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Agent-Based Modeling in Economics and Finance: Past, Present, and Future

Robert L. Axtell and J. Doyne Farmer

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Agent-Based Modeling in Economics and Finance: Past, Present, and Future∇

Robert L. Axtell*%& and J. Doyne Farmer&^§

* Computational Social Science Ph.D. Program
Department of Economics
Department of Computational and Data Sciences
George Mason University
Fairfax, Virginia

%Google Research

& Santa Fe Institute
Santa Fe, New Mexico

^ Complexity Economics Programme
Institute for New Economic Thinking at the Oxford Martin School
University of Oxford
Oxford, UK

§ Mathematical Institute
University of Oxford
Oxford, UK

Abstract

Agent-based modeling (*ABM*) is a novel computational methodology for representing the behavior of individuals in order to study social phenomena. Its use is rapidly growing in many fields. We review *ABM* in economics and finance and highlight how it can be used to relax conventional assumptions in standard economic models. In economics, *ABM* has enriched our understanding of markets, industrial organization, labor, macro, development, environmental and resource economics, as well as policy. In financial markets, substantial accomplishments include understanding clustered volatility, market impact, systemic risk and housing markets. We present a vision for how *ABM*s might be used in the future to build more realistic models of the economy and review some of hurdles that must be overcome to achieve this.

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I. Introduction: What is Agent-Based Modeling?

The emerging computational methodology of agent-based modeling (*ABM*) can improve the explanatory power of economics by permitting the relaxation of assumptions that are commonly used in mathematical models for reasons of analytical tractability. This article is an introduction to *ABM* in economics and finance, and to some of its major accomplishments so far, the current state of the art, and promising new directions.

Agent-based computing goes by many names and acronyms depending on the field in which it is employed—it is called *agent-based modeling* in most of the social sciences, *multi-agent systems* (*MAS*) in computer science, *individual-based modeling* (*IBM*) in ecology. It refers to a class of computational techniques that have proven useful as a way to represent individual behavior for purposes of studying social phenomena.[1] Models of this type feature a population of objects called *agents*, which are typically heterogeneous and situated in an economic or social environment. The individual agents are given explicit rules of behavior, which can be quite general—as in 'seek greater utility'—or very specific (e.g., 'lower prices by *5%* if inventory exceeds target levels'). The agents interact directly with one another through social, spatial, or physical networks that are either exogenously specified or endogenously generated. Such models may produce conventional agent-level equilibria (e.g., Nash equilibria), or can yield perpetual dynamics at the micro-level as agents constantly adjust their behaviors. Importantly, the aggregate level is not explicitly pre-specified. Rather it *emerges* from the myriad interactions of the agents. The majority of models of this type implement the agents as software objects, each with local state information or data, their rules of behavior being functions or methods of the objects. Economists who employ these methods sometimes use the phrase *agent-based computational economics* (*ACE*), and we shall treat this expression and its acronym as synonyms for *ABM*.[2] This survey covers the use of such computational agents in economics and finance over the past 60 years, reviewing what

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[1] At the dawn of the digital computation era economists, operations researchers, game theorists, and other social scientists were among the early adopters of computers (e.g., Mirowski, 2001). Early uses included the manipulation of large matrices in input-output models (Leontief, 1951), statistical analyses of economic data (Brown, Houthakker and Prais, 1953), estimation of macro models from aggregate data (Klein and Goldberger, 1955), linear programming (Dorfman, 1951, Dorfman, Samuelson and Solow, 1958) and other kinds of mathematical programming (Kemeny, Morgenstern and Thompson, 1956), numerical solution of analytically intractable microeconomic models (Cohen and Cyert, 1961), and even theorem proving (Newell and Simon, 1956). It is also the case that *novel* uses of digital computing—beyond numerical analysis, statistical methods, and mathematical logic—soon appeared. Specifically, Orcutt and co-workers (1961) pioneered *microsimulation* models, a type of stochastic simulation that remains in use today, primarily for policy purposes (e.g., Bergmann, Eliasson and Orcutt (1980), Urban-Brookings tax model (Rohaly, Carasso and Saleem, 2005)). At about this same time the *Carnegie School* invigorated the theory of the firm via computational models of intra-firm (organizational) behavior (Cyert and March, 1963). *System dynamics* (*SD)* tools were developed at MIT in the 1950s and '60s (Forrester, 1958, 1969), primarily as a way of modeling aggregate stocks and flows of various socially-relevant variables, often from a policy perspective. While some early applications of *SD* (e.g., Meadows *et al*., 1972) proved controversial among economists (Ridker, 1973), it has been broadly applied in a variety of contexts (Sterman, 2000). Some of these new uses of digital computation fell under the broad category of *simulation*, and within operations research (*OR*) a methodology for *discrete event simulation* (e.g., Conway, Johnson and Maxwell, 1959) arose for analyzing military and business processes. The post World War II era also gave birth to *artificial intelligence* (*AI*), a major thematic area of computer science, with strong cross-over into psychology and the behavioral sciences via the birth of cognitive science (Newell and Simon, 1972).

