UNIVERSITY OF OXFORD
Institute for New Economic Thinking
AT THE OXFORD MARTIN SCHOOL

Liquidity Spirals

Garbrand Wiersema, Esti Kemp & J. Doyne Farmer

6 Sept 2023

INET Oxford Working Paper No. 2023-16

Liquidity Spirals

Garbrand Wiersema[1,2,*], Esti Kemp[3,4,*], J. Doyne Farmer[1,2,5,6]

[1] *Institute for New Economic Thinking at the Oxford Martin School, University of Oxford, Oxford OX2 6ED, UK*
[2] *Mathematical Institute, University of Oxford, Oxford OX1 3LP, UK*
[3] *South-African Reserve Bank*
[4] *International Monetary Fund*
[5] *Oxford-Man Institute of Quantitative Finance, University of Oxford, Oxford, OX2, 6ED, UK*
[6] *Santa Fe Institute, Santa Fe, NM 87501, USA*
[*] *Corresponding author: Wiersema@maths.ox.ac.uk*

September 6, 2023

Abstract

We introduce a novel method for studying liquidity spirals and use this method to identify spirals before stock prices plummet and funding markets lock up. We show that liquidity spirals may be underestimated or completely overlooked when interactions between contagion channels are ignored, and find that financial stability is greatly affected by how institutions choose to respond to liquidity shocks, with some strategies yielding a "robust-yet-fragile" system. To demonstrate the method, we apply it to a highly granular data set on the South African banking sector and investment fund sector. We find that liquidity spirals are exacerbated when the liquidity positions of institutions worsen, and that central bank-provided liquidity can greatly dampen liquidity spirals. We also show that, depending on the market conditions, a liquidity spirals is sometimes caused by the banking and fund sectors' collective dynamics, but at other times by one sector's individual impact. The approach developed here can be used to formulate interventions that specifically target the sector(s) causing the liquidity spiral.

Keywords

Liquidity Spiral, Financial Stability, Systemic Risk, Financial Contagion, Interacting Contagion Channels, Intersectoral Contagion Channels, Multiplex Networks, Stress Test, Solvency-Liquidity Nexus

JEL Classification: G01, G17, G18, G21, G23, G28

1 Introduction

The progressive worsening of market and funding liquidity due to positive feedback loops in the financial system is referred to as a liquidity spiral, and poses a significant risk to financial stability by causing or exacerbating crises such as the Great Financial Crisis of 2008 (Brunnermeier and Pedersen, 2009). These positive feedback loops are made up of mechanisms that propagate financial shocks; so-called contagion channels (Allen and Gale, 2000). Various contagion channels have been studied in the literature (see e.g. Allen and Gale, 2000, Eisenberg and Noe, 2001, Gorton and Metrick, 2012) and the interactions between different contagion channels have been observed to severely amplify instabilities.[1] Furthermore, multiple types of institutions across various sectors may be involved in the contagion process (see e.g. Farmer et al., 2020, Wiersema et al., 2021). We capture liquidity spirals that consist of various interacting contagion channels and multiple types of institutions.

We identify liquidity spirals before they progressively depress market and funding liquidity using a *shock transmission matrix* (Wiersema et al., 2019). The matrix captures the stability of various interacting contagion channels without relying on any specific, subjective stress scenarios. When the largest eigenvalue of the matrix exceeds one, market and funding liquidity progressively worsen and a liquidity spiral emerges. We find that liquidity spirals may be severely underestimated or even completely overlooked when interactions between different types of institutions or contagion channels are ignored, and that the intensity of the spiral greatly depends on which assets institutions choose to liquidate in response to a liquidity shock. In particular, we identify liquidation strategies that yield a "robust-yet-fragile" system, which is resilient to small shocks, but may become highly unstable due to a single large shock to institutions' liquidity. Gai and Kapadia (2010) find a similar phenomenon for certain topologies of financial networks. The identification of the robust-yet-fragile tendency of financial systems across multiple dimensions underscores the importance of stability measures that assess a system's resilience to a wide range of shocks, such as the eigenvalue-based approach developed here.

To demonstrate our method, we apply it to a highly granular data set on the South African financial system. The South African financial system consists of a core of five large banks and a periphery of smaller banks and a large number of investment funds. By evaluating both the stability of the individual sectors as well as the combined system, we find that, depending on the market conditions, a liquidity spiral may emerge either as the result of the banking and fund sectors' collective dynamics or due to an individual sector's instability. Interventions that target the investment fund sector when the banking sector is the main cause of the spiral (and vice versa) have little effect. In particular, we find that

[1] See e.g. Caccioli et al. (2013), Poledna et al. (2015), Kok and Montagna (2013), Wiersema et al. (2019), Cont et al. (2020), Detering et al. (2021).

liquidity spirals are exacerbated when the liquidity of institutions falls and that central bank-provided liquidity can greatly dampen the liquidity spiral, but only when provided to the right sector. The approach developed here can be used to determine whether a specific sector is driving the instability and to formulate interventions that specifically target this sector.

1.1 Contributions

Our results complement the previous literature on market and funding liquidity crises. Various mechanisms that may progressively worsen market and funding liquidity have been studied in, e.g., Brunnermeier and Pedersen (2009), Gorton and Metrick (2012), Thurner et al. (2012) and Hałaj (2018). However, such studies only include a subset of the contagion channels that our model captures and therefore may underestimate the severity of the liquidity spiral.[2] Our approach of identifying liquidity spirals based on the contagion dynamics' largest eigenvalue rather than based on a specific, subjectively determined stress scenario further strengthens the comprehensiveness of our analysis (Borio et al., 2014, Wiersema et al., 2019).

Our main contribution is the insight that our method provides into the impact of institutions' choices on financial stability; which actions institutions choose to take in response to liquidity shocks strongly affects the potential for liquidity spirals to emerge. Institutions' liquidation strategies have been empirically studied in Kim (1998), van den End and Tabbae (2012), and Ma et al. (2020), but to the best of our knowledge, these strategies' impact on financial stability has not been explicitly studied previously. Furthermore, our finding that certain liquidation strategies may yield a robust-yet-fragile financial system complements the identification of robust-yet-fragile network topologies by Gai and Kapadia (2010), and demonstrates that financial systems show this property across multiple dimensions.

We also contribute to the literature on the interconnectedness of the South African financial system (see e.g. Kemp, 2017, Wiersema et al., 2021). Using a similar data set as we do here, Wiersema et al. (2021) study (counterparty) exposures in the South African financial system and how they are affected by institutions' solvency. The analysis presented in this paper broadens the understanding of the stability of the South African financial system by focusing on liquidity crises.

1.2 Structure

The remainder of this paper is organized as follows: Section 2 presents our method for identifying liquidity spirals and the insights it offers. In section 3, we apply the framework

[2] Caccioli et al. (2013), Kok and Montagna (2013), Poledna et al. (2015), Wiersema et al. (2019), Detering et al. (2021).

to the South African financial system. We discuss our data on the South African banks and investment funds, and present the results of the calibration of our method to the South African financial system. Section 4 concludes by discussing the implications and limitations of our results, and provides avenues for further research.

2 Identifying Liquidity Spirals

We use the framework developed in Wiersema et al. (2019) to study the conditions under which liquidity spirals emerge in the South African financial system. This framework allows us to capture many interacting contagion channels and sectors without relying on any specific stress scenario, which are often subjectively defined (Borio et al., 2014). By capturing the contagion dynamics in a linear framework, we can use the largest eigenvalue to identify a liquidity spiral before a liquidity crisis develops.

2.1 The Solvency-Liquidity Nexus

A financial system's contagion dynamics are driven by the *Solvency-Liquidity Nexus*. The culmination of a severe financial crisis is usually the default of one or more institutions (Brunnermeier, 2008, Roukny et al., 2013), where a default can be caused by *insolvency* or *illiquidity*. Insolvency occurs when asset values drop to the point where equity becomes negative – that is, when the value of an institution's liabilities exceeds that of its assets (Amini et al., 2016). Default due to illiquidity, on the other hand, occurs when an institution is unable to meet its payment obligations (Cont and Schaanning, 2017). Insolvency and liquidity can be related, but are analytically distinct: an institution can default due to a liquidity shock even when it is solvent, and vice versa. During financial crises, liquidity tends to be the more direct threat; an institution may survive temporary insolvency by maintaining liquidity and regaining its solvency at a later date. In normal economic times, a solvent institution is expected to borrow to avert a liquidity shortage. In times of economic crisis, however, this may not be possible because lending markets malfunction due to uncertainty about asset values, escalating collateral requirements, liquidity hoarding and capital flight, etc. (Rochet and Vives, 2004, Gorton and Metrick, 2012).

We can analyze the stability of the financial system in terms of its resilience to shocks, which we can classify either as liquidity shocks or valuation shocks, depending on the type of default they threaten to cause. For the purposes of this paper, we define a liquidity shock as an unexpected outflux (or cancellation of an expected influx) of liquid assets and a valuation shock as a drop in the value of an institution's assets.[3] Although we are

[3]We consider *expected* inflows and outflows of liquid assets as part of regular day-to-day liquidity management, and therefore do not classify such flows as a liquidity shock. For simplicity, we assume that shocks are non-negative. In principle, the framework could also capture negative shocks (i.e. liq-

principally interested in how liquidity shocks depress market and funding liquidity, the relevance of valuation shocks will become clear in the next section, where we show that contagion channels may convert valuation shocks to liquidity shocks.

2.2 Contagion Channels

We now demonstrate how to capture contagion channels in terms of the propagation of liquidity and valuation shocks and the conversion of one type of shock into the other. We discuss the five contagion channels most likely to contribute to the emergence of liquidity spirals. Note that this set of channels differs from the contagion channels included in Wiersema et al. (2019), which highlights the flexibility of the framework.

Overlapping Portfolio Contagion

Overlapping portfolio contagion can materialize when two institutions hold common securities and either institution sells securities, which drives prices down and lowers the securities' value[4]: If institution i suffers a liquidity shock it may be forced to sell securities to raise liquidity. This depresses their price. If institution j also has a position in these securities it experiences a valuation shock. Hence, *overlapping portfolio contagion converts liquidity shocks to valuation shocks.* By increasing the demand for liquidity on trading markets, overlapping portfolio contagion depresses market liquidity.

Funding Contagion

Funding contagion occurs when an institution depends on short-term funding to provide liquidity and runs the risk that the investor might withdraw its funding[5]: If institution i depends on a short-term funding from institution j, if j suddenly withdraws the funding to meet a liquidity shock it receives, then this causes a liquidity shock to i. Hence, *funding contagion propagates liquidity shocks* and reduces funding liquidity by decreasing the supply of short-term funding.

Shareholder Contagion

Shareholder contagion occurs whenever an institution suffers losses, as those losses are passed on to its shareholders: If a valuation shock causes institution i's asset value to fall, the value of its issued shares (which represent ownership of i's assets) falls accordingly, causing losses to the shareholders. Hence, *shareholder contagion propagates valuation shocks.*

uidity and asset value *gains*), but this would cause the framework to lose some of the convenient properties guaranteed by the Perron Frobenius theorem.

[4]See e.g. Adrian and Shin (2010), Caccioli et al. (2013, 2014, 2015), Duarte and Eisenbach (2018), Cont and Schaanning (2017, 2019).

[5]See e.g. Diamond and Dybvig (1983), Acharya and Skeie (2011), Caccioli et al. (2013), Brandi et al. (2018).

Share Redemption Contagion

When an institution such as an investment fund issues shares that are redeemable on a short-term basis (typically daily), the institution is at risk of share redemption contagion; when the institution suffers a loss and the value of its issued shares falls accordingly, shareholders may decide to redeem shares as part of risk-management or performance-based capital allocation schemes (Cont and Wagalath, 2013, Wiersema et al., 2021). Specifically, if a valuation shock decreases institution i's asset value and its shareholders decide to redeem (some of) their shares, institution i is forced to pay back the value of those shares and thus suffers a liquidity shock. Hence, *share redemption contagion converts valuation shocks to liquidity shocks*.

Deleveraging Contagion

Deleveraging contagion takes place when an institution uses borrowed funds to purchase assets.[6] Borrowing creates debt and the ratio of debt to equity is called the *leverage* λ. As part of good risk-management practices, it is common for financial institutions to target a particular leverage to control risk. If the value of assets drops, the debt burden remains constant but the equity value decreases, so leverage increases. This forces a leverage-targeting institution to pay off debt to maintain its leverage target, an action that drains the institution's liquidity.[7] Specifically, if a valuation shock decreases bank i's equity and its leverage rises accordingly, the institution must raise cash to pay off its debt to return to its target leverage. Hence, the institution essentially triggers a liquidity shock to itself, so *deleveraging contagion converts valuation shocks to liquidity shocks*. Note that institutions can also be forced to deleverage due to haircuts on collateralized debt (Brunnermeier and Pedersen, 2009); when the value of the collateral falls, the institution must pay back some of the debt (assuming that it cannot post additional collateral).

