. The conditions listed in this proposition involve either the absence of externalities, at a more or less disaggregated level, or the commonality of goals guiding individual decisions.

Proposition 3 *Assume that all functions u_i, F_i, T are continuously differentiable and concave, and that the sets X_i, S_i are all closed and convex. Then the lower set² of the utility possibility set is convex. Assume moreover that the function $U_i(x, s) := u_i(x_i, F_i(x, s))$ is non-satiable. Distinct sufficient conditions for efficiency of an interior Walras-Nash equilibrium include the following (from logically stronger to weaker, except for the second and third which are not related):*

1. (No externalities) For all $i, j \in N$ ($i \neq j$), F_j is independent of x_i, s_i .
2. (Either no externalities or common goals) For all $i, j \in N$ ($i \neq j$), F_j is independent of x_i ; and for all $h \in \{1, \dots, H\}, l \in \{1, \dots, L\}$, either (i) F_{jl} is independent of s_{ih} , or (ii) $F_{jl} = F_{il}$ and $F_{il'}$ is independent of s_{ih} for all $l' \neq l$ and u_i is monotonic in y_{il} .
3. (No satisfaction externalities) For all $i, j \in N$ ($i \neq j$), all $x, s, u_j(x_j, F_j(x, s))$ is independent of x_i, s_i .
4. (Either no satisfaction externalities or common goals) For all $i, j \in N$ ($i \neq j$), $x, s, u_j(x_j, F_j(x, s))$ is independent of x_i and for all $h \in \{1, \dots, H\}$, either $u_j(x_j, F_j(x, s))$ is independent of s_{ih} or $u_j(x_j, F_j(x, s))$ and $u_i(x_i, F_i(x, s))$ are ordinally equivalent functions of s_{ih} .
5. (Balanced externalities) $\forall i \in N, k \in \{1, \dots, \ell\}, h \in \{1, \dots, H\}$,

$$\sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0,$$

where λ_j denotes the marginal utility of money for individual j in the equilibrium.

²The lower set of a set A is the set of elements x such that $x \leq a$ for some $a \in A$.

6. (Weakly balanced externalities) $\forall i \in N, k \in \{1, \dots, \ell\}, h \in \{1, \dots, H\},$

$$\sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} \text{ is a fixed proportion of } p_k; \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0$$

Conventional economic analysis of market efficiency assumes away any externality, in the form of condition 1 in the proposition. It is worth being aware of the absurdity of modeling a society in which people have no social influence on one another. Condition 3 is less strict but also involves the absence of net externalities. Such conditions may suggest that efficiency in the economy (Walrasian subequilibrium) combined with efficiency in the social sphere (Nash subequilibrium) suffices to achieve full efficiency. However, this is incorrect because it relies on sufficient conditions for efficiency (the absence of net externalities) that are unduly restrictive. Efficiency in the economy and in the social game can be achieved under a larger set of circumstances, and this is revealed by the “balanced externalities” condition in the proposition, which means that externalities generated by every i are a mix of positive and negative externalities, and *the total marginal willingness to pay of the individuals impacted by the externalities must be zero*. The “weakly balanced externalities” condition allows for positive or negative total externalities generated by x_{ik} , but in such a way that the total marginal willingness to pay is a fixed (positive or negative) proportion of prices.

The last two conditions, which are akin to the Bowen-Lindahl-Samuelson condition for efficient production of public goods, introduce a link between the social interdependence of individuals entailed by externalities and the distribution of wealth in the economy. Such interdependence between the economic and social sides of the model will be further elucidated in the next subsections.

It should be emphasized that this analysis is about the Walras-Nash equilibrium in which individuals pursue their personal satisfaction as depicted in the model. While their personal utility may depend on other-regarding facts through the F_i function, one can imagine yet other ways of dealing with externalities that involve deviating from pursuing personal satisfaction (e.g., for moral reasons). We leave this issue for another paper.

4.2 Efficiency and inefficiency in the park model

The park model gives the simplest relation between general efficiency and efficiency in the economic sphere and the social sphere. First, it is possible to define efficiency separately for each sphere: An economic allocation x is efficient if there is no other feasible allocation x' such that $f_i(x_i) \leq f_i(x'_i)$ for all i and $f_i(x_i) < f_i(x'_i)$ for some i ; a social allocation y is efficient if there is no other feasible allocation y' such that $g_i(y_i) \leq g_i(y'_i)$ for all i and $g_i(y_i) < g_i(y'_i)$ for some i .

Since the Arrow-Debreu equilibrium is separable from the rest of the model, the First Welfare Theorem applies and under the usual assumptions about preferences, the economic equilibrium is efficient. But overall efficiency may be hard to obtain. Indeed, in this model, the first-order condition (12) boils down to

$$\forall i \in N, h \in \{1, \dots, H\}, \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial v_j}{\partial g_j} \frac{\partial g_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0,$$

and still connects the distribution of resources to the distribution of social externalities, in spite of the strong separation of the two spheres.

Proposition 4 *For an allocation (x, y) to be efficient it is necessary, but not sufficient, that x and y both be separately efficient.*

Here is an example illustrating how an allocation (x, y) which is separately efficient in x and in y can nevertheless be grossly inefficient.

Example 1 (Efficiency requires coordination between the economic and social spheres).

Consider a society with two individuals. On the economic side, there is only one good, so that there is no possibility of economic trade and every individual consumes her endowment: $x_i = \omega_i$. This is trivially efficient. On the social side, there is only one dimension (y_i is a real number, and s_i , too) and the game is defined by $y_i = \min_{j=1,2} s_j$, i.e., the veto function introduced in section 3.1.2.³ For instance, in the park the length of the conversation may be determined by the individual who stops first. Or the warmth of the relationship may be determined by the colder individual.

In this game, every individual has a preferred y_i^* which maximizes $g_i(y_i)$, and in the Nash equilibrium can play $s_i = y_i^*$, generating the social outcome $y_1 = y_2 = \min_{i=1,2} y_i^*$. Assuming there is a unique y_i^* for each i , this is efficient because one individual has his best possible outcome, and there is no way to improve the allocation for the other one without harming the former. (There are other, Pareto-inferior, equilibria in which both play the same strategy $y^{**} < y_i^*$, and no individual deviation may improve on the allocation. Here we focus on the salient efficient equilibrium.)

Figure 1 illustrates the allocation, with the economic consumption on the vertical axis and the social outcome on the horizontal axis, and the situation of the two individuals is depicted in the figure. Individual 1 is richer than individual 2 and happens to have a lower y_i^* , therefore determining the outcome of the social game. There is no way to improve the situation of both individuals by altering the economic allocation only, or the social strategies only.

Figure 2 shows that, in spite of being separately efficient in x and in y , this equilibrium is not efficient. Combining transfers of consumption from individual 2 to individual 1 with an increase in the “conversation” can make both of them better off. Referring back to condition (12), it is clear that in this example, individual 2 has a positive willingness to pay for more time with individual 1, and therefore individual 1 imposes a net externality on the rest of society.

In Fig. 2, the improvement represented by the arrows looks very much like the introduction of a market for conversation, in which individual 2 pays individual 1 for a longer chat. But commodifying the social interaction is generally not feasible. A market for the timing of friendly chats is self-defeating, since a chat that is paid is not self-motivated, as friendship requires.⁴

To illustrate this point, imagine the introduction of a market for social intercourse. Individuals may differentiate between genuine interaction y_i and paid interaction z_i , which is a new commodity created

³In the notations introduced in section 3, one would interpret s_i as being actually s_{ij} , i.e., the proposal for joint activity made by i to j , and s_{ii} would be irrelevant.

⁴For general analyses of the limits of markets, see Kanbur (2004), Satz (2012), Sandel (2012).

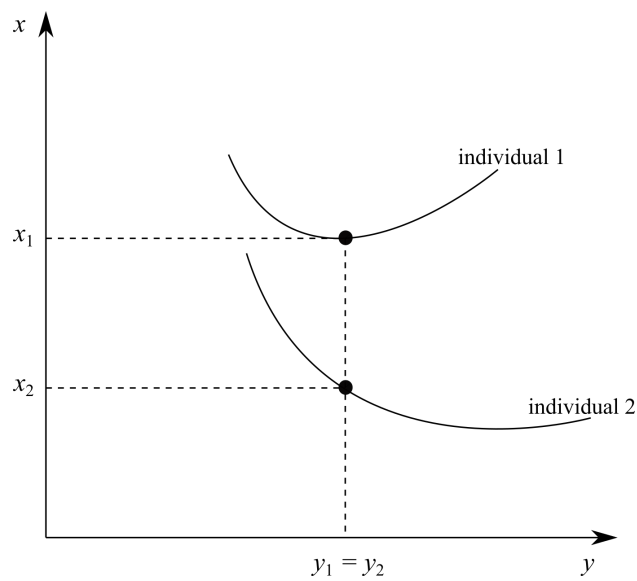


Figure 1: Equilibrium in Example 1

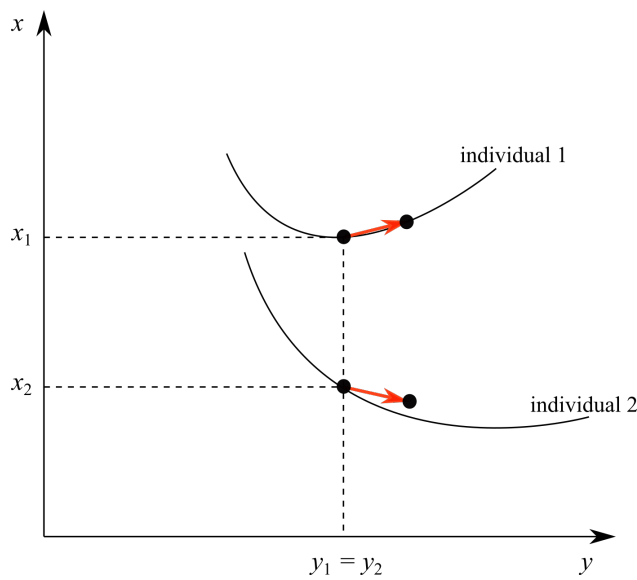


Figure 2: How to improve on the equilibrium in Example 1

by this market. Individual 2 represented in Fig. 2 may have the following preferences (taking x_2, y_2 as the quantities in the initial equilibrium of Fig. 1, and introducing z_2 as the quantity of paid interaction into the vector (x_2, y_2, z_2)), for relevant values of δ, ε :

$$u_2(x_2 - \delta, y_2 + \varepsilon, 0) > u_2(x_2, y_2, 0) > u_2(x_2 - \delta, 0, y_2 + \varepsilon).$$

In other words, this individual would rather stay at the initial equilibrium than give money for a “paid chat”, even if this individual would actually like a similar move, with the same quantities of money lost and interaction gained, if it was a genuinely friendly chat.

Similarly, a direct Coasean bargain between the individuals might not work if it is in terms of material payment, or any form of exchange requiring direct and immediate reciprocation, but it might work if it involves other, more subtle ways. For instance, rather than proposing to pay for more conversation, individual 2 in Fig. 2 might make a gift and this might induce individual 1 to “spontaneously” reciprocate by trying to be nicer and stay longer. To describe these considerations explicitly in the model, one would have to add to the social game the possibility to make free transfers of resources and introduce these transfers into the budget constraint on the economic side. This avenue is not explored in this paper.

One way to restore efficiency without involving transfers of resources consists in changing the technology of production of social outcomes. In the conversation example, norms of politeness play a key role in determining the actual length of conversation. Example 1 features a norm that gives the power to every individual to stop the interaction at any time (“sorry, I have to go”). The opposite norm would impose to stay whenever the other person still wants to chat. This would correspond to the technology $y_i = \max_{j=1,2} s_j$, also introduced in section 2. Similar inefficiency problems would arise with this technology, now with bored individuals willing to “pay” to stop the conversation earlier. Casual observation suggests that, for conversation, many people have developed a more subtle norm of politeness that involves body language. A sudden change of position or tone is meant to gently suggest that one would like to move on, without abruptly signaling an injunction to stop. Then the conversation slowly winds down, depending on the interest of the participants in the substance. Under this subtle politeness norm, the actual outcome lies between $\min_{j=1,2} s_j$ and $\max_{j=1,2} s_j$. This norm might approximate an efficient outcome, illustrated in Figure 3.⁵

Example 1 is about a joint activity (chatting) instead of a social competition, but similar results can be obtained when the social game is a competition for status or power, even considering special cases in which the competition is not wasteful per se.

This subsection provides three key lessons. First, for a given technology of the social game, the only way to improve efficiency may involve a combined alteration of the economic distribution and the social strategies. Second, extending the scope of market transactions may not be a practically effective way to address inefficiency problems. Third, changing social norms may alter the technology of the social game so as to reduce inefficiency.

⁵A similar outcome may also be obtained not through external social norms, but through internalized values, when individuals care about their behavior being conducive to optimal social outcomes. However, this requires discussing whether such internalizing process implies a revision to the measurement of individual well-being.