[2] See the *Handbook of Computational Economics,* vol. 2, edited by Tesfatsion and Judd (2006) and vol. 4, Hommes and LeBaron eds. (2018). Arguably, the modifier 'computational' is redundant in *ACE* if one understands 'agent' in *ABM* as referring to software agents.

has been accomplished, describing current practice and looking to future prospects, while assessing potential bottlenecks to further progress.[3]

This is a propitious time for *ABM* due to a confluence of factors. Continued advances in computing hardware, largely driven by Moore's law (Nagy *et al*., 2011), makes bigger models ever more feasible. (See the final section). When combined with the growing quantity and availability of high quality data (Einav and Levin, 2014) from both administrative and commercial sources, large-scale, empirically-grounded *ABM*s are becoming possible and have recently begun to appear. Furthermore, the ability to express behavior algorithmically, grounded in data from laboratory (e.g., Hommes, 2011) or field experiments, offers expanding opportunities to express economic processes computationally, with greater verisimilitude than conventional analytic models.

For these and other reasons we review below, we forecast growing utilization of *ABM* by economists. *ABM* provides a novel, flexible technology that is capable of rendering models in the conventional vocabulary of maximizing agents who face constraints. However, it also permits relaxation of assumptions commonly made in theoretical models for mathematical tractability, and so provides an alternative when realism is more important than conceptual simplicity. By facilitating the generalization of mathematical models through more realistic and behaviorally-grounded representations of human behavior, *ABM* is well-positioned for growth in all the social sciences as computing technologies progressively penetrate most spheres of research even further.

With a few caveats, we see agent-based modeling as a complement rather than a substitute for conventional economic modeling. There are many problems that are difficult to address with conventional methods that are naturally addressed by *ABM*, and vice versa. There are also many problems where the two are coming into direct competition, but *ABM*'s strengths and weaknesses are sufficiently different than those of conventional methods that it provides useful diversity in the space of solutions. Where the two are in competition, the accumulation of empirical evidence will eventually decide which, if any, of the two approaches become dominant. The time has come to make a more serious effort to develop better *ABM*s and begin accumulating such evidence.

A. *ABM* as Computationally-Enabled Economics, from the Bottom Up

Essentially all of the sciences today have been revolutionized by digital computation. Computers are used to generate numerical solutions to analytically intractable mathematical specifications, to relax unrealistic assumptions, and to systematically integrate newly available data into models. Biologists now model whole cells computationally (Karr *et al*., 2012, Waltemath *et al*., 2016, Goldberg *et al*., 2018), consisting of hundreds of thousands of chemical reactions. They simulate the role of all genes in large-scale regulatory (Zhu *et al*., 2008) and signaling (Pawson, 1995, Janes and Lauffenburger, 2013) networks . Brain models consisting of billions of digital neurons are now possible (Markram, 2006, 2012), with the hope of eventually representing cognition in biological terms. In chemistry, medically-significant compounds are modeled computationally (Lewars, 2011) in advance of being synthesized in the lab. In fluid mechanics, analytically intractable turbulent flows can now be rendered

[3] This article is *not* a tutorial on how to create an *ABM* in software. Gilbert (2008) is an overview of the agent modeling process. Good textbooks now exist on *ABM* programming (Railsback and Grimm, 2011, Wilensky and Rand, 2015). See Appendix 3 of this article for background on how to create *ABMs* in software.