2.3 The Shock Transmission Matrix

We show how to characterize the collective stability of the five described contagion channels without relying on any specific, subjective stress scenarios. The interactions of these contagion channels can be captured in a *shock transmission matrix* (Wiersema et al., 2019). Assuming discrete dynamics, the shock transmission matrix A_t is defined with respect to a specific time t as contagion equations may change along with institutions' balance sheets. Furthermore, let $x^l_{t,i}$ denote the liquidity shock suffered by institution i at time t such that the N-dimensional vector $\boldsymbol{x}^l_t$ gives the liquidity shocks to all institutions (where N is the number of financial institutions). Similarly, $x^v_{i,t}$ denotes the valuation

[6] See e.g. Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009), Adrian and Shin (2010), Geanakoplos (2010), Adrian and Shin (2014), Aymanns et al. (2016).

[7] We do not consider slower mechanisms to raise equity-capital, such as issuing new shares or retaining earnings.

shock to institution i at time t and $\boldsymbol{x}_t^v$ the N-dimensional vector of valuation shocks to all institutions. The combined shock vector $\boldsymbol{x}_t$ of length $2N$ is

$$\boldsymbol{x}_t = \begin{bmatrix} \boldsymbol{x}_t^l \\ \boldsymbol{x}_t^v \end{bmatrix}. \tag{1}$$

The shock transmission matrix A_t is the $2N \times 2N$ matrix that acts on the shock vector $\boldsymbol{x}_t$ according to

$$\boldsymbol{x}_{t+1} = A_t \boldsymbol{x}_t. \tag{2}$$

Given the distinction between the top and bottom half of $\boldsymbol{x}_t$, we decompose the shock transmission matrix into its four quadrants,

$$A_t = \begin{bmatrix} A_t^{ll} & A_t^{vl} \\ A_t^{lv} & A_t^{vv} \end{bmatrix}, \tag{3}$$

where each of the components $A_t^{ll}, A_t^{lv}, A_t^{vl}$ and A_t^{vv} are $N \times N$ matrices, so that Eq. (2) can be written in the form

$$\boldsymbol{x}_{t+1} = \begin{bmatrix} \boldsymbol{x}_{t+1}^l \\ \boldsymbol{x}_{t+1}^v \end{bmatrix} = \begin{bmatrix} A_t^{ll} \boldsymbol{x}_t^l + A_t^{vl} \boldsymbol{x}_t^l \\ A_t^{lv} \boldsymbol{x}_t^l + A_t^{vv} \boldsymbol{x}_t^v \end{bmatrix}. \tag{4}$$

Eq. (4) makes explicit how the diagonal quadrant A_t^{ll} describes the propagation of liquidity shocks and A_t^{vv} the propagation of valuation shocks. The off-diagonal quadrant A_t^{lv} gives the conversion of liquidity to valuation shocks and A_t^{vl} the conversion of valuation to liquidity shocks. Hence, each of the five described contagion channels is associated with a specific quadrant of the shock transmission matrix.

The shock transmission matrix can be used to study the system's stability and resilience to shocks. Because all its elements are non-negative, the Perron-Frobenius theorem guarantees that the matrix has a non-negative real eigenvalue greater than or equal to (the magnitude of) the matrix' other eigenvalues. This largest eigenvalue describes the dominant dynamics of the financial system[8]: If the largest eigenvalue is greater than one, shocks that are not orthogonal to the corresponding eigenvector are increasingly amplified, causing more and more funding and overlapping portfolio contagion. Hence, an eigenvalue greater that one indicates a positive feedback loop that progressively depresses market and funding liquidity, which we refer to as a liquidity spiral. When the largest eigenvalue is smaller than one, contagious propagation of shocks may still worsen market and funding liqiuidities beyond the impact of the initial shock, but liquidity eventually stabilizes as the shock is damped out.

[8]See e.g. Caccioli et al. (2014), Bardoscia et al. (2017), Cont and Schaanning (2019), Wiersema et al. (2019).

2.4 Pecking Orders

What actions institutions choose to take in response to liquidity shocks determines the contagion that institutions transmit and the corresponding entries of the shock transmission matrix. When a liquidity shock hits, an institution must liquidate assets to meet the shock. We assume that each institution has a *pecking order* that specifies the sequence in which it liquidates its assets (Kok and Montagna, 2013, Hałaj, 2018, Wiersema et al., 2019). For example, once an institution has fully sold its position in a given security, it may move on to selling another, less liquid, security. The assumption of a liquidity pecking order underpins the design of regulatory measures like the Liquidity Coverage Ratio and Net Stable Funding Ratio requirements (BIS, 2013, 2014) as well as many studies [9].

van den End and Tabbae (2012) observe Dutch banks to use a *uniform pecking order* during crises while employing *liquidity-differentiated pecking orders* in benign times. An institution with a uniform pecking order does not differentiate between liquid assets of various types and uses all simultaneously by liquidating (part of) each asset to respond to shocks. Institutions with liquidity-differentiated pecking orders, as the name suggests, distinguish between assets of various liquidities. Institutions across multiple sectors have been observed to liquidate assets in order of decreasing liquidity when responding to shocks (Kim, 1998, van den End and Tabbae, 2012, Ma et al., 2020). This pecking order typically minimizes liquidation costs as long as shocks remain small, so we refer to it as as the *optimistic pecking order*. On the other hand, institutions with the *conservative pecking order* liquidate assets in increasing order of liquidity so as to conserve their most liquid assets for the worst circumstances. Institutions may employ the conservative pecking order in anticipation of a flight to liquidity during crises (see e.g. De Haan and van den End, 2013, De Santis, 2014) or to preemptively divest from illiquid securities to avoid being forced to liquidate those securities during the worst of the crisis, when their price is well below their fundamental value (see e.g. Bernardo and Welch, 2004).

An institution with a liquidity-differentiated pecking order only liquidates the asset at the top of its pecking order in response to liquidity shocks until that asset is exhausted and the institution is forced to move on to liquidating the asset that is next in line. As long as the asset at the top of the institution's pecking order is not exhausted, that asset exclusively determines the contagion that the institution transmits in response to liquidity shocks (and accordingly determines the corresponding entries of the shock transmission matrix). Hence, for shocks small enough not to exhaust the assets at the tops of institutions' pecking orders, the contagion transmitted in response to liquidity shocks is exclusively determined by institutions' most liquid assets when all institutions have the optimistic pecking order, and by institutions' least liquid assets when all institutions have the conservative pecking order.

[9] See e.g. Allen and Gale (2000), Kok and Montagna (2013), Hałaj (2018), Wiersema et al. (2019).

Large shocks, on the other hand, exhaust the assets at the tops of institutions' pecking orders and force the institutions to liquidate assets lower on their pecking orders, which changes the dynamics of the system; when the top asset is exhausted, the asset that is next in line becomes the new asset at the top of the pecking order and the contagion transmitted change accordingly, as do the corresponding entries of the shock transmission matrix . As we will see below, pecking orders strongly affect the potential for liquidity spirals to emerge.

2.5 Stylized Example

The framework that we have described allows us to study the impact of institutions' pecking orders on the potential for liquidity spirals to emerge. As an example, consider a simple banking system where all banks hold only two types of liquid assets; deposits at the central bank and at other banks in the system, and (irredeemable) equity shares in other banks in the system. The deposits can be withdrawn at no cost, which causes funding contagion when withdrawn from other banks (but not when withdrawn from the central bank), while the shares can only be sold at a discount due to the price impact of the sale, and cause overlapping portfolio contagion when sold. Hence, deposits sit at the top of optimistic pecking orders while shares sit at the top of conservative pecking orders. Finally, assume that banks maintain their current levels of leverage.

The banks' pecking orders determine whether funding or market liquidity falls first during crises, as the contagion transmitted in response to a liquidity shock is determined exclusively by the assets at the top of an institution's pecking order until these assets are exhausted by the liquidity demand. Hence, when all banks have the optimistic pecking order, funding liquidity falls until some banks have withdrawn all their deposits and start selling shares, causing market liquidity to be reduced too. On the other hand, when all banks have the conservative pecking order, market liquidity declines until some banks have sold all their shares and start withdrawing deposits, which depresses funding liquidity. Finally, when institutions have the uniform pecking order, funding and market liquidity fall in tandem.

Pecking orders have a strong impact on financial stability, which we can illustrate using this simple setup; for example, when all banks have only deposits at the top of their pecking orders, the lower-left quadrant of the shock transmission matrix, i.e the overlapping portfolio contagion quadrant, is empty so the matrix is block-triangular which is not the case for other pecking orders. Because the matrix is block-triangular, its largest eigenvalue is determined completely by the diagonal quadrants and hence is not affected by the upper-right quadrant, i.e. the deleveraging contagion quadrant. Banks' leverages are typically on the order of ten in well-developed financial systems, and consequently deleveraging strongly amplifies shocks and tends to raise the largest eigenvalue (Wiersema et al., 2019). This makes liquidity spirals more likely to occur when the deleveraging contagion quadrant has an impact on the largest eigenvalue, i.e. when the shock transmission matrix is not block-triangular. Hence, the system is generally more resilient to (small) shocks when all banks have only deposits at the top of their pecking order, than when the banks only have equity shares, or a mix of deposits and shares, at the top of their pecking orders.

Furthermore, the specific case of all banks having only deposits at the top of their pecking orders also serves to illustrate how a large shock may destabilize the system: When the shock is large enough to exhaust (some) banks' deposits, these banks are forced to liquidate shares to meet liquidity shocks. Hence, the shock transmission matrix is no longer upper block-triangular, so its largest eigenvalue likely increases, as just explained. Under certain conditions therefore, a large shock that exhausts the assets at the top of institutions' pecking orders may cause a liquidity spiral to emerge.

A financial system which is very resilient to small shocks but becomes unstable when hit by a large shock, as may be the case when a sufficient number of institutions have the optimistic pecking order, may be referred to as "robust-yet-fragile". Gai and Kapadia (2010) find a similar phenomenon for certain topologies of financial networks. The identification of this robust-yet-fragile tendency of financial systems across multiple dimensions highlights the dangers of optimizing financial stability with respect to the small shocks that are incurred on a frequent basis; such a system may turn out to be highly fragile when a large shock eventually hits. It also underscores the risk of assessing the resilience of a financial system only to specific stress scenarios, which may cause severe instabilities to go unnoticed.

Finally, we derive two observations in section A.1 of the appendix that direct follow from the shock transmission matrix and the properties of block matrices:

1. Out of all contagion channels considered in this analysis, the funding contagion channel is the only channel that may cause a liquidity spiral to emerge in the absence of other contagion channels.

2. For the funding contagion channel to cause a liquidity spiral on its own, banks must hoard liquidity in response to shocks, i.e. they must recover liquidity in excess of the shock incurred in order to build reserves against potential future shocks.

From observations 1 and 2 follows that, absent liquidity hoarding, all liquidity spirals result from a combination of interacting contagion channels. Hence, one overlooks any liquidity spiral in even the most unstable of systems when studying contagion channels in isolation (and thus ignoring the channels' interactions, as is often done).

3 Measuring the Potential for Liquidity Spirals in the South African Financial System

We demonstrate our framework for identifying liquidity spirals by applying it to the South African financial system. South Africa is a small open economy with a relatively well-developed financial market compared to other African or emerging-market economies (Kemp, 2017). The South African debt market is liquid and well-developed in terms of the number of participants and their daily activity, and its equity market dominates the region in terms of capitalisation (Andrianaivo and Yartey, 2010). Due to the relative lack of well-developed peers in the region, South African institutions are very reliant on the domestic financial market, making it highly interconnected (Kemp, 2017).

Banking sector assets exceed GDP in aggregate terms, but are smaller than the assets held by the non-bank financial intermediation sector, which includes entities such as insurers, pension funds and collective investment schemes (the latter are henceforth referred to as "investment funds"). Since the Global Financial Crisis, the share of assets held by banks has decreased, as the growth of assets held by the non-bank financial sector – in particular investment funds - has outpaced that of banks (Kemp, 2017). Non-bank financial intermediaries are an important source of funding for banks; banks' funding provided directly by non-bank financial intermediaries other than pension funds and insurers amounts to 15% of bank assets (FSB, 2018).