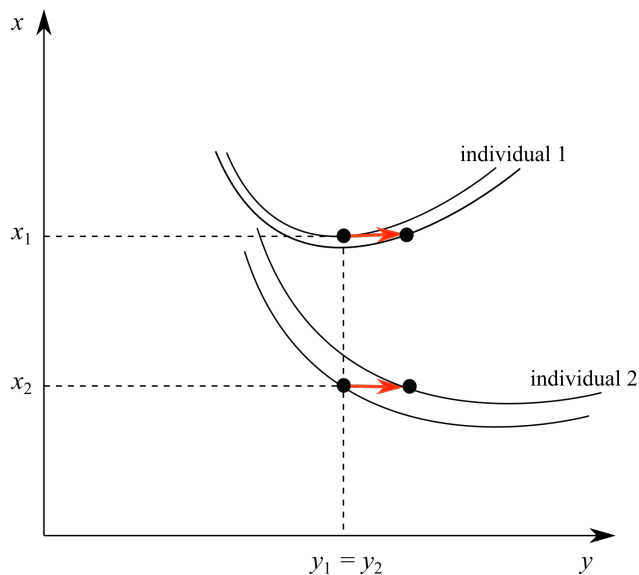


Figure 3: Efficiency through altered social norm

4.3 Further interactions between the spheres

The park model exhibits minimal interdependence between the economic and the social sphere. In particular, intervention in the economy alone has very limited impact since the social game is separate. In contrast, the backyard model and the community model have sufficient interdependence to make economic interventions impactful on the efficiency of the social situation.

In the backyard model, feasibility constraints include interactions between the economy and society, which makes it much harder for the economic equilibrium to be efficient. Consider the comprehensive utility function:

$$U_i(x_i, x_{-i}, s) = v_i(f_i(x_i), g_i(F_i((x_i, x_{-i}), s))).$$

As explained in subsection 3.2.2, the individual's behavior in the economic subequilibrium maximizes this utility function U_i by choosing x_i . The First Welfare Theorem no longer applies, due to economic externalities. When choosing x_i , individual i does not take into account that this will influence the utility $U_j(x_j, x_{-j}, s)$ of the other individuals j . These externalities are entirely due to the influence of x on the social game. This feature of the model can describe phenomena like the rat race, when people seek social status through economic prowess. Curbing the economic rat race may then be beneficial to everyone. This idea has been developed by Frank (1985) and a more technical study of optimal taxation, allowing for externalities of different signs in different parts of the income distribution, is made by Cowell and Stostad (2021).

Another interesting feature of this model is that, even in absence of such externalities, the efficiency of the whole allocation may depend on the distribution of initial resources. For instance, inequalities in the economy may hinder social relations in a way that is harmful to everyone. Redistribution can promote a more cohesive community in which the benefits of social bonding compensate the rich for their loss of economic privilege. This can be illustrated as follows.

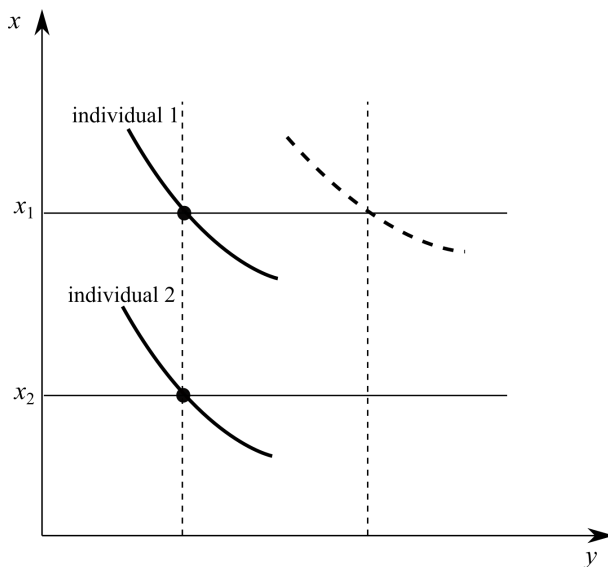


Figure 4: Equilibrium in Example 2

Example 2 (Efficiency and equity may be complementary). Consider a society with two individuals $i = 1, 2$. As in Example 1, there is only one commodity in the economy and the economic equilibrium is trivial, with $x_i = \omega_i$ for every i . In this example, the Walras subequilibrium is therefore efficient, even if this is not the case in general in the backyard model.

The individuals like to invite each other for a backyard party. But the host has to pay for the catering, and there is therefore a limited amount of invitations that each of them can extend. Moreover, out of reciprocity they maintain an equal number of parties in either backyard. The economic constraint on hosting parties is represented by the function

$$F_i(x, s) = \min \{x_1, x_2, s_1, s_2\},$$

meaning that individuals can invite less than they can afford if they wish ($s_i < x_i$) but cannot effectively invite more than they can afford.

Figure 4 illustrates the equilibrium in this example. Individual 1 is richer and would like to host more parties (dashed indifference curve), but is limited by individual 2's smaller wealth and the reciprocity norm. The vertical axis represents personal consumption, the horizontal axis the partying, and preferences over personal consumption and parties take the conventional form $u_i(x_i, y_i)$, where u_i is a quasi-concave function. This is separable in x_i and in y_i when x and y are one-dimensional.

Figure 5 shows how a Pareto-improvement is possible with redistribution of resources from individual 1 to individual 2. By giving some resources to individual 2, more parties are possible, and this compensates the loss of resources for individual 1 if the transfer is not excessive. This example features a non-differentiable F function, but the gist of condition (12) applies, since individual 1 has a positive willingness to pay for a joint increase in x_2 and s_2 (rather than either of them, which would suffice if F were differentiable), and the move illustrated in Fig. 5 reflects how an improvement can be obtained

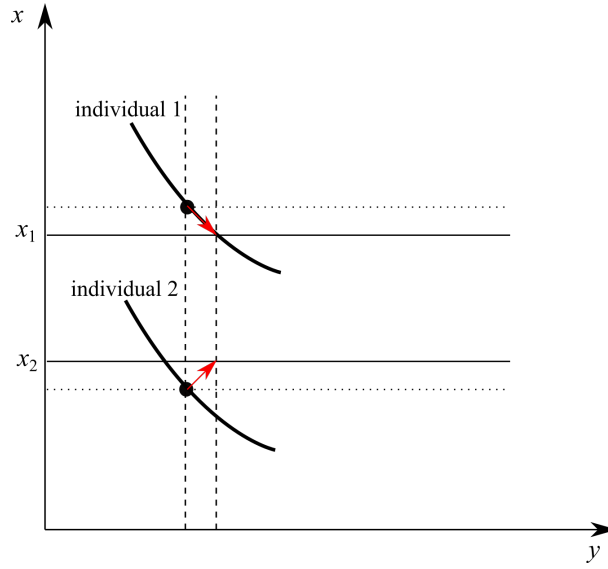


Figure 5: How to improve on the equilibrium in Example 2

along these lines.

In this example, there is no need to consider setting up a market, since an unconditional transfer of resources from individual 1 to 2 suffices to nudge the latter to extend more invitations. But, once again, it is not obvious that simply enabling individuals to make gifts in this model would provide an effective solution. A gift may be embarrassing to receive, especially if it is a disguised subsidy toward being invited more often. It would not be difficult to tweak the model to have individual 2 strictly prefer the counterfactual situation where he would have more resources to the status quo, but nevertheless strictly prefer the status quo to receiving a patronizing gift from individual 1. Coordinating the distribution of resources and social externalities is a complicated matter.

Let us turn to the community model, in which private consumption is relevant to social interactions but does not entail externalities on other people directly. In the absence of externalities originating from economic activities, the economic equilibrium is generally efficient *for a given value of the social strategies*. But the two spheres, the economic and the social, are interdependent, since s influences the economic equilibrium and x influences the social game, both through the non-separability of preferences over x_i and y_i . This introduces a new source of inefficiency in the general allocation, which can be illustrated as follows.

Example 3 (Suboptimal commodification). There are two individuals and, this time, two commodities, an all-purpose good x and a supporting service z , e.g., personal care. The latter is a substitute of social support provided through the social game, e.g., in the family. Individual utility is defined by two terms, a direct utility from consuming the two commodities and enjoying social support, and a disutility of providing social support per se as well as a disutility for not providing a similar quantity of social support, compared to the others. Formally,

$$u_i((x_i, z_i), y_i) = v(x_i, z_i + y_{i1} + y_{i2}) - c(y_{i1}, y_{i2}),$$

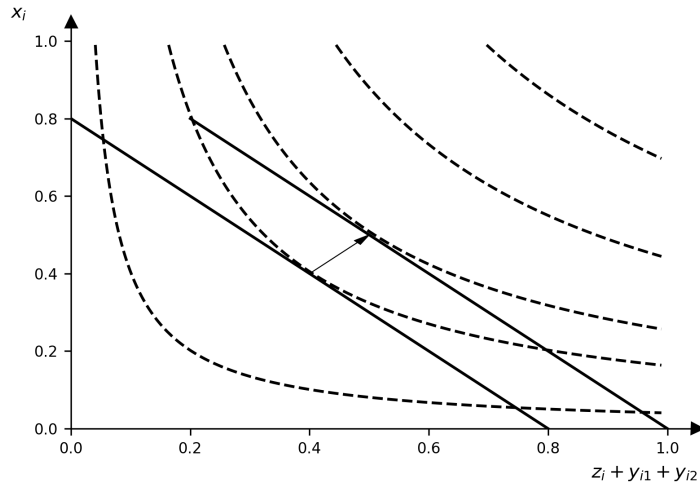


Figure 6: Effect of an increase of $y_{i1} + y_{i2}$ on the choice of (x_i, z_i) .

where $y_{i1} = s_i$, i 's contribution to the social support network, and $y_{i2} = s_j$, j 's contribution.⁶ The function v is concave and increasing in its two arguments, while c is convex in its first argument, and its first partial derivative is decreasing in its second argument—this last feature reflecting mimetic behavior. In this example, it is assumed that total consumption of x and z can vary according to a CRS technology transforming one into another at a fixed rate. The budget of an individual is then $x_i + pz_i = \omega_i$, where ω_i denotes i 's initial wealth and p is a fixed price determined by the technology.

This example depicts a situation in which social support through non-market relationships can free resources for economic activities and improve efficiency, at least if the disutility of providing social support outside the market is not too high. But social conformism can actually push individuals to devote too much energy to the social network, and therefore inefficiency can go either way, as we will show here.

When choosing the (x_i, z_i) bundle on the market, individual i is influenced by the social support situation, in such a way that an increase in $y_{i1} + y_{i2}$ is similar to a budget increase and induces an increase in x_i , assuming this good is normal, and thus a decrease in z_i (i.e., an increase in $z_i + y_{i1} + y_{i2}$ that is less than the increase in $y_{i1} + y_{i2}$). Figure 6 illustrates this effect of social support on the individual.⁷

The choice of the social strategy s_i can be represented as maximizing the gap between $v(x_i, z_i + s_i + s_j)$ and $c(s_i, s_j)$, taking account of the optimal values of (x_i, z_i) as a function of $s_i + s_j$. This choice is illustrated in Figure 7, where an increase in $y_{i2} = s_j$ induces an increase in s_i , exhibiting strategic

⁶Formally, this could also be described in the terms of the backyard model if one defined $z_i + s_i + s_j$ as a social outcome of interest for i , as this would then be a social outcome that combines economic decisions and social strategies. But this occurs here only because this term is separable in preferences, in this example, and is not a general feature of the community model.

⁷The function v used for the graphical illustrations of this example is $v = \sqrt{1 + x_i(z_i + y_{i1} + y_{i2})}$. The value of $y_{i1} + y_{i2}$ is increased from zero to 0.2 in the figure.