computationally with great verisimilitude, with eddies and vortices emerging from local conditions in flow fields (Hoffman and Johnson, 2007, Duraisamy, Iaccarino and Xiao, 2019). In astrophysics and planetary science large-scale computational models play an essential role in explaining everything from the dynamics of galaxies (Norman *et al.*, 1996, Binney and Tremaine, 2008) to the origin of Earth's moon (Canup and Asphaug, 2001, Canup, 2012). Supercomputing is an essential tool in atmospheric physics and oceanography (Gneitling and Raftery, 2005, Edwards, 2010). In all of these areas the basic processes that operate at the microscopic level are reasonably well-understood, but the emergence of novel structures at higher levels of organization is not. The digital computer has revolutionized science by giving us a tool for understanding such emergent phenomena.

This is increasingly true in economics as well. While the number of theorems, corollaries, lemmas, and formal propositions appearing in economic journals has increased monotonically over the last half century[4], it is frequently either difficult or impossible to arrive at closed-form, analytical solutions. This forces us to resort to numerical methods (Varian, 1992, Amman, Kendrick and Rust, 1996, Judd, 1998). *Numerical economics* is thus seen as a *complement* to extant economic theory (Judd, 1997). While this style of computational economics provides explicit solutions to particular problems, its ultimate utility depends on the veracity of the underlying equations being solved. Such equations are often highly idealized, assuming continuity, smoothness, perfect arbitrage, rationality, and so on, which sometimes makes the relevance of the computed solutions to real economies unclear.[5]

ABM is a different kind of computationally-enabled economics. Instead of starting with equations governing an economic process, derived from assumptions that may have been made for analytical tractability, one constructs a computer model, which imposes a very different set of constraints. Building such a model normally begins by specifying a population of agents and their behavioral repertoire. Instead of solving equations for equilibrium one simply lets the agents interact with one another; the behavioral rules produce specific agent behaviors and new agent states, and the system evolves as a dynamical system, from one state to another. Successful models produce states that are relevant to the economic process being modeled. Unsuccessful models do not. Each realization of an *ABM* is a *sufficiency theorem* (Newell and Simon, 1972): IF agents start with certain initial states, S, and engage in specific behaviors, B, THEN after some number of interactions, N, they will have definite new states, $S' = B(S, N)$; (S, B, N) are sufficient to produce S'. While *ABMs* often have pseudo-stochastic elements, each model run is typically deterministic, in the sense that a given initial condition and random number seed always results in the same behavior. Theorems of this type have limited generality but by making many runs of a specific model and allowing random seeds or other stochastic elements to vary, as in Monte Carlo simulation, the generality of the

[4] As a crude measure of this we count 45 such statements in the Jan. 2016 issue of the *American Economic Review*, 38 in the Jan. 2006 issue, 20 in the Jan. 1996 number, 18 in Jan. 1986, 6 in Jan. 1976, 4 in Jan. 1966, 2 in Jan. 1956, and none in previous Jan. issues of 1946, 1936, 1926 and 1916.

[5] For example, in Judd's (1998) textbook heterogeneous agents are only encountered in the last chapter, and only for $N = 2$ such agents.

model results can be assessed and distributional properties of agent states characterized. Below we list a few features that *ABMs* often possess:

- One or more populations of agent software objects, each agent representing an individual or group (e.g., a firm) with local state information (e.g., income);
- Agent behavioral specifications that are conditional on the state of the agent, thus making behavior heterogeneous across the population, and involving either direct or indirect interactions with other agents;
- An external environment that agents are embedded in, e.g. aggregate economic variables (e.g., prices, interest rates), interaction networks, or a spatial landscape;
- A scheme for agent updating;
- Data gathering and statistical facilities for assessing the state and behaviors of the agents and environment;
- A visualization engine or descriptive statistics to depict the activity of the agents in a way that is meaningful for assessing that the model is performing as desired;

ABMs are often run in an integrated development environment that permits the user to analyze and debug her model, providing facilities for making large numbers of runs by varying parameters and storing a record of each simulation.

It is also interesting to note what is normally *not* present in *ABMs*. Because *ABM*s are simulated at the micro level, there are typically no equations that explicitly relate aggregate states to agent-level states (even if they may exist implicitly via aggregation and accounting relations). Although it is common for individual agents to have mathematical representations of their environment as they try to decide how to behave, this is not necessary, and such representations may be quite different than rational expectations. While an *ABM* may evolve into a configuration that is close to an equilibrium, this comes about as a consequence of the behavioral rules the agents are following and agent-level interactions rather than being imposed by fiat.