3.1 Institutions

In this study, we focus on banks and investment funds domiciled in South Africa. Pension funds and insurers are not included due to data limitations, but we do not expect this to affect our results substantially as pension funds and insurers typically do not cause contagion through any of the channels included in our model. Non-financial corporates, henceforth referred to as the corporate sector, and the South African government are not modeled, but our data include the tradable securities corporates and the government issue.

3.1.1 Banks

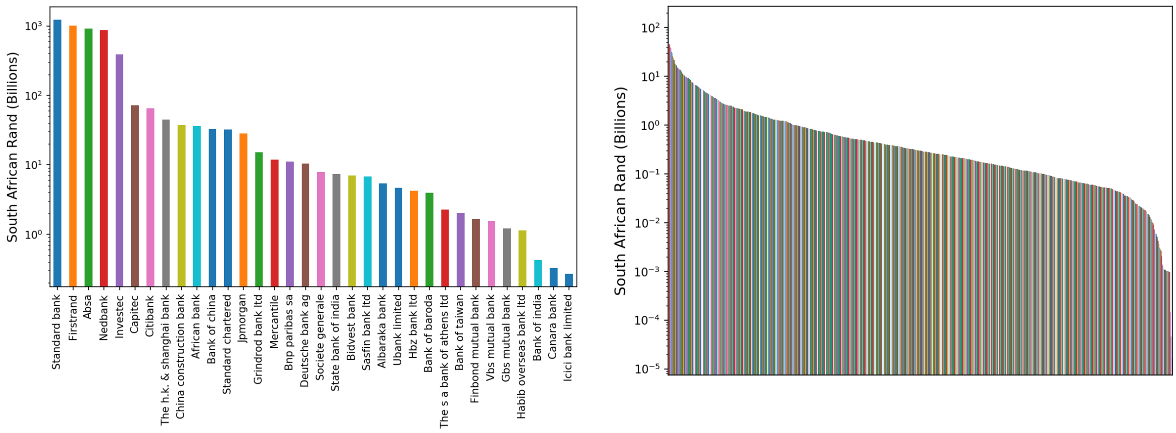
The South African banking sector comprises 34 registered banks, local branches of foreign banks and mutual banks as of Q4 2016. The sector is concentrated, with the five largest banks by assets holding more than 90% of the banking sectors' assets (SARB, 2017), as illustrated in Figure 1a. Overall, the banking sector is largely funded by deposits, but banks also issue debt instruments, such as bonds and money market instruments, and equity shares. The banks' leverages (debt-to-equity ratio) vary, with a median of 7.4. The largest banks' leverages are between 11 and 13, which is not uncommon is well-developed financial systems, and the smaller banks typically have lower leverages.

3.1.2 Investment Funds

Investment funds pool investors' money and purchase a portfolio of securities, thereby offering investors the opportunity to obtain exposure to a diverse portfolio of underlying securities, without having to purchase and trade securities directly. From the investor's perspective, investment funds provide investors with an opportunity to earn higher returns than those offered by deposits, in return for taking on greater risk. There are over 1200 open-ended investment funds registered in South Africa with assets under management of about 2 trillion South African Rand in 2016. The investment sector is highly concentrated, as show in Figure 1b.

Investors invest in funds by buying fund shares, which represent ownership of a portion of the underlying portfolio. These shares are typically redeemable on a daily basis. In extreme circumstances, funds are susceptible to "runs" – i.e. large-scale redemption requests, when investors anticipate or observe a substantial drop in their fund shares' value. When a run is initiated, the investment fund may run out of liquid assets and become unable to meet redemptions. As a result, the investment fund may have to resort to fire-selling assets (Cont and Wagalath, 2013, Wiersema et al., 2021).

The value of a fund share is given by its Net Assset Value (NAV), which is equal to the the investment fund's total asset value, divided by the investment fund's total number of shares outstanding. Fund shares can be either Constant NAV-valued (CNAV) or Variable NAV-valued (VNAV). When a investment fund makes a profit or loss, a VNAV fund adjusts the shares' NAV to reflect this while keeping the number of shares that shareholders own constant, whereas a CNAV fund adjusts the number of shares that each shareholder owns while keeping the NAV constant. While the mechanism through which VNAV and CNAV funds pass on their profits and losses to their shareholders is different, the impact on the value of an investor's share portfolio is identical. Therefore, we assume for simplicity that all fund shares in our model are VNAV-valued.



(a) Banks ranked by total assets

(b) Investment Funds ranked by total assets

Figure 1: **Distribution of South African financial institutions by asset size.** The institutions are listed on the x-axis in decreasing order of total assets size and their total assets in billions of South African Rand are on the y-axis (log-scale). Note that the funds' names are not listed because they are too numerous. The banking sector consist of a core of five large banks and a periphery of 29 smaller banks. The investment fund sector includes over 1200 funds and also shows a strong concentration in terms of asset size.

3.2 Assets

The data used are sourced from two publicly available data sets as of Q4 2016. Aggregate balance sheet data (aggregate assets, liabilities and equity) on individual banks are sourced from the BA900 data published by the South African Reserve Bank, or *SARB* (SARB, 2016). Balance sheet entries are aggregated by asset type and counterparty type (e.g. "deposits at domestic banks"). The bank data distinguish between the various asset types discussed in the next section, and between all counterparty types considered in our model (i.e. banks, funds, non-financial corporates and the government). The bank data also cover asset and counterparty types not included in our model, such as household mortgages.

Data on investment funds' assets were sourced from Morningstar Inc and are highly granular. These data report investment funds' investments per instrument type in individual counterparties (e.g. "bonds issued by Standard Bank"). The great majority of the funds' assets are covered by our model, but a small fraction (less than 5%) is excluded, e.g. because the counterparty or asset type is unknown. Note that neither the bank nor fund data include short positions in tradable securities, so only long positions are considered.

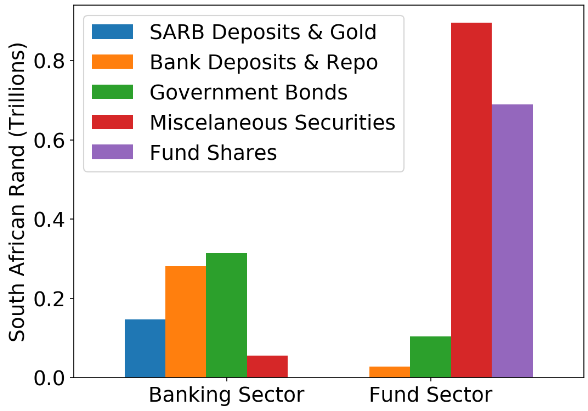


Figure 2: **Aggregate asset holdings per category of the banking sector and fund sector.** Asset holdings are aggregated into the categories set out in section 3.2.1 for the banking sector and fund sector. Banks do not hold fund shares, while funds do not hold South African Reserve Bank deposits or gold. Miscellaneous securities include non-government bonds, money market instruments, and equity shares. Note that the banks' position in bonds issued by the South African government far exceeds the funds' position, while the funds' position in miscellaneous tradable securities far exceeds that of the banks.

3.2.1 Balance Sheet Composition

We include the following assets: Gold, deposits, repurchase agreements, or *repo*, money market instruments, or *MMIs*, bonds, equity shares, and fund shares. We refer to non-government bonds, MMIs, and equity shares as *miscellaneous securities*. Figure 1 shows where these assets appear on the stylized balance sheets of banks and investment funds. Note that some investment funds may heavily invest in one type of asset while not investing in other types.

Assets		Liabilities
Repurchase agreements		Repurchase agreements
SARB and Bank Deposits		Banks' and Funds' Deposits
Tradable securities	MMIs	MMIs
	Bonds	Bonds
	Gold	Other liabilities
	Equity Shares	
Other assets		**Equity**

(a) Stylised balance sheet of a bank.

Assets		Liabilities
Bank Deposits		Fund shares
Tradable securities	MMIs	
	Bonds	
	Equity shares	
Fund shares		
Other assets		

(b) Stylised balance sheet of an investment fund.

Table 1: **Stylised balance sheets of South-African banks and investment funds**. (a) shows the stylized balance sheet of a bank and (b) of an investment fund. Note that specific subsets of investment funds may heavily invest in one type of asset while not investing in other types.

Central Bank Deposits & Gold

Banks invest in gold and make deposits at the SARB, whereas investment funds do not. We assume that both are perfectly liquid and that neither causes contagion when liquidated. This makes their dynamics in our model identical, so they are grouped together for simplicity.

Bank Deposits & Repurchase Agreements

South African banks receive deposits and issue repo, while investment funds do not (as they do not have debt). Furthermore, while both banks and funds make deposits at (other) South African banks, only the banks buy repo. The banks and funds also make deposits at foreign banks, but these make up a very small part of their portfolios. Because we do not explicitly model collateral and assume that both repo and deposits can be redeemed on a daily basis, their dynamics in our model are identical. As such, we group repo and deposits at (commercial) banks together for simplicity.

Bonds

Domestic bonds are issued by banks, the corporate sector and the South African government. Additionally, South African banks and investment funds own some bonds issued by foreign parties but these positions are relatively small. The investment fund data distinguish between bonds issued by different banks, whereas the bank data do not. Contrary to repo and deposits, bonds are tradable.

Money market instruments

MMIs are defined in line with Board Notice 90 of the Financial Sector Conduct Authority (Board, 2014), and include commercial paper, negotiable certificates of deposits, bankers acceptances and promissory notes. The data do not distinguish between these various types of MMIs so they are treated identically in our model. MMIs in our data are exclusively issued by *domestic* banks and are bought by both banks and funds. Like bonds, MMIs are tradable.

Equity shares

Funds invest in listed equity shares issued by the South African banks and corporate sector, while banks invest in listed shares issued by the corporate sector and hold unlisted shares in (other) South African banks. Furthermore, South African banks and investment funds own some listed equity shares issued by foreign parties but the great majority of shares held by the banks and investment funds are domestically issued. Listed equity shares are tradable whereas unlisted shares are not, and neither are redeemable.

Fund Shares

Fund shares are issued by investment funds and are assumed to be redeemable on a daily basis (as is almost always the case in reality). Therefore, fund shares are not traded. South African investment funds buy other funds' shares while banks do not. As explained, we assume that all fund shares are VNAV-valued for simplicity, so the shares' NAV is updated to reflect any losses that the issuing investment fund may suffer.

3.2.2 Initialization Values

We do not have data on the market prices or NAVs of the securities that institutions hold, nor the number of securities they hold, but only on the value of an institution's position in a security (i.e. the market value of a position in a tradable security, or the NAV times the number of shares of a position in fund shares). For simplicity, we normalize the initial NAV of each fund share and the initial market price of each tradable security to one South African Rand. This normalization has no effect on our results and is only for simplicity. Furthermore, we assume that the initial market price of a listed equity share is equal to its book value (i.e. equity shares' initial market values are assumed equal the issuer's accounting value of that share).

3.2.3 Interbank Asset Allocation

The contagion channels that we model require reconstruction of the counterparties of interbank deposits, repo, and unlisted equity shares as the bank data only provide banks' aggregate assets and liabilities. Due to data limitations, we do not distinguish between tradable securities of a specific type issued by different non-financial corporates, nor between tradable securities of a specific type issued by different domestic banks. Therefore, we do not reconstruct counterparties of banks' investments in these securities.

The technique used for the reconstruction of the banks' investments is similar to Kok and Montagna (2013) and Wiersema et al. (2021), and aims to reproduce the high heterogeneity of interconnections observed in financial networks. We assume that the initial market value of any security that a bank has issued is equal to the book value of that liability or equity share on the banks' balance sheets, and perform the following steps for each of the asset types
$\beta \in \{$deposits, repo, (unlisted) equity shares$\}$:

1. We subtract from each bank's aggregate liabilities (or equity) of type β the funds' investments of type β in that bank.

2. We pick a random pair of banks y and z, where bank y is the investor and bank z is the investee. Bank y is picked from the banks with nonzero aggregate assets of type β and z is picked from the banks with nonzero aggregate liabilities (or equity) of type β.

3. We pick a random number $x \in U(0,1)$ and generate an investment of type β of bank y in bank z equal in size to the product of x and the minimum of y's aggregate assets of type β and z's aggregate liabilities (or equity) of type β.[10]

4. The investment is added to the balance sheets of y and z, and the value of the investment is subtracted from y's aggregate assets of type β and z's aggregate liabilities (or equity) of type β.