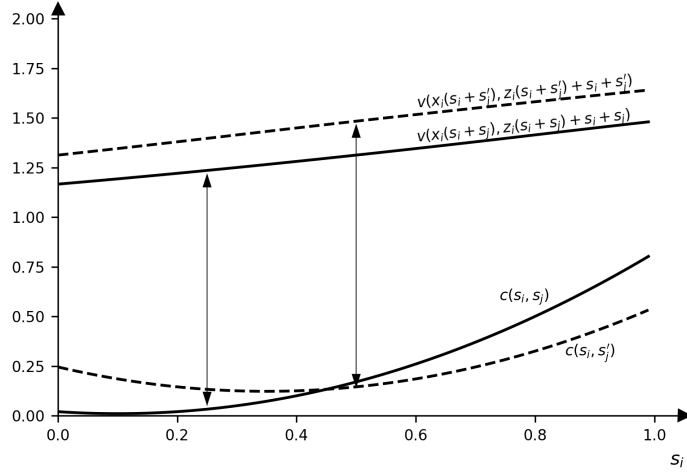


Figure 7: Effect of an increase in s_j on s_i .

complementarity in this game.⁸

The Walras-Nash equilibrium is found by solving this equation in s_i , where v_2 is the partial derivative w.r.t. the second argument, and c_1 the partial derivative w.r.t. the first argument:

$$v_2(x_i(s_i + s_j), z_i(s_i + s_j) + s_i + s_j) = c_1(s_i, s_j)$$

for $(i, j) = (1, 2)$ and $(2, 1)$.⁹ In the case of identical individuals, the symmetric equilibria solve:

$$v_2(x_i(2s), z_i(2s) + 2s) = c_1(s, s).$$

The Pareto efficient symmetric allocation, in contrast, solves

$$2v_2(x_i(2s), z_i(2s) + 2s) = c_1(s, s) + c_2(s, s).$$

Figure 8 illustrates typical configurations for the best response curves of the two individuals, in which the symmetric equilibrium is inefficient, with insufficient or excessive social support depending on the disutility of providing social support.¹⁰

One can relate the two graphs of Fig. 8 to familiar archetypes of social situations. In one situation, society is too individualistic and would be better off expanding its non-market connections. In the other situation, familiar to feminists, conformism induces people to rely too much on non-market

⁸The cost function used in this graph is $c(s_i, s_j) = 0.5s_i^2 + 0.5(s_i - s_j)^2$, and the two exogenous values for s_j are $s_j = 0.2, s'_j = 0.7$.

⁹The envelope theorem allows us to use v_2 here instead of the full derivative $\frac{\partial}{\partial s_i} v(x_i(s_i + s_j), z_i(s_i + s_j) + s_i + s_j)$.

¹⁰The two graphs differ only with respect to the cost function $c(s_i, s_j) = \beta s_i^2 + 0.5(s_i - s_j)^2$, with $\beta = 0.5$ for the left panel and 0.2 for the right panel of Fig. 8. The kinks in the best response curves represent thresholds beyond which social support crowds out commodity z completely. In both panels, the social optimum is the same and eliminates the market for supporting services.

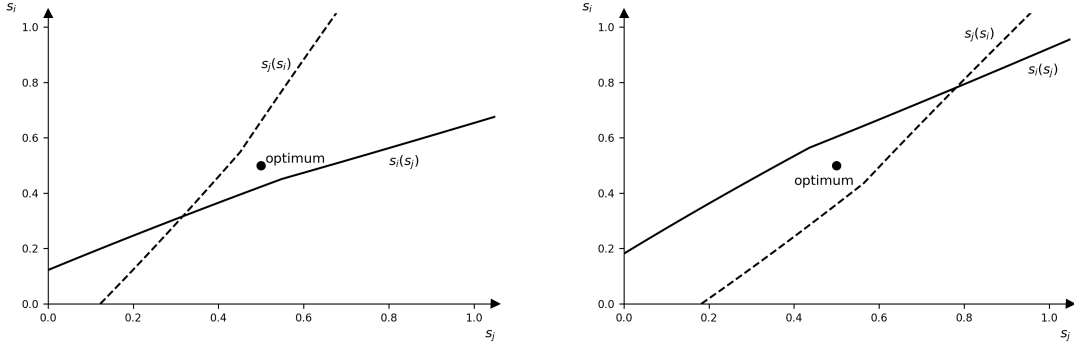


Figure 8: Equilibria in (s_i, s_j) and social optimum

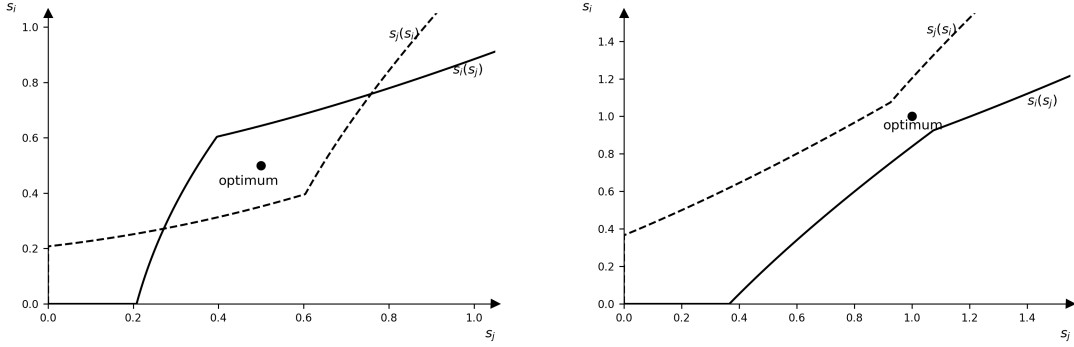


Figure 9: Multiple equilibria and technological progress

support, whereas market services would free them from social duties.¹¹

It is also possible to obtain multiple equilibria, as in the left panel of Figure 9, where two stable symmetric equilibria are obtained, including one with exclusive reliance on the market service for support.¹²

This particular configuration provides the occasion to show that technological progress that is offering better opportunities can actually harm society when it eliminates the high-support equilibrium and confines society to exclusive reliance on the market for support. The right panel of Fig. 9 shows what happens when the price of the service is halved in our numerical example. Then the only equilibrium that remains has zero social support, even though the optimal level of support would actually be greater than before.

An interesting feature of this example is that, here again, redistributing resources can influence social behavior. Reducing inequalities tends to enhance social support because the effect of wealth on contribution to social support is positive but with diminishing marginal impact. This is illustrated in

¹¹Obviously, this simple example does not represent gender inequalities and power relations in the household. But it clearly shows that it is possible to have excessive investment in non-market forms of support.

¹²In this case, the cost function is $c(s_i, s_j) = 0.3s_i + 0.01s_i^2 + 0.1(s_i - s_j)^2$. The three equilibria of the figure are Pareto-ranked in the same order as $s_i = s_j$.

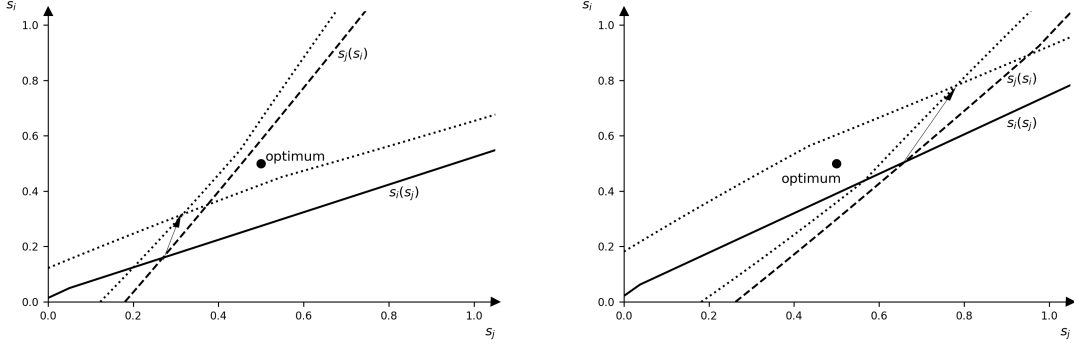


Figure 10: Redistribution enhances contributions to social support at the equilibrium.

Figure 10, where i is the poorer individual and substantially increases his contribution when wealth is equalized, whereas the richer individual is less affected. The equilibrium then has more contribution to social support from both individuals, including the rich one whose contribution is pushed upward by the increase in the other individual's one. This moves the equilibrium closer to the value of social support at the social optimum for equal wealth, when the optimum is above the equilibrium (left panel), whereas the opposite occurs when the optimum is lower than the symmetric equilibrium (right panel), although equalization might still be desirable for an inequality-averse social welfare function.¹³

4.4 Final remarks on efficiency

An extension of Prop. 4 can be formulated for the general model.

Proposition 5 *For the allocation (x, y) to be efficient, it is necessary but not sufficient that the social subequilibrium be efficient (for the given x) and that the economic subequilibrium be efficient (for the given s).*

Let us reinforce the idea that efficiency problems do not solely originate from a “messy” social sphere being plugged on the “tidy” market sphere. Although the Nash equilibrium approach is commonly considered adequate to model prisoner’s dilemma situations which can easily occur in social interactions, in some settings social interactions involve greater cooperation and commitment possibilities which are actually efficiency enhancing. Indeed, let us make the extreme assumption that the social game involves full coordination, so that in effect a social objective is maximized simultaneously by all individuals. Formally, assume that the strategy profile is selected to maximize a social objective $W(u_1(x_1, y_1), \dots, u_n(x_n, y_n))$, taking x as given. This guarantees in particular that the social sphere is efficient, provided the W function is increasing in its utility arguments.

Even in this variant, the whole allocation (x, y) can be inefficient for a similar reason as in the previous sections, i.e., due to a lack of management of the trade-offs between x and y . In order for full efficiency

¹³For Fig. 10, the cost function is as in Fig. 8, and incomes are (0.1, 1.9) in the unequal case, (1, 1) as in Fig. 8 in the equalized case.

to be guaranteed, one would need the social coordination to include the economic sphere, or at least to take account of the economic consequences of social strategies. For a given strategy profile s , one can define the corresponding Walrasian equilibrium as a function $x(s)$ if there is a unique equilibrium. Then full social coordination would maximize $W(u_1(x_1(s), F_1(x(s), s)), \dots, u_n(x_n(s), F_n(x(s), s)))$ when choosing s . One might hope that if the social sphere is able to reach perfect coordination around a common social objective, integrating the market in its scope should be possible. But this may require high expertise forecasting of the economic consequences of social arrangements.

Let us recap the main facts about efficiency in this section, in light of the lessons of the various models:

1. General efficiency is obtained in the absence of externalities in the economic and social spheres of the model. But this assumption, which is even more extreme for the social game than for the market economy, is not necessary, and it suffices to have balanced externalities in either sphere, i.e., externalities that induce a zero net willingness to pay over the whole population. This is still a very demanding assumption.
2. The efficiency of the two spheres (the economic and the social) does not guarantee full efficiency, due to interactions between the spheres even when preferences are separable and social interactions have no economic constraints (as in the park model). The interactions come from the fact that people may be willing to make trade-offs between economic and social outcomes, thus violating the zero net willingness to pay condition.
3. In general, when economic activities affect social outcomes, the economic equilibrium itself is plagued with consumption externalities, and makes the Walras subequilibrium inefficient.
4. The distribution of resources may have efficiency impacts, through the ability of individuals to obtain social outcomes as a function of their economic resources. In particular, economic equality may foster beneficial social cohesion.
5. The interaction may generate multiple equilibria, which are potentially ranked in terms of Pareto-dominance. This creates a coordination problem which involves the two spheres simultaneously.
6. Social norms that govern the social game may themselves be an impediment to achieving efficiency. In our examples, reciprocity constraints or the pressure of conformism may lock individuals in inferior allocations, and this is only one instance of how social conventions can reduce opportunities for beneficial social interactions. Altering the social norms may be a remedy.

Arguably, the most interesting source of inefficiency problems in this model is the lack of coordination across the two spheres. It is worth exploring why such a lack of coordination may occur, and in particular, why commodification of social interactions does not solve the coordination problem. There are several reasons why commodification does not work. First, commodification would undermine the essential nature of many social relations that are based on authentic feelings and deferred reciprocity, if not outright disinterested motivations. Economic theory commonly praises market trades for making every party better off, but trade is a venial type of social intercourse, where every party expects immediate reciprocation and pursues its own interest selfishly. For psychologically normal human

beings, there are higher forms of social relations and they involve selfless motivations, or at least deferred, not automatic, reciprocity. In the park example, a friendly chat could not be bought with money and still be a friendly chat with the same enjoyment. Second, a related key feature of many social relations is that they involve beliefs and feelings (as when having esteem for someone means holding certain beliefs and feelings about this person) which are inherently non-contractible and therefore cannot be subject to transactions. There is no way to pay people to believe that one is worthy of high trust, friendship or love. Truly, many economic activities are entangled with the creation of reputation and tacit reciprocity leading to more or less intimate social relations (Zelizer 2005), but the social relations themselves have no explicit price. These considerations can be reinforced by the observation that signalling takes place in social interactions (Benabou and Tirole 2003, 2006). For instance, spending time in a conversation signals that the partner is worthy of attention, and such a signal would no longer be credible if monetary incentives interfered. Insofar as individuals do care about receiving such signals of esteem, they put little value in interactions with extrinsic motivations.¹⁴

Additionally, people often have mental accounting habits which prevent them from doing the trade-offs that would be required for efficiency. Some of this mental accounting may come from conventions requiring to keep different social interactions separate lest some of them would be spoiled (e.g., by venial motivations), but there may be mental accounting above and beyond such considerations. For instance, people may keep track of reciprocity in a particular sphere (e.g., small non-monetary favors among neighbors) and ignore possible compensation through other spheres (e.g., contributing to the budget of a local association). Finally, the size of many markets for social interactions would be vanishingly small, so that specific prices could hardly form in a competitive way, thereby undermining any hope of efficiency gains in this direction.