The existence of this new approach to economic modeling comes at a time when behavioral and experimental economics have made substantial progress and offer novel ways for representing what humans actually do (Gigerenzer and Selten, 2001). *ABM* is well suited to take full advantage of our new knowledge of human behavior. Traditional models in economics begin by assigning a utility function (or some other form of preferences) to each agent and then deriving agent behavior from the assumption that each agent maximizes her utility. *ABM* takes a more agnostic approach. Agents may attempt to maximize utility but they may not succeed in doing so. Or they could have optimal response functions to given payoffs but they might make errors. Alternatively, *ABM*s often specify agent behavioral rules directly, e.g. by stipulating heuristics that agents may select from or learning algorithms that allow their behaviors to adapt. Because such rules do not require complicated optimization, they tend to be computationally more tractable (Michalewicz and Fogel, 2000). Gigerenzer (2000) and others (e.g., Kahneman, 2011) have also argued that such specifications can provide a more accurate model of human behavior. Incorporating more realistic models of human behavior offers rich new possibilities for *ABM*.

Exciting developments in the science of networks (Vega-Redondo, 2007, Jackson, 2008) provide a new lens to look at local interactions at the same time computational tools for network analysis and visualization have become available. For many applications agent-based models can be viewed as a generalization of network models. In a network model one specifies nodes, which might for example be agents, and links, which might be the agent interactions. In an *ABM* one adds agent behaviors and interaction rules so that the dynamics can be simulated. There is thus a natural progression from simple network models, to simple *ABM*s, to more complicated and realistic *ABM*s. We will provide an example of the success of this approach when we discuss systemic risk in finance in section II.B.

It is important to stress that the universe of *ABM*s is itself very heterogeneous, ranging from very simple toy models to complicated models that attempt to incorporate many features of the real world. One can distinguish three types: (1) purely theoretical models with little or no empirical ambition, illustrating a particular mechanism or shedding light on qualitative, stylized fact(s); (2) models that quantitatively reproduce aggregate economic data, and which serve to link micro and social levels; (3) models in which micro-data are used to quantitatively identify the behavior of individuals through calibration or estimation procedures in order to recreate or predict important economic phenomena. Efforts to build models at level (3) are still in their infancy, but we think this approach has huge potential to make successful economic predictions and analysis; Lustick and Miodownik (2009) make a similar point for *ABM*s in politics. We will review models at all three levels.

Increasing availability of micro-data (e.g., Mian, Rao and Sufi, 2013) potentially gives us the ability to measure the full range of heterogeneity present in populations and to incorporate it into models at level (3) (Guvenen, 2011) in ways that may not be easily achievable through more traditional approaches to macroeconomics. These new capabilities may help economists use new kinds of behavioral specifications and other abstractions in order to realize new types of models that some have called for (e.g., Kristol and Bell (1981), Kirman (1989, 2010), Hahn (1992), Stiglitz (2009), Farmer and Foley (2009), Colander et al. (2009), and Trichet (2010)).

B. Usefulness of *ABM* for Research in Economics and Finance

Models are idealizations. Ideal types are *not* exact statements about how the world works but rather approximations that are easy to write down and analyze. They facilitate modeling. An abstraction's value may derive from the closeness with which it approximates reality. Alternatively it may be such a gross approximation as to be empirically false but valuable nonetheless because it permits a model that is otherwise intractable to be solved. Each of the modeling abstractions we teach economics graduate students has a mixture of these features: it approximates reality to some degree while facilitating analysis. The status of these abstractions as imperfect but necessary for progress is well understood, for much of research in economics amounts to the replacement of an idealization by something with higher fidelity. Indeed, the research literature in economics is largely composed of work that replaces one or two of the standard assumptions, generating conclusions that encompass but are different from those produced by the usual specifications.