5. Steps 2-4 are repeated until all banks' assets of type β are allocated.

After step 5, the counterparties of all (relevant) assets and liabilities are defined.

3.3 Contagion Equations

We now derive representative formulas for each of the contagion channels that acts on the described asset classes. Note that these forms are chosen for simplicity. More elaborate contagion models may be considered when studying individual contagion channels, but

[10] We set $x = 1$ when the minimum of y's aggregate assets of type β and z's aggregate liabilities (or equity) of type β is less than or equal to 500 South African Rand.

these forms suffice for our present purposes of capturing the collective stability of these interacting contagion channels.

3.3.1 Valuation Shock-Induced Contagion

We discuss the contagion caused by valuation shocks first, as liquidity shock-induced contagion requires a more elaborate discussion. We focus on valuation shocks that do not exceed banks' equity as we are interested in how liquidity spirals emerge, while (major) institutions are not expected to default before the liquidity spiral has grown into a systemic crisis.

Deleveraging Contagion

Suppose bank i maintains a leverage target λ_i (i.e. the ratio of debt to equity). If i receives a valuation shock $x^v_{i,t}$ and its equity is reduced, i must pay off debt to return to its target. The amount by which it must reduce debt is $\lambda_i x^v_{i,t}$, so i experiences a liquidity shock $A^{vl}_{ii,t} x^v_{i,t}$ at time $t+1$, where

$$A^{vl}_{ii,t} = \lambda_i, \tag{5}$$

when i is a bank. The leverage target λ_i is given by the data and is assumed to be kept constant by bank i.

Shareholder Contagion

Suppose institution i at time t holds a number $s_{ij,t}$ shares in fund j of the total number $S_{j,t}$ of shares issued by j. A valuation shock suffered by j is distributed proportionally across its shareholders through a markdown of the shares' NAV, so

$$A^{vv}_{ji,t} = \frac{s_{ij,t}}{S_{j,t}}, \tag{6}$$

when j is a fund.

Let us now consider shareholder contagion for equity shares issued by banks. Unlisted equity shares issued by banks are marked-to-book, i.e. they are valued based on the accounting equity of the issuing bank. When bank j incurs a valuation shock, the accounting equity of an unlisted share is reduced by $x^v_{j,t}/S_{j,t}$ where $S_{j,t}$ is the total number of (listed and unlisted) shares issued by bank j. Hence, when i holds $s_{ij,t}$ unlisted shares in bank j, the shareholder contagion i suffers on these shares equals $s_{ij,t}x^v_{j,t}/S_{j,t}$ and so equation (6) also holds for unlisted equity shares issued by banks.

Since the listed equity shares issued by banks are traded, modern accounting practices require them to be marked-to-market rather than marked-to-book. Nevertheless, an efficient market price reflects the issuer's performance. We assume for simplicity that if bank j incurs a valuation shock, the value of its issued listed shares falls by the same amount as its unlisted shares (i.e. the shares' market value and book value fall by the same amount) such that equation (6) also holds for listed shares.

Share Redemption Contagion

Suppose that fund i suffers a valuation shock $x^v_{i,t}$ at time t, which depresses the NAV of shares issued by i and may prompt i's shareholders to withdraw liquidity from the fund by redeeming shares. Funds' shares are held by other investment funds in our data and "external holders", i.e. any party other than the banks and funds that we model. We assume that other investment funds that hold shares in i do not withdraw liquidity from i in response to the valuation shock $x^v_{i,t}$ that i suffered (but these funds may decide to withdraw liquidity from i when they themselves suffer a liquidity shock, i.e. through funding contagion). Furthermore, we assume for simplicity that all external holders withdraw liquidity from i proportionally to the loss i suffered at the same *redemption rate* R[11], which implies that

$$A^{vl}_{ii,t} = \epsilon_{i,t} R, \tag{7}$$

where $\epsilon_{i,t}$ denotes the fraction of fund i's shares held by external holders. As the aggregate value of all i's outstanding shares equals i's total asset value, the fraction $\epsilon_{i,t}$ is found by subtracting the aggregate value of shares in i held by other funds in our data from i's total asset value (and dividing the resulting difference by i's total asset value). The redemption rate R is a nondimensional constant of order one, which we assume for simplicity to be the same across all funds i from which shares are withdrawn.

3.3.2 Liquidity Shock-Induced Contagion

The contagion that an institution transmits in response to a liquidity shock is determined by its pecking order. To distinguish between various liquidity-differentiated pecking orders, we group assets in decreasing order of liquidity as follows:

1. Central bank deposits and gold
2. Deposits at commercial banks, repo, and fund shares
3. Government bonds
4. Miscellaneous tradable securities (MMIs, listed equity, bank bonds, and corporate bonds)

Hence, institutions with the optimistic pecking order liquidate assets in order from group 1. to 4., and the conservative pecking order is the reverse. Note that any institution

[11] We show in section A.2 in the appendix that this assumption implies that the *number* of shares withdrawn is a convex function of the NAV loss, as one would reasonably expect (see e.g. Cont and Wagalath, 2013, Wiersema et al., 2021).

that does not own any assets of the types in the group at the top of the pecking order liquidates assets from the group that is next in line.

Given the limited empirical research into the pecking orders that institutions employ under various circumstances, we consider two more pecking orders for completeness. The *short-term funding pecking order* is the optimistic pecking order but with group 1. moved to the bottom and hence group 2. at the top, and the *government bonds pecking order* is the optimistic pecking order both groups 1. and 2. moved to the bottom and hence group 3 at the top. Hence, institutions with the short-term funding pecking order use deposits, repo and fund shares as their first line of defense against liquidity shocks, which in practice are indeed a common source of liquidity for many institutions, and institutions with the government bonds pecking order use their bonds for this purpose, which is also often observed in practice. The main motivation for including these two additional pecking orders is that each of the four groups of liquid assets we distinguish is now at the top of one of the four liquidity differentiated pecking orders we consider. Table 2 summarizes the various pecking orders that we consider.

	Optimistic pecking order	Short-term funding pecking order	Government bonds pecking order	Conservative pecking order	Uniform pecking order
Top	Central bank deposits and gold	Deposits at commercial banks, repo, fund shares	Government bonds	Miscellaneous tradable securities	All liquid assets
	Deposits at commercial banks, repo, fund shares	Government bonds	Miscellaneous tradable securities	Government bonds	
	Government bonds	Miscellaneous tradable securities	Central bank deposits and gold	Deposits at commercial banks, repo, fund shares	
Bot-tom	Miscellaneous tradable securities	Central bank deposits and gold	Deposits at commercial banks, repo, fund shares	Central bank deposits and gold	

Table 2: **Pecking orders**

An institution may have multiple assets at the top of its pecking order. For example, institutions with the uniform pecking order have all of their liquid assets at the tops of their pecking orders simultaneously. Furthermore, an institution with e.g. the short-term funding pecking order may have various deposits, repo and fund shares in multiple institutions at the top of its pecking order. When an institution has multiple assets at the top of its pecking order, we assume that the institution liquidates a vertical slice across all these assets, i.e. the institution recovers an amount of liquidity from each asset proportional to that asset's total value. As a consequence, an institution with the uniform pecking order reduces each of its liquid assets by the same proportion in response to a liquidity shock and hence the shock does not change the institution's pecking order, leaving the institution's response to liquidity shocks unchanged. We assume for simplicity that the vertical slice used to respond to the liquidity shock $\boldsymbol{x}_t^l$ is based on the asset values at the start of round t (i.e. before the liquidity and valuation shocks $\boldsymbol{x}_t^l$ and $\boldsymbol{x}_t^v$ are taken into account, which may reduce the value of some securities).

We now derive contagion equations for the liquidation of each type of asset that may be at the top of the pecking order, under the assumption that the liquidity shock does not exceed the top layer. Note that we do not consider liquidity hoarding, so institutions recover liquidity equal to the shock incurred.

Funding Contagion

Suppose institution i has deposits at and/or has bought repo or fund shares issued by institution j with a total value of $d_{ij,t}$. Let $T_{t,i}$ denote the total asset value of the top layer of i's pecking order. When these deposits, repo and/or fund shares are part of the top layer, on receiving a liquidity shock $x_{i,t}^l$, institution i withdraws a total value of $x_{i,t}^l d_{ij,t}/T_{t,i}$ of these deposits, repo and/or fund shares (i.e. i liquidates a vertical slice across the assets in its top pecking order layer). This transmits a liquidity shock $A_{ji,t}^{ll} x_i^l$ to institution j, where

$$A_{ji,t}^{ll} = \frac{d_{ij,t}}{T_{i,t}}. \tag{8}$$

We assume that the withdrawal of deposits from the SARB or foreign institutions does not cause funding contagion and hence "dilutes" liquidity shock-induced contagion (as less liquidity is required to be recovered from other sources).

Overlapping portfolio contagion

Suppose institution i holds $n_{\sigma i,t}$ shares of security σ with price $p_{\sigma,t}$, which are in i's top pecking order layer. When i experiences a liquidity shock $x_{i,t}^l$, it sells shares in security σ to raise an amount of liquity $x_{i,t}^l n_{\sigma i,t} p_{\sigma,t}/T_{i,t}$ (i.e. a vertical slice). The sale depresses the price of security σ by $\Delta p_{\sigma,t} = p_{\sigma,t} - p_{\sigma,t+1}$, which causes losses to all institutions that have a position in the security. We assume that the price impact $\Delta p_{\sigma,t}$ is linear in the amount of liquidity to be raised by selling shares in security σ[12] (so price impacts are additive

[12] We show in section A.3 in the appendix that this assumption implies that the price impact is a concave function of the number of securities sold. Empirical evidence suggests that the price impact is indeed a concave function although the particular shape may depend on the context (Gatheral, 2010). We do not aim to perfectly replicate any of the empirically observed functional forms, as the current approximation suffices for our present purposes. For more accurate modeling of the overlapping portfolio contagion channel see e.g. Bouchaud and Cont (1998), Bouchaud et al. (2009).

when multiple institutions i decide to sell shares in security σ at time t);

$$\Delta p_{\sigma,t} = \sum_{i \in \mathcal{S}_t} \frac{x^l_{i,t} n_{\sigma i,t} p_{\sigma,t}}{T_{i,t} D_\sigma}. \tag{9}$$

The market depth D_σ (discussed below) is a non-dimensional constant which gives the liquidity recovered per unit drop in the securities' market price. and $\mathcal{S}_t$ the set of institutions that sell security σ at time t. We also assume for simplicity that all shares sold at time t are sold against the new price $p_{\sigma,t+1}$, such that any institution, including institutions $i \in \mathcal{S}_t$, suffers a valuation shock of $\Delta p_{\sigma,t} n_{\sigma j,t}$[13]. This implies that

$$A^{lv}_{ji} = \sum_{\sigma \in \mathcal{T}_{i,t}} \frac{n_{\sigma i,t} p_{\sigma,t} n_{\sigma j,t}}{T_{i,t} D_\sigma}, \tag{10}$$

where $\mathcal{T}_{i,t}$ denotes the set of all tradable securities σ at the top of i's pecking order at time t.

Note that the market price of listed equity shares issued by South African banks is depressed by both overlapping portfolio and shareholder contagion. We assume for simplicity that the overlapping portfolio and shareholder contagion channels' impact on the market price is additive such that equation (10) also holds for listed shares issued by banks. The combined impact of the overlapping portfolio and shareholder contagion channels on listed shares should not drive their price below zero, as the shares are subject to limited liability. This is guaranteed in our results, as explained in section A.4 in the appendix.

Market depths: For each security class σ, we explore various market depths D_σ by dividing the *baseline estimate* $\hat{D}_\sigma$ by *market depth divisor* δ_σ,

$$D_\sigma = \frac{\hat{D}_\sigma}{\delta_\sigma}. \tag{11}$$

The baseline market depth $\hat{D}_\sigma$ is a non-dimensional constant calibrated to the market capitalization of security σ, as explained in appendix A.5, and the market depth divisor δ_σ is a non-dimensional constant which we vary from one to four to explore the sensitivity of our results to the market depth. Hence, when the market depth divisor $\delta_\sigma = 1$, the market depth of σ is equal to the baseline estimate, and when the market depth divisor $\delta_\sigma = 0$, the market depth of σ is infinite and the overlapping portfolio contagion channel

[13] In reality, some institutions may decide to sell shares incrementally, while others may under- or overestimate the amount of liquidity they will recover from selling securities, and still others may intentionally liquidate more than the required amount of securities out of conservatism (which is referred to as "liquidity hoarding"). However, for our present purposes, it suffices to simply assume that each institution liquidates the correct number of securities against the new price $p_{\sigma,t+1}$ to recover the required liquidity. Note that the price impact suffered by institutions $i \in \mathcal{S}_t$ implies that the diagonal component of $A^{lv}_{ii,t}$ for $i \in \mathcal{S}_t$ in equation (10) is positive.

is effectively turned off.