Looking beyond commodification, what obstacles prevent a more general integration of the two spheres? The fact that the economic sphere is governed by market rules that are anonymous (i.e., the rules can be specified without regard to the people following them) makes it quite difficult to devise an integration that neither destroys the market (the personalized social way) nor expands it to cover social interactions (the commodification way). The celebrated effectiveness of the economic sphere, a signature achievement of the modern era (although it was prefigured in earlier periods of history), stems precisely from anonymization, i.e., its relative separation from the rest of social interactions. The only way in which a successful integration that preserves the autonomy of the economic sphere could lead to full efficiency would, in all likelihood, go through something similar to the maximization of the global social objective alluded to earlier in this subsection. This would require the social sphere to achieve full coordination of strategies among the individuals, a feat that is very far from reality, and a perfect anticipation of the general equilibrium economic consequences of social strategies, another feat which appears at least as farfetched.

In conclusion, societies are probably condemned to suffer from inefficiencies which are neither due to market failures nor inefficient social interactions, but come from the lack of coordination across the various spheres of interaction, as illustrated here with the economic and the social sphere. However,

¹⁴Our simple model allows for all players' strategies to enter individuals' utilities, but not their beliefs as in psychological games introduced by Geanakoplos et al. (1989). However, as shown by Segal and Sobel (2007), making room for strategies as arguments of utility corresponds to a particular class of psychological games. Our simple model could be extended to accommodate a richer set of phenomena involving beliefs.

some tacit coordination may occur through altered social norms that enable individuals to express the intensity of their preferences as would be measured by their “willingness to pay” (i.e., the trade-offs in their preferences, not their willingness to engage in commodified relations) for the quality of social interactions.

5 Equity

This model enables us to analyze equity in more dimensions than resource equality. Indeed, inequalities in social relations, in terms of status or power, can also be explicitly examined here. In this section, we first examine how economic equity matters, depending on the degree of interaction between the two spheres, and then study how to define equity in a comprehensive way.

5.1 Economic equity

When the economic and the social spheres interact, two things are likely to happen. First, economic inequalities may become more important because they may reinforce or foster social inequalities. On the other hand, social inequalities may have a life of their own which can either provide a decorrelation from economic inequalities (one may be a highly respected poor teacher) or entrench inequalities in a way that makes economic equality harder to achieve and a limited remedy to the general stratification problem.

In this subsection, we study two questions. First, we examine the conditions under which one can expect a strong correlation between economic and social inequalities, implying that redressing economic inequalities is an important social policy objective. Second, we show that even in the presence of a strong correlation, one should be careful in pushing for economic equality, as we highlight a possible efficiency-equity trade-off that is distinct from the familiar trade-off due to disincentives.

5.1.1 Economic and social inequalities

Consider the case in which x_i and y_i are each associated with strict partial orders,¹⁵ both denoted \succ since no confusion is possible, which are identical across individuals and serve to compare individuals and track the morally relevant inequalities. For instance, x_i may be ordered by vector dominance of commodity bundles, or by market value (for a set of possible market prices), and y_i may be ordered in terms of number of contacts (for social inclusion) and/or status (for social inequality). The orderings are partial and each can be thought of as the intersection of special orderings for particular dimensions in the space of resources and in the space of social outcomes, respectively. For instance, one may fail to have $y_i \succ y_j$ when i is more popular than j in the neighborhood but has a social network that is less extended geographically. Let $[y_i, y_j] \succ y_k$ be an abbreviation for “ $y_i \succ y_k$ and $y_j \succ y_k$ ”.

Consider the following properties for the social game form F . The first stipulates that economic advantage always offers opportunities to be successful in the social sphere as well.

¹⁵A strict partial order, or strict preorder, is an irreflexive, transitive, asymmetric binary relation.

Economic edge: For all $i, j \in N$, all x_i, x_j , if $x_i \succ x_j$, then for all $s_{-i} \in S_{-i}$ there is $s_i \in S_i$ such that $F_i(x, s) \succ F_j(x, s)$.

The second property implies that when social success is possible for an individual, this is not limited to counterfactual situations that are not comparable to the individual's best response but can actually be done with a dominated strategy.

Robust edge: For all $i, j \in N$, for all $s_{-i} \in S_{-i}$, if there is $s_i \in S_i$ such that $F_i(x, s) \succ F_j(x, s)$, then for every best response s_i^* to s_{-i} , there is $s'_i \in S_i$ such that $F_i(x, s_i^*, s_{-i}) \succ F_i(x, s'_i, s_{-i}) \succ F_j(x, s'_i, s_{-i})$.

The second depicts a game in which social competition is negative, in the sense that helping others succeed better is always detrimental to oneself.

Negative competition: For all $i, j \in N$, all $s \in S, s'_i \in S_i$, if $F_j(x, s'_i, s_{-i}) \succ F_j(x, s)$, then $F_i(x, s) \succ F_i(x, s'_i, s_{-i})$.

The third property is that a change in i 's strategy can never have a greater impact on others than on her own ranking. Since the partial orders do not have a defined cardinality, this is meaningful only in the case in which the social positions enjoyed by i under the two strategies are bracketed by the positions of another individual.

Self-impact: For all $i, j \in N$, all $s \in S, s'_i \in S_i$, one never has either $F_j(x, s'_i, s_{-i}) \succ [F_i(x, s'_i, s_{-i}), F_i(x, s)] \succ F_j(x, s)$ or $F_j(x, s) \succ [F_i(x, s'_i, s_{-i}), F_i(x, s)] \succ F_j(x, s'_i, s_{-i})$.

The next proposition lays out how such properties can exclude equilibrium situations in which an individual with an economic advantage is less successful in the social game than another individual.

Proposition 6 *Let the two strict partial orders \succ over individual outcomes of the economy and the social game be given, and assume that the latter is compatible with individual preferences, i.e., for all i and all $x_i, y_i \succ y'_i$ implies $u_i(x_i, y_i) > u_i(x_i, y'_i)$. Assume that the social game form F satisfies the economic edge and robust edge properties, and either negative competition or self-impact. Then, for all $i, j \in N, x_i \succ x_j$ and $F_j(x, s) \succ F_i(x, s)$ cannot occur simultaneously at any Walras-Nash equilibrium. Moreover, each triple of conditions is necessary in the sense that the result no longer holds if one condition in the triple is dropped.*

This proposition highlights various conditions that favor the correlation between economic and social standing. Three of them can be seen as mild. Economic edge is an obvious condition, in absence of which reversals are likely to occur. It mostly says that other things equal, economic advantage never hurts in the social competition. Robust edge is primarily a richness ancillary condition with no deep meaning, although it does force the social competition to be present at the vicinity of best-response strategies. Self-impact does not seem strong, it only reduces the strength of social externalities and makes every individual the first factor in her own success. Negative competition is an outlier. It corresponds to a situation in which helping others is never good for oneself, and this is a very demanding, unrealistic, and unappealing condition. But it does reinforce the correlation between economic and social standing because it implies that by pursuing their own success, individuals will

be led to undermining the others' success, and this, combined with economic edge and robust edge, enables the rich to transform their economic advantage into a secure social domination. Note that the assumption that preferences over y_i are consistent with \succ implies a noxious form of egoism under negative competition, which is perhaps unrealistic, since it means that individuals always pursue their own success whatever the harm this entails for others.

This result suggests that, in order to reduce correlation between economic and social standing, it is not enough to curb severe forms of competition as exemplified by the negative competition condition. One must also target the economic edge property. Economic edge is a property that can hold only when the F_i functions include x as an argument. The park and the community models in which this linkage between the economic and the social spheres is missing cannot display this property, and are less prone to the correlation between economic and social standing than the backyard model, for instance. However, Example 3 shows that wealth may affect social strategies and social success via its effect on preference trade-offs. This can be another channel for the correlation, and it is not covered by the above proposition.

One should also emphasize that this analysis has been dealing with merely ordinal patterns of rankings, not with the level of inequalities. This means that, although eliminating the economic edge may appear impossible in its “qualitative” form, it should make sense to seek to reduce the “quantitative” social inequalities that economic inequalities may induce. And this model enables us to analyze the channels by which economic resources shape social inequalities, as analyzed in the next subsection.

5.2 General equilibrium and social multiplier effects of economic inequalities

By combining the economic model with a social game, this model makes it possible to distinguish and compare general equilibrium effects and social multiplier effects. The former are channeled by changes in prices, while the latter appear through interactions in the social game and depend on whether this game exhibits strategic complementarity or substitutability. Disentangling the two effects can be done for any change in the parametric data of the model, and is illustrated here for the distribution of economic endowments.

Consider a change in the distribution of social outcomes y . Assuming full differentiability of the relevant functions and effects, one can disentangle the components of this change in y . As a general notation, let the expression $\left[\frac{\partial f_{ij}}{\partial z_{kl}}\right]$, where the indices run as $i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K, l = 1, \dots, L$ denote the $IJ \times KL$ matrix where rows are ordered as $ij = 11, \dots, 1J, \dots, I1, \dots, IJ$, and columns are ordered similarly. The expression $\left[\frac{\partial f_{ij}}{\partial z_k}\right]$ is an $IJ \times K$ matrix similarly constructed, and so on. When i, k are drawn from the same list, the expression $\left[\frac{\partial f_{ij}}{\partial z_{il}}\right]$ means that the two i indices run independently through the I values, not that they are equal in every cell of the matrix. The expression $\left[\frac{\partial f_{ij}}{\partial z_{il}}\right]_0$ means that the cells of the “diagonal” type $\frac{\partial f_{ij}}{\partial z_{il}}$ are put to zero in the matrix.

In our model, we have $x = (x_{ik})_{i=1, \dots, n, k=1, \dots, \ell}, y = (y_{id})_{i=1, \dots, n, d=1, \dots, m}, s = (s_{ijd})_{i,j=1, \dots, n, d=1, \dots, m}$.

One computes:

$$dy = \left[\frac{\partial F_{id}}{\partial x_{ik}} \right] dx + \left[\frac{\partial F_{id}}{\partial s_{ijd}} \right] ds.$$

The bundle x_i and strategy s_i are both functions of $(\omega_i, p, x_{-i}, s_{-i})$. One then has:

$$\begin{aligned} \begin{pmatrix} dx \\ ds \end{pmatrix} &= \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right] \\ \left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right] \end{pmatrix} d\omega + \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial p_k} \right] \\ \left[\frac{\partial s_{ijd}}{\partial p_k} \right] \end{pmatrix} dp + \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix} \begin{pmatrix} dx \\ ds \end{pmatrix} \\ &= \left(I - \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix} \right)^{-1} \left(\begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right] \\ \left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right] \end{pmatrix} d\omega + \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial p_k} \right] \\ \left[\frac{\partial s_{ijd}}{\partial p_k} \right] \end{pmatrix} dp \right), \end{aligned}$$

where I denotes the identity (i.e., diagonal) matrix. Putting these equations together, one obtains a useful decomposition:

$$\begin{aligned} dy &= \begin{pmatrix} \left[\frac{\partial F_{id}}{\partial x_{ik}} \right] & \left[\frac{\partial F_{id}}{\partial s_{ijd}} \right] \end{pmatrix} \left(I - \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix} \right)^{-1} \left(\begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right] \\ \left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right] \end{pmatrix} d\omega + \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial p_k} \right] \\ \left[\frac{\partial s_{ijd}}{\partial p_k} \right] \end{pmatrix} dp \right) \\ &= \begin{pmatrix} \left[\frac{\partial F_{id}}{\partial x_{ik}} \right] & \left[\frac{\partial F_{id}}{\partial s_{ijd}} \right] \end{pmatrix} \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right] \\ \left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right] \end{pmatrix} d\omega \quad (\text{direct effect}) \\ &\quad + \begin{pmatrix} \left[\frac{\partial F_{id}}{\partial x_{ik}} \right] & \left[\frac{\partial F_{id}}{\partial s_{ijd}} \right] \end{pmatrix} \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial p_k} \right] \\ \left[\frac{\partial s_{ijd}}{\partial p_k} \right] \end{pmatrix} dp \quad (\text{general equilibrium effect}) \\ &\quad + \begin{pmatrix} \left[\frac{\partial F_{id}}{\partial x_{ik}} \right] & \left[\frac{\partial F_{id}}{\partial s_{ijd}} \right] \end{pmatrix} (II - I) \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right] & \left[\frac{\partial x_{ik}}{\partial p_k} \right] \\ \left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right] & \left[\frac{\partial s_{ijd}}{\partial p_k} \right] \end{pmatrix} \begin{pmatrix} d\omega \\ dp \end{pmatrix} \quad (\text{social multiplier}) \end{aligned}$$

where

$$II = \left(I - \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix} \right)^{-1}.$$

The matrix II has the typical form of a social multiplier, stacking up the successive iterative effects of the matrix of social influences, as shown in the following formula:

$$II - I = \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix} + \begin{pmatrix} \left[\frac{\partial x_{ik}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial x_{ik}}{\partial s_{ijd}} \right]_0 \\ \left[\frac{\partial s_{ijd}}{\partial x_{ik}} \right]_0 & \left[\frac{\partial s_{ijd}}{\partial s_{ijd}} \right]_0 \end{pmatrix}^2 + \dots$$

Such a decomposition makes it possible to analyze properties of the institutions embodied in the social game. In particular, let us use the above formula to examine what would happen to social inequalities if economic endowments were more equally distributed. The direct effect depends on how individuals react, consuming less ostentatious goods, $\left[\frac{\partial x_{ik}}{\partial \omega_{ik}} \right]$, or adopting more inclusive strategies, $\left[\frac{\partial s_{ijd}}{\partial \omega_{ik}} \right]$, and how the game form transforms such actions through $\left[\frac{\partial F_{id}}{\partial x_{ik}} \right]$, $\left[\frac{\partial F_{id}}{\partial s_{ijd}} \right]$ into social inequalities. The general

equilibrium effects may reinforce or attenuate the reduction in budget inequalities, thus modulating the direct effect. Finally, the social multiplier comes from the mutual influence of individual decisions on the others, both through x and through s . Here again, the social multiplier may reinforce or attenuate the effects, depending on whether strategic complementarity or substitutability prevails in such mutual influences.