Table 1 below lists important features of economic models in the first column and standard neoclassical abstractions in the second, roughly in line with what is taught (e.g., Mas-Colell, Whinston and Green, 1995). The third column, labeled "increased realism", gives some of the ways the standard idealizations have been relaxed or generalized in the economics literature. While any particular entry in that column may have its own moniker—e.g., behavioral economics for the fourth row—the column *as a whole* ranges over topics that are foundational to the study of *complex adaptive systems* (CAS); see Anderson, Arrow and Pines (1988), Mirowski (1996), Arthur, Durlauf and Lane (1997), Blume and Durlauf (2005), Kirman (2011).[6]

Model feature	Neoclassical conception	Increased realism
Number of agents	representative (1, 2, *N*, infinite)	many (preferably full-scale)
Diversity of agents	homogeneous, a few 'types'	heterogeneous, idiosyncratic agents
Agent goals, objectives	static, scalar-valued utility	evolving, other-regarding
Agent behavior	rational, maximizing	purposive, adaptive, biased, myopic
Learning	individual, fictitious play	derived from AI or behavioral science
Information	centralized, possibly uncertain	distributed, tacit
Beliefs	coordinated for free	uncoordinated, costly to coordinate
Interaction topology	equal probability, well-mixed	social networks, fixed or changing
Markets	auctioneer, global price vector	decentralized, local prices
Firms and institutions	unitary actors, production functions	multi-agent groups and organizations
Selection	single level	multi-level, group selection
Governance	benevolent planner, median voter	self-governance, incentive problems
Temporal structure	static, impulse tests, 1-shot	dynamic, full transient paths
Source of dynamism	exogenous, outside economy	endogenous to the economy
Properties of dynamics	smooth, differentiable	irregular, volatile, heavy-tailed
Character of dynamics	Markovian, path is forgotten	path-dependent, history matters
Solution concepts	equilibrium at agent level	macro steady-state (stationarity)
Multi-level character	Neglected or simple micro-macro	many levels, higher ones emerge
Methodology	deductive, mathematical	abductive, computational
Ontology	utilitarian agents who optimize	ecology of purposive interacting agents
Policy stance	designed from the top down	evolved from the bottom up

Table 1*: Contrast between standard economic abstractions and more realistic ones*

In economics and finance, standard idealizations take the form of things like rational or representative agents, global price vectors, and Nash equilibrium. What keeps economists from moving from the middle column to the right are a variety of conceptual, mathematical and econometric difficulties that can make richer models intractable.[7] Agent computing offers a way to explore the right column by removing analytical barriers. For most rows there exist *ABMs* relevant to the right column. *ABMs* are a new tool in the economist's toolbox that can be used to move from the center toward the right column. Unfettered by the mathematical strictures that constrain economic models that

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[6] For succinct introductions to *CAS* see Holland (1998, 2012, 2014) or Bak (1996); for a social science point-of-view there are Miller (2015) and Miller and Page (2007); Boccara (2010) is a more mathematical treatment with a natural science focus; for a computer science perspective consult Mitchell (2009) while Downey (2012) provides working *ABM* code; Krugman's (1996) take is very readable, albeit dated; Durlauf (2012) expresses skepticism of the relevance of *CAS* to economics despite earlier views to the contrary; others are optimistic (cf. Colander (2000), Potts (2000), Berry, Kiel and Elliott (2002), Kirman (2004), Rosser (2004), Beinhocker (2006), Axtell (2007), Farmer and Foley (2009), Farmer and Geanakoplos (2009), Rosser (2010), Holt, Rosser and Colander (2011), Gallegati and Kirman (2012), Colander and Kupers (2014), Arthur (2015), Haldane and Turrell (2018)). Earlier versions of this table appear in Axtell *et al*. (2016), Axtell (2017). For a similar but more evolutionary perspective see Bowles (2004: 479).

[7] In his masterful *Engine Not a Camera* Donald Mackenzie (2006) has argued that the goal of the middle column was never descriptive when it came to finance. Rather, when markets did not conform to the tenets of mathematical economics, the *normative* quickly displaced the *positive* in the guise of financial engineering.

must be written down and solved analytically, *ABMs* serve as a potent new methodology for accelerating progress in economics.