Note that our data include tradable securities issued by foreign institutions and gold. Because these securities are predominantly held by foreign institutions, we do not model the overlapping portfolio contagion caused by the sale of these securities, so we effectively assume that these securities have infinite market depth. Consequently, overlapping portfolio contagion only spreads across domestic securities. Hence, foreign securities and gold "dilute" liquidity shock-induced contagion, because the liquidity recovered from foreign securities and gold does not cause contagion but reduces the liquidity required to be recovered from other sources.

3.4 Results

Here, we study the contribution of the banking and investment fund sectors to the emergence of liquidity spirals for various pecking orders. We do so by comparing the stability of the full system to the stability of each sector individually, where the largest eigenvalue of an individual sector is calculated from the shock transmission matrix that only includes institutions belonging to that sector. The shock transmission matrices are calculated for time $t = 1$, to which securities' market prices, book values, as NAVs are normalized. Note that all results present the means over 1000 random generations of the interbank assets reconstruction. As these interbank assets form a small part of banks' total balance sheets, they do not affect our results significantly and therefore the standard errors of the means are negligible and not visible in the plots.

3.5 Uniform Pecking Orders

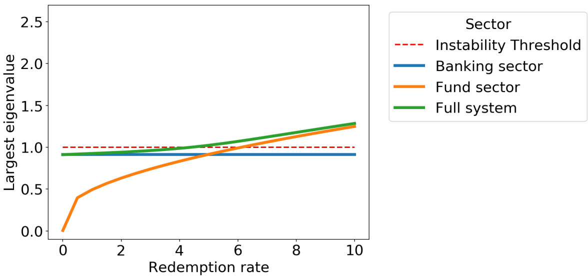
To understand the stability of the uniform pecking order, we plot in Figure 3 the largest eigenvalue of the full system, and of the banking sector and fund sector individually, under the assumption that all institutions have the uniform pecking order. In Figure 3a we set the market depth divisor to its baseline value of $\delta_\sigma = 1$ for all securities and vary the redemption rate R to explore the impact on stability, and find that a liquidity spiral emerges for about $R = 5$. Note that the banking sector is not affected by the redemption rate R, so increases in the redemption rate affect stability only through the fund sector.

In Figure 3b, we set the redemption rate to its baseline value of $R = 1$ and vary the market depth divisor δ_σ for all securities. We find that the stability is greatly affected by the market depth, and that a liquidity spiral emerges as soon as the market depth divisor δ_σ increases beyond one. Hence, a liquidity spiral emerges almost as soon as market depths fall below their baseline values, while only particularly high redemption rates cause a liquidity spiral.

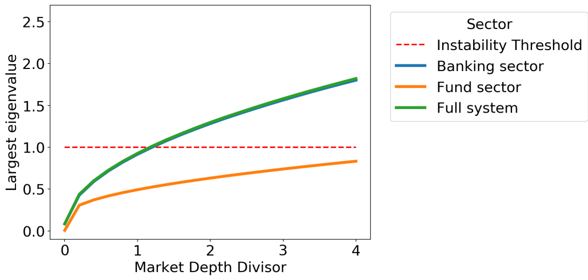
In Figure 3c and Figure 3d we consider how the South African Reserve Bank, acting as a lender of last resort, may attempt to dampen the liquidity spiral by injecting cash into the banking sector. Because banks use the uniform pecking order and therefore liquidate proportionally to asset values, raising the banks' central bank deposits increases the banks' reliance on their central bank deposits to meet liquidity shocks and decreases their reliance on other liquid assets. Withdrawing central bank deposits does not cause contagion, while the liquidation of other assets typically does. Hence, increasing the banks' central bank deposits stabilizes the system.

In Figure 3c we set the market depth divisor to $\delta_\sigma = 2$ for all securities and the redemption rate to its baseline value of $R = 1$, such that the liquidity spiral is mainly driven by the banking sector. Conversely, in Figure 3d we set the market depth divisor to its baseline value of $\delta_\sigma = 1$ for all securities and the redemption rate $R = 5$, such that the liquidity spiral is mainly driven by the fund sector. The increase in banks' central bank deposits as a fraction of their total liquid assets is presented on the x-axis in both figures. Although the liquidity spiral dissipates in both cases, Figure 3d shows that the stabilizing effect of the cash injection into the banking sector is minimal when the fund sector is the main driver of the liquidity spiral, while Figure 3c shows that the cash injection is more effective when the banking sector is the main driver of the liquidity spiral. Hence, to intervene effectively, the regulator must understand which sector is the main driver of the spiral.

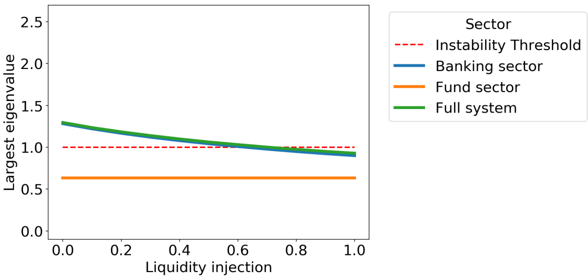
Finally, comparison of Figure 3b and Figure 3c suggests that, compared to the impact of the market depth, the effect of the cash injection on the largest eigenvalue is modest even when the banking sector is the main driver of the liquidity spiral. However, note that the x-axis Figure 3b ranges from an infinitely deep market ($\delta_\sigma = 0$), up to a reduction of the baseline market depth by up to a factor of $\delta_\sigma = 4$, while Figure 3c only covers up to a doubling of banks' liquid assets. This explains, at least partially, why the eigenvalue is more strongly impacted in Figure 3b than in Figure 3c.



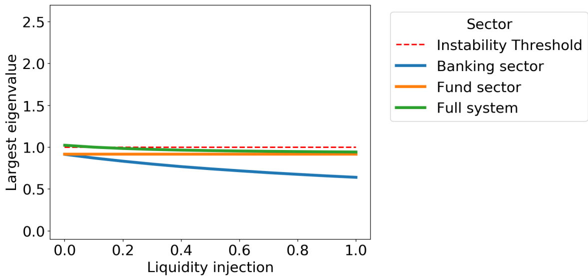
(a) Eigenvalue vs. Redemption Rate R



(b) Eigenvalue vs. Market Depth Divisor δ_σ



(c) Liquidity Provision for $\delta_\sigma = 2$



(d) Liquidity Provision for $R = 5$

Figure 3: **Eigenvalue Dependency for Uniform Pecking Orders.** All institutions are assumed to have the uniform pecking order. We explore when the largest eigenvalue exceeds one and a liquidity spiral emerges for various redemption rates R in (a) and for various market depth divisors δ_σ in (b). (c) shows the impact of a central bank cash injection into the banking sector when $\delta_\sigma = 2$ and $R = 1$, and (d) shows this impact when $R = 5$ and $\delta_\sigma = 1$.

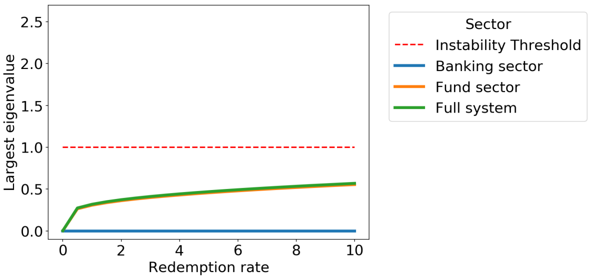
3.6 Liquidity-Differentiated Pecking Orders

We now consider the stability of the four liquidity-differentiated pecking orders, under the assumption that shocks do not exhaust the assets at the top of any institution's pecking order. As such, only the assets at the top of each institution's pecking order need to be considered when calculating the dynamics' largest eigenvalue. As Figure 2 gives the aggregate values of all assets that may be at the top of institutions' pecking orders, the figure gives an indication of how large shocks may be before the assets at the top of institutions' pecking orders are exhausted. For example, Figure 2 shows that the assets at the top of funds' government bonds pecking orders are generally exhausted quickly, while for banks this is the case for the assets at the top of conservative pecking orders. The assets at the top of other pecking orders are considerably more substantial. After understanding how the liquidation of each type of asset affects stability, we consider shocks that exceed the assets at the top of institutions' pecking orders (so institutions must liquidate the assets that are next in line) in section 3.7.

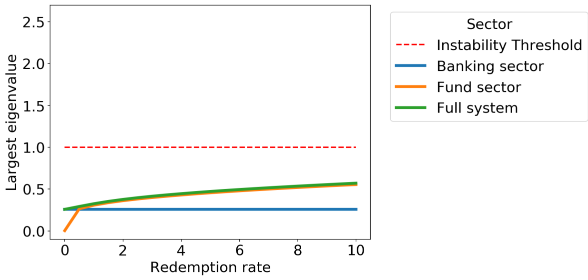
In Figure 4, we plot the largest eigenvalue of the banking sector and fund sector, and of the full system, for the four liquidity-differentiated pecking orders. We set the market depth divisor to its baseline value of $\delta_\sigma = 1$ for all securities and vary the redemption rate R. In Figure 4a, all institutions are assumed to have the optimistic pecking order and in Figure 4d the conservative pecking order. In Figure 4b, all institutions have the short-term funding pecking order, and in Figure 4c all institutions have the government bonds pecking order.

The results show that the optimistic and short-term funding pecking orders are very stable, because the liquidation of the assets that are at the top of these pecking orders does not cause high levels of contagion; even for very high redemption rates no liquidity spiral emerges. Conversely, when government bonds are at the top of the pecking order, as shown in 4c, the banking sector is highly unstable. The main reason for this is that the majority of the government bonds in our data are held by banks, so the price impact caused by selling the bonds is predominantly suffered by the banks, and that the banks strongly amplify any valuation shocks that they incur through their high leverages (Wiersema et al., 2019). Furthermore, 4c shows that for high redemption rates the largest eigenvalue of the full system is substantially higher than the largest eigenvalue of than either of the individual sectors. Hence, the intensity of the liquidity spiral would be underestimated when the interactions between the banking and fund sector are ignored, which highlights the importance of capturing these sectors' combined dynamics.

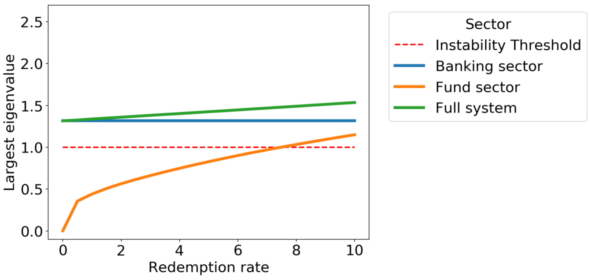
Figure 4d shows that the conservative pecking order is generally more stable than the government bonds pecking order in Figure 4c, but is also more strongly affected by the redemption rate R. The main reason for this is that the miscellaneous tradable securities in our data are mainly held by investment funds, so the funds suffer most of the price impact caused by selling the miscellaneous tradable securities, and contrary to banks, funds only amplify valuation shocks when the redemption rate R is high.