This type of decomposition can be illustrated with an example where the intensity and quality of social relations depends on the gap between classes and is therefore affected by redistribution of endowments. This example also provides an interesting illustration of the model to the case of joint economic and social inequalities, with complex effects on welfare due to the public good feature of social proximity between social groups.

Example 4 (costly socialization with homophily). Consider a society with two classes of equal size, each conveniently described by a representative agent, i and j , the former being richer than the latter. There are three goods in the economy, an all-purpose good x that is produced out of labor time l , and a good z that is complementary to socialization activities (going out, joining clubs...). The individual budget is $x_i + pz_i = w_i l_i + p\omega_i$, where w_i is the fixed productivity of labor and ω_i the endowment in good z , which cannot be produced and is available in quantity Ω . Individual “wealth”, for the purpose of social stratification, is $m_i = w_i l_i + p\omega_i$. This is questionable because it does not take account of leisure, but it conforms with the prevailing culture in modern societies nowadays.

Individual preferences for z depend on the probability of meeting people like themselves in the socialization activities. The greater the average social distance to the people they meet, the less they are interested in socializing. Their social strategy $s_i \in \mathbb{R}_+$ simply consists in devoting time and energy to socialization. Their utility is

$$u_i((x_i, l_i, z_i), y_i) = v(x_i, z_i, y_{i1}, D_i) - c(l_i, y_{i1}),$$

where $y_{i1} = s_i$ is their socialization level, and D_i is the average difference between their own social status y_{i2} and the social status of people they are likely to meet in society. A greater D_i , or a lower y_{i1} , reduces the willingness to pay for the “socialization” good z_i . The distances to people met when going out include the probabilities of meeting someone from the other class as a function of the relative degree of socialization of the groups, and can be computed as the expected gap in social status with the person one may meet when socializing:

$$D_i = \frac{s_j}{s_i + s_j} (y_{i2} - y_{j2}), \quad D_j = \frac{s_i}{s_i + s_j} (y_{i2} - y_{j2}).$$

The definition of social status y_{i2} is naturally circular if one wants to capture the fact that people do not like to meet with people who are not liked. As we have two classes here, social status can be defined as the solution to the following system:

$$\begin{cases} y_{i2} = m_i - \alpha D_i \\ y_{j2} = m_j - \alpha D_j, \end{cases}$$

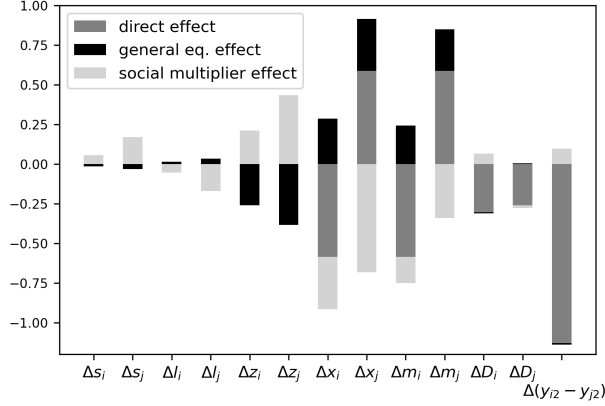


Figure 11: Decomposing the effects of redistributing endowments in Ex. 4

where α is a coefficient scaling a social mirror effect, meaning that the social status of a class depends not just on its wealth but also on the average distance with the people it encounters, and suffers when these encounters are not very congenial. As a consequence, a greater socialization by the upper class increases the social gap because the lower class is then more often meeting people who dislike them, whereas the opposite occurs when the lower class socializes more, thus imposing its own dislike on the upper class when the latter goes out.

In the end, one obtains

$$y_{i2} - y_{j2} = \frac{m_i - m_j}{2} \left/ \left(1 + \alpha \frac{s_j - s_i}{s_i + s_j} \right) \right.$$

It is assumed that every individual takes the distance D_k for her class $k = i, j$ as given when deciding her socialization strategy and choosing her economic bundle, since every individual has negligible influence over the general social game.

One can define response functions for x_i, l_i, z_i, s_i as functions of (ω_i, w_i, p, D_i) , and proceed with a decomposition as follows, taking x_i and a redistribution of endowments as an example:

$$\begin{aligned} \Delta x_i &= x_i(\omega_i + \Delta\omega_i, w_i, p, D_i) - x_i(\omega_i, w_i, p, D_i) \quad (\text{direct effect}) \\ &+ x_i(\omega_i + \Delta\omega_i, w_i, p + \Delta p, D_i) - x_i(\omega_i + \Delta\omega_i, w_i, p, D_i) \quad (\text{general equilibrium effect}) \\ &x_i(\omega_i + \Delta\omega_i, w_i, p + \Delta p, D_i + \Delta D_i) - x_i(\omega_i + \Delta\omega_i, w_i, p + \Delta p, D_i) \quad (\text{social multiplier effect}) \end{aligned}$$

Figure 2 illustrates a decomposition of the effect of a reduction of inequalities in endowments ω_i, ω_j , for a particular specification of this model¹⁶

The figure can be understood as follows. As a quasi-linear specification has been adopted for this

¹⁶The specification retained for this computation has $v(x_i, l_i, z_i, y_{i1}, D_i) = x_i + \frac{5}{D_i}(z_i(y_{i1} - 1))^{0.2}$, $c(l_i, y_{i1}) = 0.5(l_i + y_{i1})^2$. The initial endowments are $(\omega_i, \omega_j) = (2, 0)$ and are equalized into $(1, 1)$. As a result, the price p increases from 0.59 to 0.78. Wages are $(w_i, w_j) = (3, 2)$ and $\alpha = 0.5$.

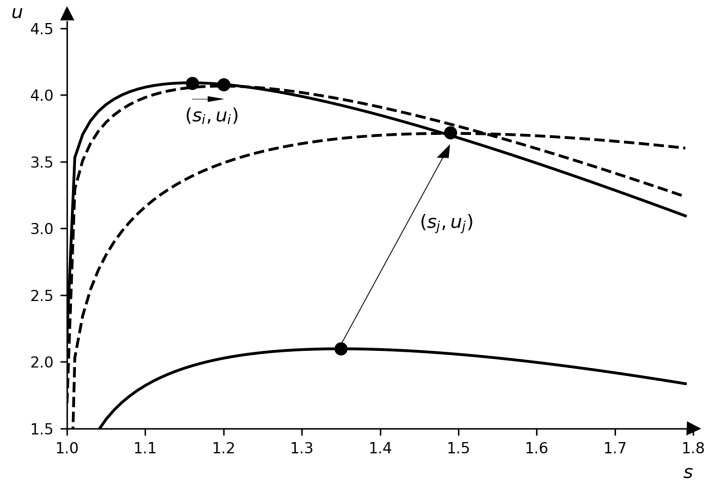


Figure 12: Choice of s by the two classes, before and after redistribution

example, the direct effect of redistribution is felt only on x and on wealth, not on labor and z , or socialization. The redistribution of endowments mechanically reduces the social distance between classes, and contributes to raising socialization. Additionally, the share of the lower class in socialization increases, which contributes to reducing the social status gap and encourages socialization a little more. This increase in socialization is tempered by the induced increase in the demand for z , which pushes its price up and indirectly makes socialization more costly, as can be seen in the GE effect column. On the other hand, the increased socialization of the lower class makes it lose ground on labor income, which ultimately increases the social status gap in the social multiplier effect. In this last effect, the upper class sees an increased social distance with people met when socializing, because they are relatively poorer and relatively more present in socialization places, whereas the lower class does not see such an increase because it enjoys meeting more of its own class.

An interesting feature of this example is that socialization is largely driven by a common factor, the status gap, but also displays a grain of strategic substitutability. As the other class goes out more, it becomes less pleasant to socialize because the probability of meeting someone different increases, and therefore, the component of the social multiplier that is determined by the socialization shares of the classes has opposite effects on the two classes.

We focus here on the effect of redistribution on economic and social outcomes, but the impact on welfare would also be interesting to study, because both classes may enjoy increased socialization in a context of reduced social distance for both. The welfare consequences of redistribution heavily depend on the weight of the second term in function v . Redistribution can be beneficial for both classes if it is important enough, thus providing a positive link between economic equity and efficiency. In the example at hand, Figure 11, where solid curves represent utility as a function of s_i, s_j before redistribution and dashed curves the post-redistribution situation, shows that the upper class barely loses utility in the process, whereas the lower class gains a lot.

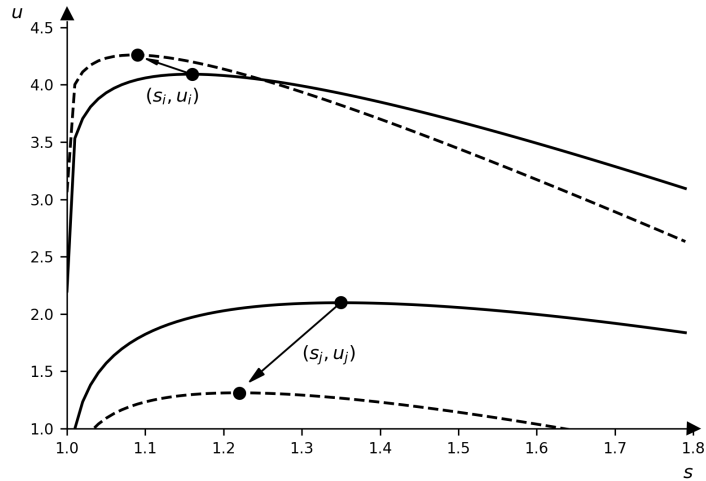


Figure 13: Choice of s , before and after productivity increase for the upper class

This example can also serve to show that technological progress that benefits the most qualified may be socially disruptive. An increase in the wage of the most productive, without any change to the productivity of the less productive, in this example, increases social distance and reduces socialization so much so that the lower class loses substantially in welfare terms, while the upper class does not gain that much because its economic gain is paired with separation from the rest of society. This is illustrated in Figure 12.¹⁷

5.2.1 A new efficiency-equity trade-off

Economic equity can improve efficiency via increased social cohesion that benefits everyone, but adding a social dimension to the economic model also creates the possibility a trade-off between efficiency and equity that is different from the familiar one in economics, which involves the disincentive effects of redistribution. The new trade-off comes from the fact that, if individual preferences over the trade-off between economic standing and social status are heterogeneous, the principle that reducing economic inequality, other things equal, is good for social welfare overall may be incompatible with the Pareto principle. In other words, the classical Pigou-Dalton transfer principle, a cornerstone of social welfare analysis, needs to be applied with caution in the presence of the social sphere.

To simplify the presentation, consider the case in which x_i is one-dimensional, and is interpreted as income (or wealth). A natural extension of the Pigou-Dalton transfer principle to this model would recommend reducing economic inequalities among individuals sharing the same social outcome. The restriction to individuals having the same social status is sensible, because a transfer from a disreputed rich to a popular poor would not be obviously contributing to general socio-economic inequality. This cautious application of the transfer principle is encapsulated in the following requirement for social evaluation of allocations:

¹⁷For the computations in the figure, the wages rates of the two classes move from $(3, 2)$ to $(3.5, 2)$.

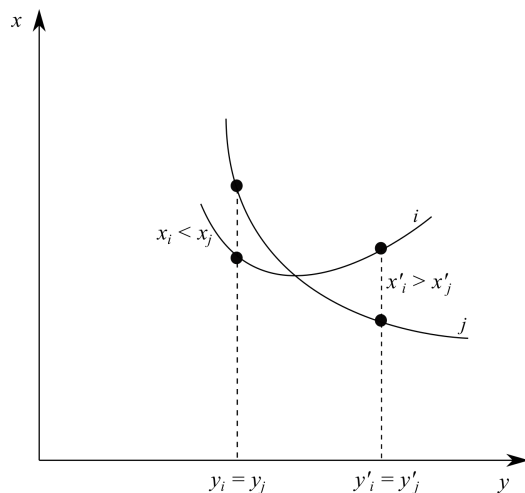


Figure 14: Economic equity clashes with the Pareto principle

Economic equity For all allocations $(x, y), (x', y')$ and all $i, j \in N$, such that $y_i = y_j = y'_i = y'_j$ and $x_i > x_j$, if $(x, y), (x', y')$ differ only by a regressive transfer $x'_i = x_i + \delta, x'_j = x_j - \delta$, with $\delta > 0$, then (x, y) is better than (x', y') .

As is well known in the theory of fair social orderings (Fleurbaey and Maniquet 2011), this type of principle, in a multi-dimensional context, runs afoul of the Pareto principle when individuals may have different preferences about trading off x_i against y_i . This is because, as illustrated in Fig. 13, situations with equally low $y_i = y_j$ and $x_i < x_j$ may be Pareto indifferent to situations with equally high $y'_i = y'_j$ and $x'_i > x'_j$. According to the Economic equity principle, the former situation could be improved by a transfer from i to j , whereas the latter could be improved by a transfer from j to i . Since individuals are Pareto indifferent, respect for the Pareto principle should treat these two situations as equivalent, hence a clash.

This observation confirms the general message of this paper, which is that the social sphere cannot be ignored. The Economic equity principle is not cautious enough because it fails to reckon with the possibility that the two individuals have different preferences over the trade-off between x_i and y_i . A rich may have the same social status as a poor, but is not necessarily advantaged overall if she is more dissatisfied with their common social status. The proper way to deal with this issue is to rely on measures of socio-economic advantage rather than economic standing for the purpose of inequality analysis and redistributive policy evaluation. Developing such measures is the topic of the next subsection

5.3 Measuring socio-economic inequalities

The current model is similar to contexts in which individual preferences bear on market commodities and non-market aspects of quality of life. For this type of context, one can follow Fleurbaey and Blanchet (2013) and restrict the application of the Economic equity principle to situations in which the non-market aspect of life is at its best for every individual. This restriction eliminates the tension

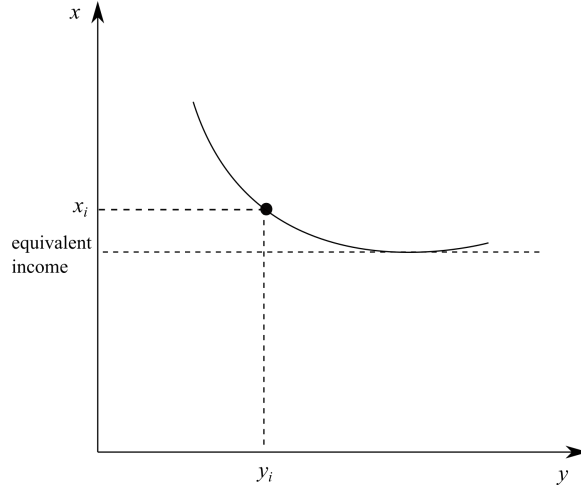


Figure 15: Equivalent income

with the Pareto principle. Let us say that y_i is ideal for i given x_i when y_i maximizes $u_i(x_i, y_i)$ among the possible values of y_i .

Economic equity under ideal social outcomes For all allocations $(x, y), (x', y')$ and all $i, j \in N$, such that y_i, y_j, y'_i, y'_j are ideal for i, j given x_i, x_j, x'_i, x'_j respectively,

and $x_i > x_j$, if $(x, y), (x', y')$ differ only by a regressive transfer $x'_i = x_i + \delta, x'_j = x_j - \delta$, with $\delta > 0$, then (x, y) is better than (x', y') .

Although the tension with Pareto is alleviated, combining this equity principle with the Pareto principle seriously narrows down the set of acceptable approaches. Let us first state the Pareto principle and introduce the notion of equivalent income.

Strong Pareto For all allocations $(x, y), (x', y')$ such that $u_i(x_i, y_i) \geq u_i(x'_i, y'_i)$ for all $i \in N$, (x, y) is at least as good as (x', y') ; and if the inequality is strict for at least one i , then (x, y) is better than (x', y') .

The equivalent income is a utility representation defined as the minimal x_i that is needed to bring i to the current utility level, when full adjustment of social outcomes is possible:

$$\min \left\{ z \mid \max_w u_i(z, w) \geq u_i(x_i, y_i) \right\}.$$

It is illustrated on Figure 14. The equivalent income obtains at a situation in which y_i is ideal given this level of income.

Finally, let us say that an ordering over n -vectors of real numbers is monotonic increasing if an increase in a component moves the vector up the ordering, and inequality averse if a regressive transfer between two components moves the vector down. One then obtains a proposition similar to Willig's (1981) approach.

Proposition 7 *If an ordering of allocations (x, y) satisfies Economic equity under ideal social outcomes and Strong Pareto, the ordering is entirely defined by a monotonic increasing and inequality*

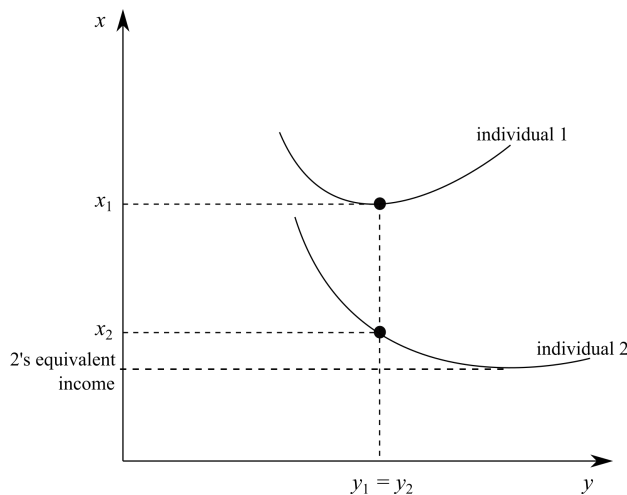


Figure 16: Inequality in equivalent incomes vs. incomes

averse ordering on the distribution of equivalent incomes, for the allocations for which equivalent incomes are well defined.

The above proposition is silent for allocations for which the equivalent income is not defined for some individuals. This is likely to be rare in practice, as income is a necessary good, implying that in Figure 12, indifference curves near the horizontal axis are likely to be close to horizontal, meaning that economic subsistence becomes a priority over social outcomes. But this may be debated, as poor people do complain that the worst of their condition is not so much deprivation as the lack of respect and dignity in their social interactions with the rest of society. This might mean that certain social deprivations may be worse than falling below the subsistence level on the economic front.

Let us provide a few illustrations of the equivalent-income approach. In example 1, the economic inequality between individuals 1 and 2 is accompanied with an additional inequality due to the frustration of individual 2 not having his full lot of chat. This can be assessed by looking at the inequality in equivalent incomes, which is, as can be seen from Fig. 15, greater than inequality in resources, because individual 2's equivalent income is below her income, whereas individual 1's equivalent income is equal to her income.

In example 1, efficiency is achieved when the willingness to accept of the less chatty individual equals the willingness to pay of the chattier individual. In Fig. 3, the gap in equivalent incomes between the two individuals is reduced when norms of politeness makes the richer, less chatty one concede more time to the other. Interestingly, if, as in example 1, the former is richer, this individual is likely to be choosier, so that the efficient allocation will have a lower gap between the actual and the preferred quantity of interaction for this individual (this is illustrated in Fig. 3). In a nutshell, efficiency would justify that the rich could be less polite than the others, in terms of concessions with respect to the preferred chatting time. It was suggested in section 3 that social norms of politeness tend to reduce inefficiency by letting individuals subtly express their wishes. But norms of politeness do not refer to willingness to pay and are therefore likely to produce more egalitarian results in terms of quantitative

concessions. Therefore, they are likely to further reduce the inequality in equivalent incomes, compared to what the efficient allocation would be in absence of transfers of resources.

In example 2, both individuals suffer from the impossibility to have as many parties as they would wish, so that their equivalent incomes are lower than their ordinary incomes. Moreover, if partying is a normal good, the gap between income and equivalent income is larger for the richer person (who is further constrained by the lack of resources of her neighbor).

The case of Faustian socio-economic bargains provides another illustration of the approach. The threat of economic duress, which affects a large share of the population since most people cannot survive without selling something, can lead the most disadvantaged among them to accept sacrifices on their social status or their autonomy in order to get by. Economists have long been interested in the analysis of what exactly is exchanged in the labor market. Adam Smith, in the *Theory of Moral Sentiments*, offered a (not-so-well-known) invisible hand perspective on trades that transfer money from rich employers to poor employees: “They are led by an invisible hand to make nearly the same distribution of the necessaries of life, which would have been made, had the earth been divided into equal portions among all its inhabitants; and thus, without intending it, without knowing it, advance the interest of the society, and afford means to the multiplication of the species.” (1759, p. 188) Karl Marx believed he uncovered the secret of profit in the idea that employers only paid the value of the labor force but could then extract the full value of labor. Neoclassical economists emphasized the fact that everyone benefits from the trade, compared with their initial endowment. Labor economists noted that leisure has a value for people and offered various ways to account for the disutility of lost leisure, which include the equivalent income proposed here and other variants of the money-metric approach (Preston and Walker 1999).

What is missing from all of this is an explicit account of social status and autonomy. Yet this was a rather prominent concern for the Founding Fathers of the US Republic. “Although most Americans in 1776 believed that not everyone in a republic had to have the same amount of property . . . all took for granted, that a society could not long remain republican if a tiny minority controlled most of the wealth and the bulk of the population remained dependent servants or poor landless laborers.” (Gordon S. Wood, *Empire of Liberty: A History of the Early Republic, 1789–1815*, cited in Blasi et al. 2013, p. 7). More recent surveys of job satisfaction point to the importance of autonomy for many employees (Freeman and Rogers 2006), although they generally ignore comparisons of status with independent workers and employers, since employee status has become the norm rather than the exception. Incorporating the loss of independence and autonomy into the computation of the equivalent income of employees should capture these aspects of their situation, at least to the extent that their preferences have not come to accept their inferior position as a matter of indifference.

This last remark raises the important issue of whether questionable social conventions may make a preference-based measure like equivalent income problematic for analyzing inequalities. It is of course possible to “correct” preferences to eliminate biases (as is commonly done to treat biases, such as present bias, in behavioral economics), before they are applied to the measurement of equivalent income. If some individuals come to like their servitude, social analysis can still measure how their situation fares according to more acceptable preferences.

5.4 Distinguishing economic and social contributions to socioeconomic inequality

The distribution of equivalent incomes can be used for a decomposition analysis of inequalities. In particular, a decomposition of the respective contributions of economic and of social inequalities to the overall socio-economic inequality can be performed, and further elements can be added, such as the contribution of the heterogeneity of preferences. A methodology for the decomposition of inequalities of equivalent income measures of well-being, focused on measuring a contribution for preference heterogeneity and a contribution for the correlation between preferences and living conditions, is developed in Decancq et al. (2017) and can be adapted to our framework.

The general principle of such a decomposition is as follows, and actually can be applied to any interpersonally comparable well-being measure of the type $u_i(x_i, y_i)$, not only equivalent income, which is but one representation of preferences among many. Compute the average vectors \bar{x}, \bar{y} and the average function \bar{u} , as well as a large sample of randomly reshuffled distributions of x and y , denoted \tilde{x}, \tilde{y} , where \tilde{x}_i is obtained by a permutation of bundles among individuals, and similarly for \tilde{y}_i . For any chosen inequality index, one can write, letting $u(x, y)$ denote $(u_i(x_i, y_i))_{i \in N}$, and using a similar convention when actual bundles are replaced by reshuffled bundles or average vectors, and an upper bar means that an average of the sample of reshuffled distributions is taken:

$$\begin{aligned}
 I(u(x, y)) &= I(u(x, y)) - \overline{I(u(\tilde{x}, \tilde{y}))} \quad \text{correlation of preferences with situations} \\
 &\quad + \overline{I(u(\tilde{x}, \tilde{y}))} - I(\bar{u}(x, y)) \quad \text{preference heterogeneity} \\
 &\quad + I(\bar{u}(x, y)) - \overline{I(\bar{u}(\tilde{x}, \tilde{y}))} \quad \text{correlation between } x \text{ and } y \\
 &\quad \quad + \overline{I(\bar{u}(\tilde{x}, \tilde{y}))} - I(\bar{u}(\bar{x}, y)) \quad \text{inequality in } x \\
 &\quad \quad \quad + I(\bar{u}(\bar{x}, y)) \quad \text{inequality in } y
 \end{aligned}$$

Such a decomposition also singles out the contribution to socioeconomic inequality coming from the correlation between economic and social standing studied in section 5.1. Other orders for this decomposition can be followed, such as this one, in which outcome inequalities come before preference heterogeneity:¹⁸

$$\begin{aligned}
 I(u(x, y)) &= I(u(x, y)) - \overline{I(u(\tilde{x}, \tilde{y}))} \quad \text{correlation between } x \text{ and } y \\
 &\quad + \overline{I(u(\tilde{x}, \tilde{y}))} - I(u(\bar{x}, y)) \quad \text{inequality in } x \\
 &\quad + I(u(\bar{x}, y)) - I(u(\bar{x}, \bar{y})) \quad \text{inequality in } y \\
 &\quad \quad + I(u(\bar{x}, \bar{y})) \quad \text{preference heterogeneity}
 \end{aligned}$$

¹⁸Along this path, there is no way to distinguish a contribution from the correlation between preferences and individual situations.

or this one, in which inequality in y comes before inequality in x :

$$\begin{aligned}
 I(u(x, y)) &= I(u(x, y)) - \overline{I(u(x, \tilde{y}))} \quad \text{correlation between } x \text{ and } y \\
 &+ \overline{I(u(x, \tilde{y}))} - I(u(x, \bar{y})) \quad \text{inequality in } y \\
 &+ I(u(x, \bar{y})) - I(u(\bar{x}, \bar{y})) \quad \text{inequality in } x \\
 &+ I(u(\bar{x}, \bar{y})) \quad \text{preference heterogeneity}
 \end{aligned}$$

Such a path dependence is commonplace in decompositions, and can be dealt with either by laying out the results for different paths, or averaging contributions over different paths. It is also possible to adopt coarser decompositions, e.g., if correlations are not deemed relevant.