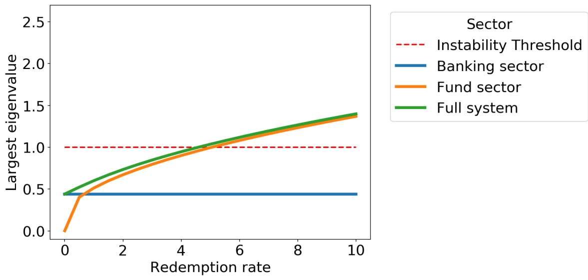
(a) Optimistic Pecking Order



(b) Short-Term Funding Pecking Order



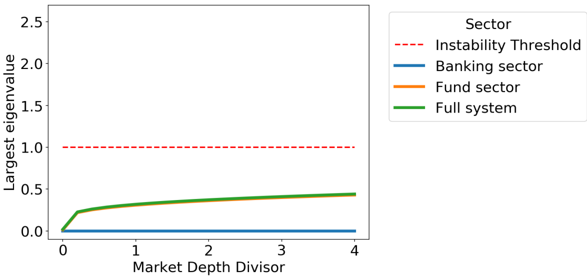
(c) Government Bonds Pecking Order



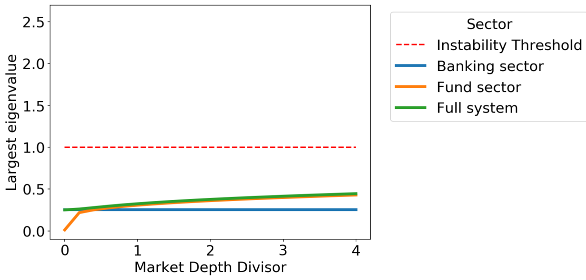
(d) Conservative Pecking Order

Figure 4: **Eigenvalue Dependency on Redemption rates for Liquidity-Differentiated Pecking Orders.** We explore when the largest eigenvalue exceeds one and a liquidity spiral emerges for various redemption rates R and various liquidity-differentiated pecking orders. The market depth divisor is set to its baseline value of $\delta_\sigma = 1$. In (a) all institutions are assumed to have the optimistic pecking order, in (b) the short-term funding pecking order, in (c) the government bonds pecking order and in (d) the conservative pecking order.

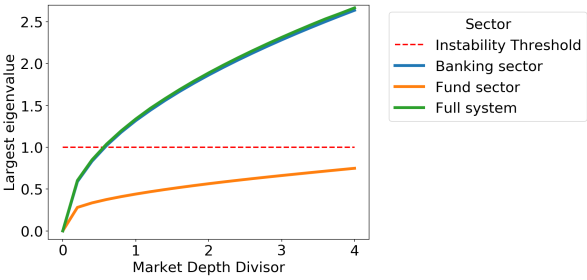
Figure 5 is analogous to Figure 4, but here we vary the market depth divisor δ_σ (for all securities) rather than the redemption rate, which is set to its baseline value of $R = 1$. The results again show the standard and short-term funding pecking orders to be very stable as no liquidity spiral emerges in Figure 5a and Figure 5b even for very illiquid markets (i.e. high δ_σ). When government bonds are at the top of the pecking order, as shown in Figure 5c, the banking sector is again highly unstable, and is strongly affected by the market depth. Similar to what we found in Figure 4, Figure 5d shows that the conservative pecking order is generally more stable than the government bonds pecking order. Furthermore, the eigenvalue of the full system in Figure 5d shows that when markets are very illiquid (i.e. high δ_σ), *a liquidity spiral emerges which is completely overlooked when ignoring the interactions between the banking sector and fund sector* (as neither individual sector has an eigenvalue greater than one).



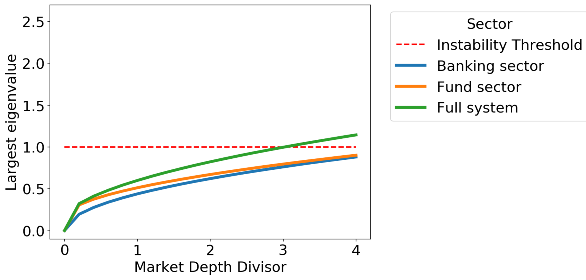
(a) Optimistic Pecking Order



(b) Short-Term Funding Pecking Order



(c) Government Bonds Pecking Order



(d) Conservative Pecking Order

Figure 5: **Eigenvalue Dependency on Market Depths for Liquidity-Differentiated Pecking Orders.** We explore when the largest eigenvalue exceeds one and a liquidity spiral emerges for various market depth divisors δ_σ and various liquidity-differentiated pecking orders. The redemption rate is set to its baseline value of $R = 1$. In (a) all institutions are assumed to have the optimistic pecking order, in (b) the short-term funding pecking order, in (c) the government bonds pecking order and in (d) the conservative pecking order.

3.7 Liquidity Shocks

In Figure 6, we consider how the stability of the system is impacted by a large liquidity shock when all institutions have the optimistic pecking order (which empirical evidence suggests to be the most commonly employed pecking order[14]). As we have seen, the system is very resilient to the liquidation of the assets at the top of the optimistic pecking order, so when a liquidity shock exhausts these assets and forces institutions to liquidate assets that are next in-line on their pecking orders, we can expect the system to become less stable. Figures 6a and 6b show the sensitivity of the largest eigenvalue to the shock size and Figure 6c and Figure 6d show which asset types are at the top of institutions' pecking orders after the shock has exhausted part of their liquid assets.

We consider the marginal impact on stability of an increase in the shock size, by calculating the largest eigenvalue of the system according each institution's assets lowest on its pecking order that it is forced to liquidate in response to the liquidity shock. For simplicity, we assume that the liquidity shock reduces each institution's total liquid asset holdings by the same proportion. Although we can reasonably expect an institution's total liquid asset holdings to be (at least somewhat) calibrated to the magnitude of the

[14]See e.g. Kim (1998), van den End and Tabbae (2012), Ma et al. (2020).

liquidity shocks that the institution expects to incur, liquiditiy shocks come in various distributions and magnitudes in reality. Future research should therefore explore the stability of financial systems across a wide range of liquidity shocks.

In Figure 6a, the redemption rate and market depth divisor are set to their baseline values of $R = 1$ and $\delta_\sigma = 1$. The reduction in institutions' liquid assets due to the liquidity shock is presented as a proportion of the institution's total liquid asset holdings on the x-axis. Note that the proportion cannot exceed one as we do not model illiquidity defaults. Figure 6a shows that the liquidity shock has the potential to greatly destabilize the system, as a liquidity spiral emerges when about half of institutions' liquid assets are exhausted and institutions are forced to liquidate assets lower on their pecking orders.

Figure 6a also shows that the liquidity spiral that emerges when institutions consume their pecking orders is predominantly driven by the banking sector. Comparison with Figure 6c allows us to understand how the gradual depletion of banks' pecking orders causes the liquidity spiral to emerge. Figure 6c shows that all but one bank initially have central bank deposits and/or gold at the top of their pecking orders (as shown by the blue line), and hence the system is initially very stable, but the number of banks with these assets at the top of their pecking order drops off quickly as the shock size increases. Nevertheless, this does not immediately cause instabilities as most of these banks start liquidating interbank deposits and/or repo instead (as shown by the orange line), which we have seen in previous sections to yield a relatively stable system too. The liquidity spiral in 6a only emerges when the number of banks with interbank deposits and/or repo at the top of their pecking orders starts falling too and more and more banks start liquidating government bonds instead (as shown by the green line in 6c). Indeed, we have seen in previous sections that the liquidation of government bonds drives the emergence of liquidity spirals.

Figure 6a also shows that the liquidity spiral dissipates when the shock size approaches the point of fully exhausting institutions' pecking orders. Figure 6c shows that this dissipation coincides with a strong drop in the number of institutions that liquidate interbank deposits and/or repo and a steep rise in the number of institutions that sell miscellaneous tradable securities (as shown by the red line). Interestingly, we previously showed for $R = 1$ and $\delta_\sigma = 1$ that the short-term funding pecking order (which has bank deposits, repo and fund shares at the top) is more stable than the conservative pecking order (which has miscellaneous tradable securities at the top) in a system where all institutions have the same pecking order. Conversely, Figure 6 suggests that in a system where a substantial number of banks have government bonds at the top of their pecking order, withdrawing interbank deposits and repo is more destabilizing than selling miscellaneous tradable securities to meet a liquidity shock. This is a prime example of *emergence*, as the interactions between institutions with different pecking orders yields dynamics the one would not expect based on the individual pecking orders, and highlights the importance of capturing the complex nature of financial systems.

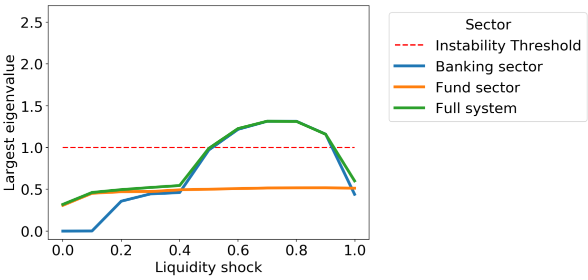
The dissipation of liquidity spirals when institutions' pecking orders are close to exhaustion would be highly advantageous, as it could stabilize the system before institutions default through illiquidity. However, this dissipation may not materialize during adverse market conditions. To illustrate this in Figure 6b, we set the redemption rate to $R = 5$ and the market depth divisor to $\delta_m = 2$ for miscellaneous tradable securities, but to $\delta_g = 1$ for government bonds (i.e. the market depth of miscellaneous tradable securities is halved, while the market depth of government bonds remains at its baseline value). For these market conditions, the liquidity spiral is predominantly driven by the fund sector, rather than the banking sector.

Figure 6d shows that almost all funds initially have bank deposits and/or fund shares at the top of their pecking orders (as shown by the orange line). However, this number falls off sharply as the shock size reaches just 10% of institutions' total liquid asset holdings. This drop coincides with a strong increase in the number of funds that have miscellaneous tradable securities at the top of their pecking orders (as shown by the red line), and with the emergence of a liquidity spiral as show in Figure 6b. The number of funds with bank deposits and/or fund shares at the top of their pecking orders continues to fall, and the number of funds that sell miscellaneous tradable securities continues to rise, until the shock size reaches 100%. The liquidity spiral, rather than dissipate, grows steadily in intensity until the maximum shock size is reached.

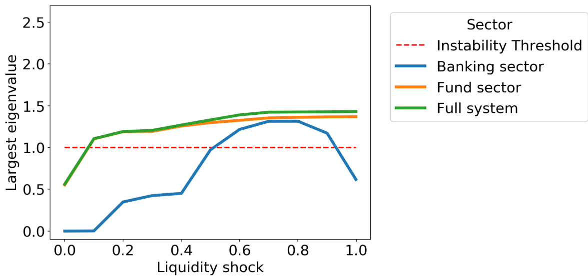
Finally, let us compare the optimistic pecking order in 6a to the uniform pecking order in Figure 3. Comparing largest eigenvalues for the same redemption rate $R = 1$ and market depth modifier $\delta_\sigma = 1$, we find that the optimistic pecking order yields a system more resilient to small shocks than the uniform pecking order. However, the uniform pecking order is not affected by liquidity shocks, as explained in section 3.3.2, while a liquidity spiral emerges in response to large shocks when institutions have the optimistic pecking order. Relative to the uniform pecking order, the optimistic pecking order therefore yields a "robust-yet-fragile" system, which is very resilient to small shocks but may be greatly destabilized by a single, large shock. This is similar to the robust-yet-fragile network topologies identified by Gai and Kapadia (2010), but manifests here in terms of pecking order configurations, and highlights the importance of evaluating the resilience of financial systems against a wide range of shocks.

Note that by focusing on time $t = 1$, we have only considered how a liquidity spiral emerges but not how it evolves as shocks continue to propagate. Even when all institutions have the uniform pecking order, which does not change in response to shocks, the dynamics of the system evolve as shocks propagate and tradable securities change hands (which changes how overlapping portfolio contagion is distributed), shares are redeemed (which dampens the shareholder contagion channel), and additional cash flows enter or leave the system (which may affects institutions' pecking orders). However, modeling such changes to the system would require additional assumption and/or (empirical) research beyond the scope of this paper.

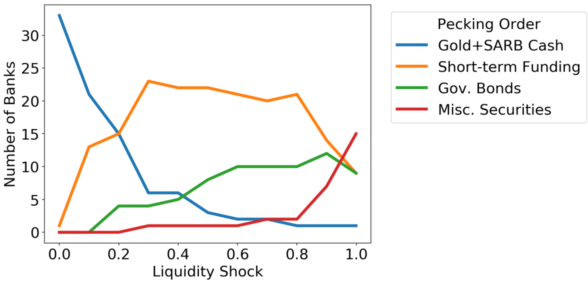
Furthermore, Figure 6 shows that a large liquidity shock may raise the largest eigenvalue above one and cause a liquidity spiral to emerge. However, a smaller shock that pushes the eigenvalue close to, but not outright above one, may continue to propagate and drain institutions' pecking orders until a spiral eventually emerges. Although Figure 6 indicates how the dynamics would evolve as liquidity losses accumulate and pecking orders are progressively depleted by propagating shocks, this does not take into account the changes to the system mentioned in the previous paragraph that are unrelated to the pecking order. To accurately assess how the stability evolves as shocks continue to propagate, more comprehensive modeling approaches are required.



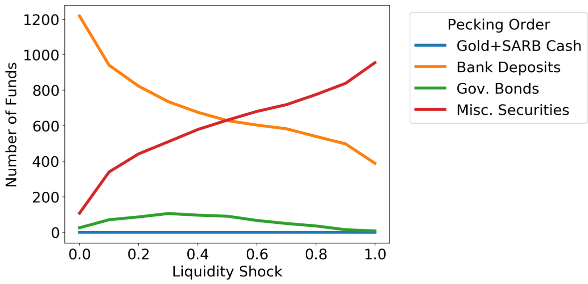
(a) Eigenvalue vs. Liquidity Shock ($R = 1$)



(b) Eigenvalue vs. Liquidity Shock ($R = 5$)



(c) Banks' Top Layer of Pecking Order



(d) Funds' Top Layer of Pecking Order

Figure 6: **Impact of Liquidity Shock on Largest Eigenvalue.** We explore how a large liquidity shock affects stability and may potentially cause the largest eigenvalue to exceed one and a liquidity spiral to emerge. All institution are assumed to have the optimistic pecking order and the reduction in institutions' liquid assets due to the liquidity shock is presented as a proportion of the institution's total liquid assets pool on the x-axis. In (a) the redemption rate is set to its baseline value of $R = 1$ an the market depth divisor to its baseline value of $\delta_\sigma = 1$ for all securities. In (b), the redemption rate is set to $R = 5$. The market depth divisor is set to $\delta_g = 1$ for government bonds g and to $\delta_m = 1$ for all miscellaneous tradable securities m. In (c) and (d) we show the number of banks respectively funds per asset type at the top of their pecking orders depending on the size of liquidity shock incurred. The blue line shows the number of institutions with gold and/or central bank deposits at the top of their pecking order, the orange line the institutions with commercial bank deposits, repo, and/or fund shares at the top of their pecking order, the green line government bonds, and the red line miscellaneous tradable securities.

4 Discussion

Liquidity spirals progressively worsen market and funding liquidity (Brunnermeier and Pedersen, 2009). We have studied liquidity spirals that consist of various interacting contagion channels and/or multiple types of institutions. To accurately assess these spirals, models are required that can take the interactions between multiple types of contagion and institutions into account. We use the framework developed in Wiersema et al. (2019), which allows us to identify liquidity spirals before market and funding liquidity fall progressively. Wiersema et al. (2019)) show in a general setting that financial stability may be greatly overestimated when ignoring the interactions between contagion channels. Here, we demonstrate that liquidity spirals may be completely overlooked when interactions between different types of institutions or contagion channels are ignored.

The framework allows us to evaluate the impact of institutions' pecking orders on the potential for liquidity spirals to emerge without relying on any specific, subjective stress scenario. We show that institutions' pecking orders strongly affect finnacial stability, with some pecking orders yielding a "robust-yet-fragile" system. The robust-yet-fragile tendency of financial systems has been previously observed by Gai and Kapadia (2010) for certain network topologies. Here, we have seen it manifested for specific pecking order configurations. The identification of robust-yet-fragile tendencies of financial systems across multiple dimensions highlights the dangers of optimizing stability with respect to the small shocks that are incurred on a frequent basis; a financial system that has been optimized to be highly resilient against small shocks may turn out to be highly fragile to large shocks once one eventually materializes. Moreover, it underscores the importance of stability measures that assess a system's resilience to a wide range of shocks, such as the eigenvalue-based approach developed here.

We demonstrate our method by applying it to a highly granular data set on the South African financial system and capture the combined dynamics of the banking and investment fund sector. Wiersema et al. (2021) show that exposures in the South African financial system are underestimated when the interactions between the banking and fund sector are ignored. Here, we identify market conditions for which a liquidity spiral emerges that cannot be identified without taking the interactions between the banking and fund sector into account. These results highlight that comprehensive modeling approaches such as the one presented here are vital for understanding financial stability. We also identify market conditions that yield a liquidity spiral which is predominantly driven by one of the two sectors. This greatly affects the effectiveness of interventions such as liquidity injections into the banking sector. Hence, policy makers may employ the model developed here to decide on what strategies may be most effective at combating liquidity spirals.

We have explored how the system's stability changes in response to a large liquidity shock. We show that when institutions sell their most liquid assets first, the system is very resilient against small liquidity shocks. However, a liquidity spiral may emerge as soon as a substantial part of institutions' pecking orders are exhausted by a sizable liquidity shock. This robust-yet-fragile tendency may appear in any financial system where institutions liquidate assets in order of decreasing liquidity, as contagion typically worsens as institutions are forced to liquidate assets of lesser and lesser liquidity. This highlights the importance of exploring financial stability across all layers of institutions' pecking orders.

The evolution of the system in response to the shock depends strongly on the distribution and magnitude of the shock. As we have only considered liquidity shocks that are distributed proportionally to institutions' pecking order depths, future research should aim to formulate realistic stress scenarios and investigate how the system evolves over time in response to these. Furthermore, as we have established the important role that pecking orders play in the potential for liquidity spirals emerge, further empirical investigation into the pecking orders that financial institutions employ under various market conditions is warranted.

References

Acharya, V. V. and Skeie, D. (2011), 'A model of liquidity hoarding and term premia in inter-bank markets', *Journal of Monetary Economics* **58**(5), 436–447.

Adrian, T. and Shin, H. S. (2010), 'Liquidity and leverage', *Journal of financial intermediation* **19**(3), 418–437.

Adrian, T. and Shin, H. S. (2014), 'Procyclical leverage and value-at-risk', *Review of Financial Studies* **27**(2), 373–403.

Allen, F. and Gale, D. (2000), 'Financial contagion', *Journal of political economy* **108**(1), 1–33.

Amini, H., Cont, R. and Minca, A. (2016), 'Resilience to contagion in financial networks', *Mathematical finance* **26**(2), 329–365.

Andrianaivo, M. and Yartey, C. A. (2010), 'Understanding the growth of african financial markets', *African Development Review* **22**(3), 394–418.

Aymanns, C., Caccioli, F., Farmer, J. D. and Tan, V. W. (2016), 'Taming the basel leverage cycle', *Journal of financial stability* **27**, 263–277.

Bardoscia, M., Battiston, S., Caccioli, F. and Caldarelli, G. (2017), 'Pathways towards instability in financial networks', *Nature Communications* **8**(1), 1–7.

Bernardo, A. E. and Welch, I. (2004), 'Liquidity and financial market runs', *The Quarterly Journal of Economics* **119**(1), 135–158.

BIS (2013), Basel iii: The liquidity coverage ratio and liquidity risk monitoring tools, Technical report, Bank of International Settlements.

BIS (2014), Basel iii: the net stable funding ratio, Technical report, Bank of International Settlements.

Board, F. S. (2014), Board notice 90 of 2014, Technical report.

Borio, C., Drehmann, M. and Tsatsaronis, K. (2014), 'Stress-testing macro stress testing: does it live up to expectations?', *Journal of Financial Stability* **12**, 3–15.

Bouchaud, J.-P. and Cont, R. (1998), 'A langevin approach to stock market fluctuations and crashes', *The European Physical Journal B-Condensed Matter and Complex Systems* **6**(4), 543–550.

Bouchaud, J.-P., Farmer, J. D. and Lillo, F. (2009), How markets slowly digest changes in supply and demand, *in* 'Handbook of financial markets: dynamics and evolution', Elsevier, pp. 57–160.

Brandi, G., Di Clemente, R. and Cimini, G. (2018), ‘Epidemics of liquidity shortages in interbank markets’, *Physica A: Statistical Mechanics and its Applications* **507**, 255–267.

Brunnermeier, M. K. (2008), Deciphering the liquidity and credit crunch 2007-08, Technical report, National Bureau of Economic Research.

Brunnermeier, M. K. and Pedersen, L. H. (2009), ‘Market liquidity and funding liquidity’, *Review of Financial studies* **22**(6), 2201–2238.

Caccioli, F., Farmer, J. D., Foti, N. and Rockmore, D. (2013), ‘How interbank lending amplifies overlapping portfolio contagion: A case study of the austrian banking network’, *arXiv preprint arXiv:1306.3704* .

Caccioli, F., Farmer, J. D., Foti, N. and Rockmore, D. (2015), ‘Overlapping portfolios, contagion, and financial stability’, *Journal of Economic Dynamics and Control* **51**, 50–63.

Caccioli, F., Shrestha, M., Moore, C. and Farmer, J. D. (2014), ‘Stability analysis of financial contagion due to overlapping portfolios’, *Journal of Banking & Finance* **46**, 233–245.

Cont, R., Kotlicki, A. and Valderrama, L. (2020), ‘Liquidity at risk: Joint stress testing of solvency and liquidity’, *Journal of Banking & Finance* **118**, 105871.

Cont, R. and Schaanning, E. (2017), Fire sales, indirect contagion and systemic stress testing, Technical report, Norges Bank Working Paper.

Cont, R. and Schaanning, E. (2019), ‘Monitoring indirect contagion’, *Journal of Banking & Finance* **104**, 85–102.

Cont, R. and Wagalath, L. (2013), ‘Running for the exit: distressed selling and endogenous correlation in financial markets’, *Mathematical Finance* **23**(4), 718–741.

De Haan, L. and van den End, J. W. (2013), ‘Banks’ responses to funding liquidity shocks: Lending adjustment, liquidity hoarding and fire sales’, *Journal of International Financial Markets, Institutions and Money* **26**, 152–174.

De Santis, R. A. (2014), ‘The euro area sovereign debt crisis: Identifying flight-to-liquidity and the spillover mechanisms’, *Journal of Empirical Finance* **26**, 150–170.

Detering, N., Meyer-Brandis, T., Panagiotou, K. and Ritter, D. (2021), ‘An integrated model for fire sales and default contagion’, *Mathematics and Financial Economics* **15**(1), 59–101.

Diamond, D. W. and Dybvig, P. H. (1983), 'Bank runs, deposit insurance, and liquidity', *The Journal of Political Economy* **91**(3), 401–419.

Duarte, F. and Eisenbach, T. M. (2018), 'Fire-sale spillovers and systemic risk', *FRB of New York Staff Report* (645).

Eisenberg, L. and Noe, T. H. (2001), 'Systemic risk in financial systems', *Management Science* **47**(2), 236–249.

Farmer, J. D., Kleinnijenhuis, A. M., Nahai-Williamson, P. and Wetzer, T. (2020), 'Foundations of system-wide financial stress testing with heterogeneous institutions', *Bank of England Staff Working Paper No. 861* .

Fostel, A. and Geanakoplos, J. (2008), 'Leverage cycles and the anxious economy', *The American Economic Review* **98**(4), 1211–1244.

FSB (2018), Global shadow banking monitoring report 2017, Technical report, Financial Stability Board.

Gai, P. and Kapadia, S. (2010), Contagion in financial networks, *in* 'Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences', The Royal Society, p. rspa20090410.

Gatheral, J. (2010), 'No-dynamic-arbitrage and market impact', *Quantitative finance* **10**(7), 749–759.

Geanakoplos, J. (2010), 'The leverage cycle', *NBER macroeconomics annual* **24**(1), 1–66.

Gorton, G. and Metrick, A. (2012), 'Securitized banking and the run on repo', *Journal of Financial economics* **104**(3), 425–451.

Hałaj, G. (2018), 'System-wide implications of funding risk', *Physica A: Statistical Mechanics and its Applications* **503**, 1151–1181.

Kemp, E. (2017), 'Measuring shadow banking activities and exploring its interconnectedness with banks in south africa', *SARB Occassional Papers* .

Kim, C. E. (1998), 'The effects of asset liquidity: Evidence from the contract drilling industry', *Journal of financial Intermediation* **7**(2), 151–176.

Kok, C. and Montagna, M. (2013), Multi-layered interbank model for assessing systemic risk, Technical report, European Central Bank.

Ma, Y., Xiao, K. and Zeng, Y. (2020), 'Mutual fund liquidity transformation and reverse flight to liquidity', *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper* .

Poledna, S., Molina-Borboa, J. L., Martínez-Jaramillo, S., Van Der Leij, M. and Thurner, S. (2015), 'The multi-layer network nature of systemic risk and its implications for the costs of financial crises', *Journal of Financial Stability* **20**, 70–81.

Rochet, J.-C. and Vives, X. (2004), 'Coordination failures and the lender of last resort: was bagehot right after all?', *Journal of the European Economic Association* **2**(6), 1116–1147.

Roukny, T., Bersini, H., Pirotte, H., Caldarelli, G. and Battiston, S. (2013), 'Default cascades in complex networks: Topology and systemic risk', *Scientific reports* **3**, 2759.