6 Conclusion

This paper offers a versatile model which can be used as a useful umbrella to encapsulate many aspects uncovered in the economic literature on social interactions. The stylized depiction of the economy and the society that it contains helps fleshing out how economic activities are part of a broader social setting and why they should not, in general, be studied separately. Ideally, a model such as this one should replace the canonical models that shape how people, and especially experts and policymakers, view the economy and its rules.

The literature on social interactions often refers to *externalities* as the key phenomenon brought in by the mutual interdependence in which people navigate their social lives. Externalities, in economic theory, are normally seen as a problem for the market, in the form of an afterthought. In the standard outlook of economic analysis, as well as in usual teaching curricula, one starts with the analysis of the perfect market and only later consider market failures. What the literature on social interactions suggest, and the model of this paper vividly affirms, is that externalities are actually the primary factor, and that market transactions can at most carve a space in social interactions but never escape from the general social interdependence between people. People's preferences and their social success are in substance made of externalities, and these externalities do not disappear when people interact in markets.

We have seen, however, that even though the social game seems well represented by an influence model, it does not always by itself generate inefficiency, as in prisoner's dilemma situations. In particular, veto and claim (as well as in-between) technologies and social norms for joint activities tend to produce outcomes that lie between the preferred options of the various players and are therefore efficient, though not necessarily equitable. But inefficiency can still emerge from the lack of coordination between the economic and the social spheres. People might be dissatisfied with the social outcomes and would be willing to spend resources to improve them, but resources may be the wrong currency under the spontaneity and sincerity rules that govern many social interactions. Many people would be willing to be less affluent and have a better social environment, but there may be no available technology that transforms resources into decency, friendship, esteem, genuine reciprocity and the

like. Therefore, societies must find ways to tend their social garden with appropriate social norms. Resources can, however, be devoted to education, to sharing information, and to raising awareness about the importance of social interdependence. They can also be used, sometimes, in subtle exchanges that do not undermine the value of social interactions (Zelizer 2005, 2017). Moreover, in the economic sphere, a mix of economic incentives and norms can be mobilized to enhance responsible behavior that takes account of the social consequences of economic decisions.

The thrust of this paper has been to show that a narrow focus on the economic sphere misses important dimensions of efficiency and equity. To this effect, the Arrow-Debreu model was a good starting point, as it embodies the most effective type of economic coordination and the most clearcut notion of economic standing with complete markets. But we propose that the general method of adding a social game to any economic model should become the new standard, in order to make economic analysis more comprehensive and robust. Doing so for a variety of economic models can provide additional insights into economy-society interactions. In particular, one general theme that deserves its own research program is that social interactions and social norms can help alleviate market failures. This is a well-trodden theme, of course, and many authors have emphasized how economic outcomes crucially depended on social mechanisms. Weber (1905) described how various branches of Protestantism steered the faithful toward economic activities to unequal degrees. Coase (1937) and Williamson (1981) saw market transaction costs, due to informational problems and returns to scale, as a main reason for social associations (business firms) taking over certain production activities. Arrow (1973, 1974) has been a prolific author on the interplay between ethics, trust and economic efficiency (Cato and Lutz 2018). The huge literature on economic and institutional development and on social capital (see, e.g., Keefer and Knack 2005 for a review) has developed such insights in multiple directions and provided crucial empirical evidence on variations across cultures, countries and social groups. We hope that the approach laid out in this paper can also be useful in formalizing these notions, and more.

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Appendix: Proofs

Proof of Prop. 1

The “only if” part follows directly from the definitions and the fact that the maximum of a multivariate function must be a maximum in every subset of dimensions.

The “if” part comes from the fact that condition WN-i is satisfied when x is a Walras subequilibrium for a given s and y is a Nash subequilibrium for the same x and associated to the same s . Indeed, individual utility is equal to

$$u_i(x_i, y_i) = u_i(x_i, F_i((x_i, x_{-i}), (s_i, s_{-i}))).$$

If the right-hand side is concave and continuously differentiable in (x_i, s_i) and is maximized separately in x_i and in s_i , then it is maximized in the pair (x_i, s_i) . But one does not need concavity of this function. It is sufficient that it is the monotonic transform of a concave function.¹⁹

¹⁹It is necessary to introduce differentiability, otherwise the result would not hold. Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

Proof Prop. 2

The assumptions are:

- The function $U_i(p, x, s) := u_i(x_i, F_i(p, x, s))$ is continuous in (p, x, s) and non-satiable in x_i ;
- The set X_i is closed and convex;
- The set S_i is compact and convex;
- The individual endowment $\omega_i \gg 0$;
- The cone Q is closed;
- For every p and (x_{-i}, s_{-i}) , the set of (x_i, s_i) maximizing $U_i(p, x, s)$ such that $px_i \leq p\omega_i$ is convex.

Since production is limited by the available inputs $\sum_i \omega_i$, there is a compact and convex truncation of Q , denoted Q^* , in which every feasible allocation takes its production plan. Likewise, there is a compact and convex truncation of X_i , denoted X_i^* , in which every feasible allocation takes i 's consumption. The truncation must be large enough (by going beyond the set spanned by feasible allocations, in the relevant directions) so that whenever $q \in \arg \max \{pq | q \in Q^*\}$ and belongs to a feasible allocation, then $q \in \arg \max \{pq | q \in Q\}$. Likewise, whenever $x_i \in \arg \max \{U_i(p, x, s) | x_i \in X_i^*, px_i \leq p\omega_i\}$ belongs to a feasible allocation, then $x_i \in \arg \max \{U_i(p, x, s) | x_i \in X_i, px_i \leq p\omega_i\}$. Let individual 1 be declared the owner of Q^* , and receive the profit pq (in equilibrium, this profit is null, therefore this is without loss of generality, and it is never negative).

Let $P = \{p \in R_+^\ell | \|p\| = 1\}$, where $\|\cdot\|$ is the L_1 norm $\sum_k |p_k|$, $X = \prod_i X_i$, $S = \prod_i S_i$.

Consider the correspondence over $P \times X \times S$ defined as follows: it associates p, q, s to p', q', s' such that

- $p' \in \left\{ \frac{p + \sum_i (x_i - \omega_i) - q}{\|p + \sum_i (x_i - \omega_i) - q\|} | q \in \arg \max \{pq | q \in Q^*\} \right\}$;
- for all $i \neq 1$, $x'_i \in \arg \max \{U_i(p, \hat{x}_i, x_{-i}, s) | \hat{x}_i \in X_i^*, p\hat{x}_i \leq p\omega_i\}$;
- $x'_1 \in \arg \max \{U_1(p, \hat{x}_1, x_{-1}, s) | \hat{x}_1 \in X_1^*, p\hat{x}_1 \leq p\omega_1 + pq\}$;
- for all i , $s'_i \in \arg \max \{U_i(p, x, \hat{s}_i, s_{-i}) | \hat{s}_i \in S_i\}$.

This correspondence, in each of its components, is upper hemicontinuous. In particular, $\arg \max \{pq | q \in Q^*\}$ is also upper hemicontinuous in p , while

$$\frac{p + \sum_i (x_i - \omega_i) - q}{\|p + \sum_i (x_i - \omega_i) - q\|}$$

defined by:

$$f(x, y) = \begin{cases} -x + 2y & \text{if } x \geq y \\ 2x - y & \text{if } x \leq y. \end{cases}$$

This function is concave, and at any point where $x = y$ it is maximal with respect to x and y separately, but this is not a maximum.

is continuous in (p, x, q) . The correspondence

$$\arg \max \{U_i(p, x, s) \mid x_i \in X_i^*, px_i \leq p\omega_i\}$$

is upper hemicontinuous since $X_i^* \cap \{x_i \in \mathbb{R}_+^\ell \mid px_i \leq p\omega_i\}$ is compact and continuous in p (i.e., both upper and lower hemicontinuous, the latter depending on the assumption $\omega_i \gg 0$) while $U_i(p, x, s)$ is continuous in (p, x, s) .

The images of the correspondence are convex for each component. For p , this comes from the fact that $\arg \max \{pq \mid q \in Q^*\}$ is convex, and thus

$$\left\{ \frac{p + \sum_i (x_i - \omega_i) - q}{\|p + \sum_i (x_i - \omega_i) - q\|} \mid q \in \arg \max \{pq \mid q \in Q^*\} \right\}$$

is also convex, as it is the projection of the convex set

$$\left\{ p + \sum_i (x_i - \omega_i) - q \mid q \in \arg \max \{pq \mid q \in Q^*\} \right\}$$

on the convex set P . For x_i and s_i this directly comes from the assumptions, and the truncation via X_i^* does not invalidate this assumption.

Therefore, Kakutani's fixed-point theorem can be applied, implying that this correspondence has a fixed point (p^*, x^*, s^*) . For p^* , given that

$$p \left(\sum_i (x_i - \omega_i) - q \right) = 0$$

by non-satiation, this obtains only if

$$\sum_i (x_i - \omega_i) - q = 0$$

for some

$$q^* \in \arg \max \{p^*q \mid q \in Q^*\},$$

i.e., if

$$\sum_i (x_i^* - \omega_i) - q^* = 0.$$

To see why, consider the two possible cases.

First case: $\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| = 1$. In this case, one has

$$p^* = p^* + \sum_i (x_i^* - \omega_i) - q^*,$$

implying $\sum_i (x_i^* - \omega_i) - q^* = 0$.

Second case: $\|p^* + \sum_i (x_i^* - \omega_i) - q^*\| \neq 1$. In this case, one has

$$p^* \left(\left\| p^* + \sum_i (x_i^* - \omega_i) - q^* \right\| - 1 \right) = \sum_i (x_i^* - \omega_i) - q^*,$$

implying

$$p^* \cdot p^* \left(\left\| p^* + \sum_i (x_i^* - \omega_i) - q^* \right\| - 1 \right) = p^* \left(\sum_i (x_i^* - \omega_i) - q^* \right) = 0,$$

which is impossible since $p^* \cdot p^* > 0$ by construction.

Thus, this allocation is feasible, so it also satisfies

$$q^* \in \arg \max \{p^* q \mid q \in Q\},$$

so that $p^* q^* = 0$, and thus for all $i \in N$,

$$x_i^* \in \arg \max \{U_i(p^*, x, s^*) \mid x_i \in X_i, p^* x_i \leq p^* \omega_i\},$$

implying that it is an equilibrium.

Proof of Prop. 3

The function $U_i(x, s) = u_i(x_i, F_i(x, s))$ is concave because both u_i and F_i are. The vector function $U(x, s) = (U_i(x, s))_{i \in N}$ is then concave. The set $Q = \{q \mid T(q) \leq 0\}$ is convex since T is concave, and the set $X^* = \{x \in \prod_{i \in N} X_i \mid \exists q \in Q, \text{ and } \sum_i x_i = \sum_i \omega_i + q\}$ is convex since Q and all X_i are convex. Likewise, the set $S = \prod_{i \in N} S_i$ is convex since each S_i is. The lower set of the utility possibility set is the hypograph of U on the domain $X^* \times S$, i.e., it is the set

$$U^* = \{u \in \mathbb{R}^n \mid \exists (x, s) \in X^* \times S, u \leq U(x, s)\}.$$

Since the hypograph of a concave function on a convex domain is convex, this proves the first statement of the proposition.

Efficient allocations can then be found by maximizing $\sum_i \alpha_i u_i$, over the convex set U^* , spanning $\alpha \succeq 0$. Given the smoothness and interiority assumptions, the first-order conditions of the Lagrangian program:

$$\sum_i \alpha_i u_i(x_i, F_i(x, s)) - \lambda \left(\sum_i x_i - \sum_i \omega_i - q \right) - \mu T(q)$$

are necessary and sufficient. They consist in finding $\alpha \in \mathbb{R}_+^n, \lambda \in \mathbb{R}_{++}^\ell, \mu \in \mathbb{R}_{++}, x, q$ and s satisfying (1)-(4).

To determine the efficiency of an interior Walras-Nash equilibrium, first note that condition (4) is satisfied at an equilibrium. The first-order conditions of competitive behavior can be written as follows, for every individual i : There exists $\lambda_i \in \mathbb{R}_{++}^\ell$ (measuring i 's marginal utility of money) such that:

$$\forall k \in \{1, \dots, \ell\}, \frac{\partial u_i}{\partial x_{ik}} + \sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial x_{ik}} = \lambda_i p_k, \quad (5)$$

$$\forall i \in N, h \in \{1, \dots, H\}, \sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial s_{ih}} = 0. \quad (6)$$

And profit maximization implies that there is $\mu' \in \mathbb{R}_{++}$ such that:

$$\forall k \in \{1, \dots, \ell\}, p_k = \mu' \frac{\partial T}{\partial q_k}. \quad (7)$$

Let us insert (5)-(7) into (1)-(3). One obtains that there must exist $\alpha \in \mathbb{R}_+^n, \mu'' \in \mathbb{R}_{++}$:

$$\forall i \in N, k \in \{1, \dots, \ell\}, \alpha_i \lambda_i p_k + \sum_{j \neq i} \alpha_j \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = \mu'' p_k, \quad (8)$$

$$\forall i \in N, h \in \{1, \dots, H\}, \sum_{j \neq i} \alpha_j \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0. \quad (9)$$

And since rescaling the vector α and the scalar μ'' is inconsequential, one can drop μ'' and rewrite (8) as

$$\forall i \in N, k \in \{1, \dots, \ell\}, \alpha_i \lambda_i p_k + \sum_{j \neq i} \alpha_j \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = p_k. \quad (10)$$

- **Necessary and sufficient condition:** Under the assumptions (5)-(7), and the other background assumptions stated in the proposition, the (9)-(10) conditions jointly provide a necessary and sufficient condition for efficiency. This condition can be further simplified as follows. Let $\beta_i = \alpha_i \lambda_i$. Then (10) can be rewritten as

$$\forall i \in N, k \in \{1, \dots, \ell\}, \beta_i p_k + \sum_{j \neq i} \frac{\beta_j}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = p_k, \quad (11)$$

or in matrix form,

$$\forall i \in N, k \in \{1, \dots, \ell\}, M_{ik} \beta = p_k 1_n \quad (12)$$

where, letting $w_{jik} = \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}}$ denote j 's willingness to pay for x_{ik} ,

$$M_{ik} = \begin{pmatrix} p_k & w_{j1k} & w_{n1k} \\ & \ddots & \\ w_{1ik} & p_k & w_{nik} \\ & & \ddots \\ w_{1nk} & w_{jnk} & p_k \end{pmatrix}; 1_n = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Similarly, (9) can be written as

$$\forall i \in N, h \in \{1, \dots, H\}, M_{ih} \beta = 0, \quad (13)$$

where, letting $w_{jih} = \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}}$ denote j 's willingness to pay for s_{ih} ,

$$M_{ih} = \begin{pmatrix} 0 & w_{j1h} & w_{n1h} \\ & \ddots & \\ w_{1ih} & 0 & w_{nih} \\ & & \ddots & \\ w_{1nh} & w_{jnh} & 0 \end{pmatrix}.$$

Combining (8')-(9), one obtains the following necessary and sufficient condition: there exists $\beta \in \mathbb{R}_+^n$ such that

$$M\beta = b, \quad (14)$$

where M is the $n^2(K+H) \times n$ matrix that is the transpose of

$$M^\top = \left((M_{ik}^\top)_{i=1, \dots, n; k=1, \dots, K}; (M_{ih}^\top)_{i=1, \dots, n; h=1, \dots, H} \right)$$

and b is the $n^2(K+H)$ vector that is the transpose of

$$b^\top = (\underbrace{1, \dots, 1}_{n^2 K}, \underbrace{0, \dots, 0}_{n^2 H}).$$

By Farkas' Theorem, an equivalent necessary and sufficient condition is that for all $a \in \mathbb{R}^{n^2(K+H)}$, $M^\top a \geq 0$ implies $b^\top a \geq 0$.

- Sufficient conditions:

First, consider the special case in which the vector $\beta \in \mathbb{R}_+^n$ is uniform: $\beta = (\beta_0, \dots, \beta_0)$.²⁰ In this case, (11) reads

$$\forall i \in N, k \in \{1, \dots, \ell\}, \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = \frac{1 - \beta_0}{\beta_0} p_k, \quad (15)$$

and (9) becomes

$$\forall i \in N, h \in \{1, \dots, H\}, \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0. \quad (16)$$

In the particular subcase in which $\beta_0 = 1$, (15) further simplifies into

$$\forall i \in N, k \in \{1, \dots, \ell\}, \sum_{j \neq i} \frac{1}{\lambda_j} \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = 0, \quad (17)$$

Let us now briefly prove that the first four sufficient conditions in the statement of the proposition entail (16)-(17)—the fifth one is indeed the (16)-(17) pair itself, while the sixth one is the (15)-(16) combination.

²⁰This brings up Negishi's (1972) weights equal to the inverse of the marginal utility of money: an efficient allocation maximizes a weighted sum of utilities for which the weighted marginal utility of money is equalized across individuals.

1. In this case, $\frac{\partial F_{jl}}{\partial x_{ik}} \equiv 0$ and $\frac{\partial F_{jl}}{\partial s_{ih}} \equiv 0$ for all i, j, k, h, l , directly implying (16)-(17).
2. One again has $\frac{\partial F_{jl}}{\partial x_{ik}} \equiv 0$ for all i, j, k, h, l . Pick any i, j, h, l . Either $\frac{\partial F_{jl}}{\partial s_{ih}} \equiv 0$ or $\sum_{l'} \frac{\partial u_i}{\partial y_{il'}} \frac{\partial F_{il'}}{\partial s_{ih}} = \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial s_{ih}}$ because $\frac{\partial F_{il'}}{\partial s_{ih}} \equiv 0$ for all $l' \neq l$, and since $\sum_{l'} \frac{\partial u_i}{\partial y_{il'}} \frac{\partial F_{il'}}{\partial s_{ih}} = 0$ and $\frac{\partial u_i}{\partial y_{il}} \neq 0$, this implies $\frac{\partial F_{il}}{\partial s_{ih}} = 0$. As $F_{jl} = F_{il}$, this implies $\frac{\partial F_{jl}}{\partial s_{ih}} = 0$. Thus, for all i, j, h, l , one has $\frac{\partial F_{jl}}{\partial s_{ih}} = 0$. Condition (16)-(17) are then satisfied.
3. One has $\sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = \sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0$ for all i, j, k, h , directly entailing (16)-(17).
4. One has $\sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial x_{ik}} = 0$ for all i, j, k . Pick any i, j, h . One either has $\sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = 0$ or $u_j(x_j, F_j(x, s)) \equiv \varphi(u_i(x_i, F_i(x, s)), x, s_{-ih})$ for some function φ that is increasing in its first argument. By individual optimization at the equilibrium, one has $\sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial s_{ih}} = 0$. This implies that $\sum_l \frac{\partial u_j}{\partial y_{jl}} \frac{\partial F_{jl}}{\partial s_{ih}} = \frac{\partial \varphi}{\partial u_i} \sum_l \frac{\partial u_i}{\partial y_{il}} \frac{\partial F_{il}}{\partial s_{ih}} = 0$. Thus, (16)-(17) are satisfied.

Proof of Prop. 4

If either x or y is inefficient, it is possible to find a Pareto-dominating allocation, which will increase u_i for some individual i because v_i is increasing in each of its arguments $f_i(x_i), g_i(y_i)$, and no other individual will be harmed. Therefore joint efficiency of x and y is necessary for general efficiency of (x, y) .

The counterexample proving that this is not sufficient is provided in Example 1.

Proof of Prop. 5

Suppose that, for the given x , the social subequilibrium strategy profile s is not efficient. Then it is possible to find another profile s' such that

$$u_i(x_i, F_i(x, (s_i, s_{-i}))) \leq u_i(x_i, F_i(x, (s'_i, s'_{-i})))$$

for all i , with a strict inequality for some i . Let $y'_i = F_i(x, (s'_i, s'_{-i}))$. The allocation (x, y') is thus such that it Pareto-dominates the allocation (x, y) .

Suppose that, for the given s , the economic subequilibrium is not efficient. Then it is possible to find another allocation x' such that

$$u_i(x_i, F_i(x, s)) \leq u_i(x'_i, F_i(x', s))$$

for all i , with a strict inequality for some i . The allocation (x', y') , where $y'_i = F_i(x', s)$, is thus such that it Pareto-dominates the allocation (x, y) .

The fact that sufficiency does not hold is proven by the examples provided in section 4, since these are special cases of this model.

Proof of Prop. 6

Let i, j be such that $x_i \succ x_j$ and consider equilibrium strategies s^* such that $F_j(x, s^*) \succ F_i(x, s^*)$. We show that this entails a contradiction.

By the economic edge property, there is s_i such that $F_i(x, s_i, s_{-i}^*) \succ F_j(x, s_i, s_{-i}^*)$. By the robust edge property, one can pick s_i so as to also have $F_i(x, s^*) \succ F_i(x, s_i, s_{-i}^*) \succ F_j(x, s_i, s_{-i}^*)$.

Assume that negative competition is also satisfied. In this case, since $F_j(x, s^*) \succ F_i(x, s^*) \succ F_j(x, s_i, s_{-i}^*)$, by negative competition one must have $F_i(x, s_i, s_{-i}^*) \succ F_i(x, s^*)$. But this is impossible, because of the assumption that $y_i \succ y'_i$ implies $u_i(x_i, y_i) > u_i(x_i, y'_i)$ and the fact that s_i^* is an optimal choice for i facing s_{-i}^* .

Instead of negative competition, assume that self-impact is satisfied. In this case, observe that since $F_j(x, s^*) \succ F_i(x, s^*) \succ F_i(x, s_i, s_{-i}^*) \succ F_j(x, s_i, s_{-i}^*)$, we have that $F_j(x, s^*) \succ [F_i(x, s_i, s_{-i}^*), F_i(x, s^*)] \succ F_j(x, s_i, s_{-i}^*)$. But this is excluded by self-impact.

Necessity of economic edge: Consider a social game such that one (poor) individual's y_i always dominates the others whose social outcome y_j is equal and fixed. Assume that y_i depends only on s_i , and that i has various strategies producing two possible levels for y_i , one dominating the other. This game satisfies all the properties except economic edge, and produces reversals between economic and social rankings at the equilibrium.

Necessity of robust edge: Consider a social game for a two-agent population in which whenever $x_i \succ x_j$, i has more "social chips" than j . Individual i is the only one to have decisions to make, and can decide to transfer his chips to j , having to relinquish two chips for every chip received by j . If i stays in this type of strategy, then y_i, y_j are simply the number of chips that each ends up with. In addition, i has a "joker" strategy which entails $y_i \prec y_j$, but these outcomes are not comparable to those induced by the chips strategies. For some preference profiles, the joker strategy is the best for i . This game satisfies all the properties except robust edge.

Necessity of negative competition or self-impact: Consider a social game for a two-agent population in which whenever $x_i \succ x_j$, i has a strategy s_0 (e.g., slander) which destroys y_j , but such that the only way for every i to maximize her own status is to choose strategy $s_1 \succ s_0$ which brings j at $y_j \succ y_i$. This game satisfies economic edge and robust edge but not the other properties, and it does produce reversals.

Proof of Prop. 7

Consider an allocation for which the individual equivalent incomes are well defined. By the Pareto principle, one can move every individual to the equivalent income level and the associated ideal social outcome, and this yields an allocation that is as good as the initial allocation (Note: such a move may not be feasible, but the ordering of allocations is not limited to feasible allocations).

The ordering of allocations therefore has to coincide with the ordering of these "equivalent" allocations. I.e., (x, y) is at least as good as (x', y') if and only if for the equivalent allocations, (x^*, y^*) is at least

as good as (x'^*, y'^*) , where x_i^* is the equivalent income of (x_i, y_i) for i , associated with the ideal social outcome y_i^* , and likewise for the primed allocation.

Now, the Pareto condition requires this ordering to be monotonic increasing and the equity condition requires it to be inequality averse.