SARB (2016), Banks ba900 economic returns, Technical report, South African Reserve Bank.

SARB (2017), Annual report 2016, Technical report.

Thurner, S., Farmer, J. D. and Geanakoplos, J. (2012), 'Leverage causes fat tails and clustered volatility', *Quantitative Finance* **12**(5), 695–707.

van den End, J. W. and Tabbae, M. (2012), 'When liquidity risk becomes a systemic issue: Empirical evidence of bank behaviour', *Journal of Financial Stability* **8**(2), 107–120.

Wiersema, G., Kleinnijenhuis, A. M., Kemp, E. and Wetzer, T. (2021), 'Higher-order exposures', *Available at SSRN 3914076* .

Wiersema, G., Kleinnijenhuis, A. M., Wetzer, T. and Farmer, J. D. (2019), 'Scenario-free analysis of financial stability with interacting contagion channels'.

A Appendix

A.1 Liquidity Spirals Driven by Funding Contagion

Here, we show that the funding contagion channel, given by the upper-left quadrant of the shock transmission matrix, is the only contagion channel out of all channels considered in this analysis that can cause a liquidity spiral in the absence of other contagion channels: From the properties of block matrices, we know that the largest eigenvalue is zero when the only non-empty contagion quadrant is an off-diagonal quadrant. Hence, neither deleveraging contagion (upper-left quadrant) nor overlapping portfolio contagion (lower-right quadrant) can individually drive the emergence of a liquidity spiral. Furthermore, when the lower-right quadrant, i.e. the share redemption and shareholder contagion quadrant, is the only non-empty quadrant, the largest eigenvalue is positive but only valuation shocks propagate (while liquidity shocks dissipate immediately, as can be seen from eq. 4). Hence, the only quadrant of the shock transmission matrix that can cause a liquidity spiral when all other quadrants are zero is the upper-left quadrant, i.e. the funding contagion channel.

Furthermore, in the absence of other contagion channels, the funding contagion channel can only cause a liquidity spiral when banks hoard liquidity in response to shocks; when a bank does *not* hoard liquidity, it only withdraws deposits to meet the liquidity shock it incurred up to the magnitude of the shock. Hence, the aggregate liquidity that the bank withdraws from other banks does not exceed the liquidity shock incurred, so the bank does not amplify the propagating shock. The sum of a column in the funding contagion quadrant of the shock transmission matrix gives the aggregate liquidity that a bank withdraws from other banks as a fraction of the liquidity shock that the bank incurred (see eq. 4). Hence, absent liquidity hoarding, column sums in the funding contagion quadrant cannot exceed one. Furthermore, the Perron-Frobenius theorem guarantees that the largest eigenvalue of shock transmission matrix does not exceed the largest column sum. Therefore, absent other contagion channels, i.e. the largest eigenvalue is given by the funding contagion quadrant, and absent liquidity hoarding, i.e. the column sums in the funding contagion quadrant cannot exceed one, the largest eigenvalue is upper-bounded by one and no liquidity spiral can emerge. Hence, when banks do not hoard liquidity, studying contagion channels in isolation, and thus ignoring their interactions, as is often done, will overlook any liquidity spiral even in the most unstable of systems.

A.2 Withdrawal of Fund Shares

When we discussed the share redemption contagion channel in section 3.3, we assumed that the amount of liquidity withdrawn by external investors from the investment fund is linear in the NAV loss of the fund's shares. Here, we show that this assumption implies that the number of shares withdrawn is a convex function of the fund's NAV loss.

When investment fund i suffers a loss $x^v_{i,t}$, the NAV of the fund's shares falls by

$$\Delta NAV_{i,t} = x^v_{i,t}/S_{i,t}, \tag{12}$$

where $S_{i,t}$ denotes i total number of outstanding shares at time t. Furthermore, the amount paid out per share that investors redeem is given by

$$NAV_{i,t+1} = NAV_{i,t} - \Delta NAV_{i,t}, \tag{13}$$

and we denote the number of shares withdrawn by the external investors in response to the NAV loss $\Delta NAV_{i,t}$ as $\Delta S_{i,t}$. The amount paid out for the redeemed shares gives the liquidity withdrawn from the fund;

$$(NAV_{i,t} - \Delta NAV_{i,t})\,\Delta S_{i,t} = \epsilon_{i,t} R x^v_{i,t}, \tag{14}$$

where we have used the factor $\epsilon_{i,t}R$ from equation (7) to express the amount of liquidity withdrawn in terms of the loss $x^v_{i,t}$. Using (12), we find that

$$\Delta S_{i,t} = \min\left\{\frac{\epsilon_{i,t} R S_{i,t} \Delta NAV_{i,t}}{NAV_{i,t} - \Delta NAV_{i,t}}, \epsilon_{i,t} S_{i,t}\right\}, \tag{15}$$

where we have used that $\Delta S_{i,t} \leq \epsilon_{i,t} S_{i,t}$ (as only the fraction of i's shares held by external holders can be withdrawn through shareholder contagion). Hence, the number of shares withdrawn in response to the NAV loss is linear for small NAV losses and convex for larger NAV losses (until the upper bound of $\Delta S_{i,t}$ is fixed).

A.3 Price Impact of Number of Shares Sold

When we discussed the overlapping portfolio contagion channel in section 3.3, we assumed that the price impact $\Delta p_{\sigma,t}$ is linear in the liquidity recovered from the sale. Here, we show that this assumption implies that the price impact is concave in the number of shares sold.

Let us assume for simplicity that institution i is the only institution that sells shares in security σ. The derivation generalizes straightforwardly to the case when multiple institutions sell shares in σ at the same time. For notational convenience, we assume that security σ is the only asset at the top of institution i's pecking order (and that the shock $x^l_{i,t}$ does not exhaust the asset), such that the price impact (9) reduces to

$$\Delta p_{\sigma,t} = \frac{x^l_{i,t}}{D_\sigma} = \frac{(p_{\sigma,t} - \Delta p_{\sigma,t})\,\Delta n_{\sigma i,t}}{D_\sigma}, \tag{16}$$

where $\Delta n_{\sigma i,t}$ denotes the number of shares in security σ that i sells at time t to raise liquidity $x^l_{i,t}$ and we have used the assumption that all shares are sold against the new price $p_{\sigma,t+1} = p_{\sigma,t} - \Delta p_{\sigma,t}$. Rewriting equation (16), we find the price impact as a function of the number of shares sold:

$$\Delta p_{\sigma,t} = \frac{p_{\sigma,t}\Delta n_{\sigma i,t}}{D_\sigma + \Delta n_{\sigma i,t}}, \tag{17}$$

which is linear in the number of shares sold when $\Delta n_{\sigma i,t}$ is small (similar to e.g. Cont and Schaanning, 2019 and Wiersema et al., 2019) and concave in the number of shares sold when $\Delta n_{\sigma i,t}$ is large (see e.g. Gatheral, 2010). Furthermore, note from equation (17) that $\Delta p_{\sigma,t} < p_{\sigma,t}$ so the price cannot become negative.

A.4 Market Price of Listed Equity Shares

The overlapping portfolio and shareholder contagion mechanisms derived in section 3.3 should not drive the market price of a listed equity share below zero, as the shares are subject to limited liability. This is guaranteed when the contagion mechanisms act in isolation. Here, we show that the combined impact of the two contagion channels also cannot the market price of listed equity shares issued by South African banks below zero.

Remember that we have assumed that the overlapping portfolio contagion channel and shareholder contagion channels are additive. Therefore, the market price of a tradable security σ evolves according to

$$p_{\sigma,t+1} = p_{\sigma,t} - \Delta p^s_{\sigma,t} - \Delta p^o_{\sigma,t}, \tag{18}$$

where $\Delta p^s_{\sigma,t}$ denotes the drop in market price due to the shareholder contagion channel, and $\Delta p^o_{\sigma,t}$ denotes the drop in market price due to the overlapping portfolio contagion channel. To demonstrate that neither channel can drive the market price below zero when both channels interaction, we first show that $\Delta p^o_{\sigma,t} \leq p_{\sigma,t} - \Delta p^s_{\sigma,t}$, i.e. overlapping portfolio contagion does not drive the market price below zero even when it has already been depressed by shareholder contagion. Second, we discuss why $\Delta p^s_{\sigma,t} \leq p_{\sigma,t}$, i.e. shareholder contagion does not drive the market price below zero even when the market price has been depressed by overlapping portfolio contagion at previous times.

Because we have assumed that all tradable securities σ sold at time t are sold against the new price $p_{\sigma,t+1}$, the liquidity recovered from the sale is reduced by the shareholder contagion $\Delta p^s_{\sigma,t}$. Hence, equation (16) becomes

$$\Delta p^o_{\sigma,t} = \frac{p_{\sigma,t+1}\Delta n_{\sigma i,t}}{D_\sigma} = \frac{\left(p_{\sigma,t} - \Delta p^s_{\sigma,t} - \Delta p^o_{\sigma,t}\right)\Delta n_{\sigma i,t}}{D_\sigma}, \tag{19}$$

and rewriting yields

$$\Delta p^o_{\sigma,t} = \frac{\left(p_{\sigma,t} - \Delta p^s_{\sigma,t}\right) \Delta n_{\sigma i,t}}{D_\sigma + \Delta n_{\sigma i,t}}, \tag{20}$$

so we find that $\Delta p^o_{\sigma,t} \leq p_{\sigma,t} - \Delta p^s_{\sigma,t}$. Hence, the shareholder contagion channel depresses the overlapping portfolio contagion channel by reducing the price impact per share sold, similar to Wiersema et al. (2021).

To guarantee that the shareholder contagion channel cannot drive the market price of equity shares below zero when the price has already been depressed by the overlapping contagion channel at a previous time, we should multiply equation (6) by the shares' market-to-book ratio; let $E_{\sigma,t}$ denote the equity at time t of the institution that issued the shares, and $S_{\sigma,t}$ the total number of shares that the institution issued, such that a share's book value is given by $E_{\sigma,t}/S_{\sigma,t}$ and market-to-book ratio by $p_{\sigma,t}S_{\sigma,t}/E_{\sigma,t}$. Multiplying equation (6) with the market-to-book ratio yields

$$A^{vv}_{\sigma i,t} = s_{i\sigma,t}\frac{p_{\sigma,t}}{E_{\sigma,t}}, \tag{21}$$

such that when institution σ suffers a loss $x^v_{\sigma,t} = E_{\sigma,t}$, the shareholder contagion suffered by i equals $s_{i\sigma,t}p_{\sigma,t}$. Hence, when a shock exhausts institution σ's equity, institution i loses the current market value of its position in shares issued by σ.

The market-to-book ratio reflects that the overlapping portfolio contagion channel has depressed the equity shares' market value below their book value, such that the impact of the shareholder contagion channel is reduced (Wiersema et al., 2021). However, the results in this paper are derived for time $t = 1$, when the shares' market values are assumed to be equal to their book values, so the market-to-book ratio $p_{\sigma,t}S_{\sigma,t}/E_{\sigma,t} = 1$ and can be omitted from equation (6) for simplicity.

A.5 Baseline Market Depth Estimates

We estimate baseline market depths $\hat{D}_\sigma$ for six different classes σ of domestic tradable securities: Government bonds, listed equity shares and bonds issued by the non-financial corporate sector, and MMIs, listed equity shares and bonds issued by the banking sector. Due to data limitations, we do not distinguish between tradable securities of a specific type issued by different non-financial corporates, nor between tradable securities of a specific type issued by different domestic banks. For example, all domestic bank bonds are assumed to have the same market depth, and selling a bank bond is assumed to cause the same price impact across all bonds issued by any domestic bank.

We set our baseline estimate of a security's market depth equal to its market capitalization divided by its initial price (similar to e.g. Wiersema et al., 2019, 2021). As initial prices are normalized, division by the initial price simply serves to make the baseline market depth estimate dimensionless. For (domestic) government bonds, we use the market capitalization of South African government bonds at the end of 2016[15]. Note that the banks and funds own 21% of the market capitalization of the South African government bonds. Due to data limitations, we estimate market capitalizations by assuming that the banks and funds own the same fraction of 21% of other securities' market capitalizations. Hence, the market capitalization of any security class is given by banks' and funds' aggregate holdings of the security class divided by 21%.

[15]The market capitalization of South African bonds as of Q4 2016 is sourced from the Q1 2017 SARB Quarterly Bulletin; https://www.resbank.co.za/content/dam/sarb/publications/quarterly-bulletins/quarterly-bulletin-publications/2017/7718/07Statistical-tables—Public-Finance.pdf.