ABM facilitates movement towards the right column by combining the expressiveness of computer code with a computational methodology that does not need to explicitly pre-specify aggregate outcomes. Agent computing permits one to write economic specifications that have high fidelity with the real-world, whether behavioral, institutional, or administrative. For example, production decisions can often be represented as recipes of nested IF…THEN…ELSE statements, as in IF the inventories are below a threshold THEN ramp up production by 10%. The expressiveness and behavioral suppleness that is characteristic of *ABM* means that more realistic models can be built that do not contain assumptions made for mathematical expediency. One may profitably consider using *ABM* in lieu of or in addition to mathematical analysis whenever relaxation of standard specifications produces analytical difficulties.

At present ABM is not widely used in mainstream economics. It is our thesis that the time has come for this to change. There are many important problems where *ABM* has already made substantial contributions. As we will discuss, in finance these include practical applications for understanding systemic risk and theoretical explanations for key phenomena such as clustered volatility and the functional form of market impact. In economics these include better understanding of firm dynamics, macroeconomics with heterogeneous and behavioral agents, and in a variety of other areas. We believe that with the ever-increasing availability of computational power, behavioral knowledge, and micro-data, *ABM* has the potential to make substantial contributions in most areas of economics.

II. *ABM* Antecedents and Exemplars

Like many technological innovations, *ABM* is not the result of a single ground-breaking idea but is rather a non-trivial recombination of many pre-existing technologies, permitting a new kind of computational model. In this section, to properly situate *ABM* within wider advances in computational science, we briefly describe key prior developments as they arose historically. While there are important connections to physics, ecology, and evolutionary biology, the main touchstones for modern agent computing in economics and finance lie in long-standing connections to computer science, specifically *AI*, and *OR*.[8] We then go on to describe some of the pioneering *ABMs* that suggested its potential usefulness for economics and finance.

The notion of using digital computation to model individual households or firms and study their aggregate behavior grew up in the late 1950s among several distinct groups of economists (Orcutt, 1957, Clarkson and Simon, 1960, Shubik, 1960b). While their motivations were varied—some saw computation as a way around mathematical

[8] Mirowski (2001) analyzes interactions between these fields but fails to distinguish agent computing. For perspectives closer to ours see Builder and Bankes (1991), Bankes (2002), Bonabeau (2002), or Axelrod (2003). The rise of digital computation gave rise to a new field of simulation science, primarily in operations research and industrial engineering (Conway, Johnson and Maxwell, 1959, Conway, 1963); see Macal (2016) for an overview of *ABMs* in management science and operations research and the book of North and Macal (2007) for applications to business. Several specialized languages appeared for creating simulation models, including *GEMS* (from General Electric), *GPSS* (General Purpose Simulation System), *SimScript*—developed by future Nobelist Harry Markowitz—and *SIMULA*, the first object-oriented computer programming language (Banks and Carson III, 1984, Fishwick, 1995).

difficulties of aggregation, others thought the key advance was the ability to treat decision-making as dynamic. All were optimistic that digital computers would facilitate the creation of models having greater veracity than conventional ones, e.g., "…the simulation approach has emerged, as a practical means of studying and using more nearly realistic models of economic systems" (Orcutt, 1960).

This led to an effort to move beyond Marshall's (1920) representative firm and the typical consumer to build models having empirically-justified heterogeneity. This came to be known as *microsimulation* (Orcutt, 1957, 1960, Orcutt *et al.*, 1961), which is similar spirit to *ABM* in many ways. In microsimulation it is typical to model the behavior of many distinct households using more or less conventional constrained optimization, but realized computationally (Bergmann, 1980, Bennett and Bergmann, 1986). These models explicitly simulate consumption and work decisions (Bergmann, 1990), savings levels, tax compliance (Rohaly, Carasso and Saleem, 2005), and so on. Early researchers in this area would have benefitted greatly from today's programing languages and hardware. This is clear from Orcutt (1960), who lamented that the building blocks used in microsimulation models all had to be coded individually, while seeming to call for automated replication as is now common in all object-oriented languages. In essence, the pioneers of microsimulation had many of the same motivations as modern *ABM* researchers but had inadequate tools, both too little hardware and too rudimentary software.

Contemporaneous with these developments, computational theories of the firm were developed in the late 1950s by the Carnegie School of Herbert Simon, Richard Cyert and James March, all of the Graduate School of Industrial Administration (*GSIA*) at the Carnegie Institute of Technology. Their research program aimed to better understand how firms actually behaved by studying a particular firm in great depth. Their path-breaking book *A Behavioral Theory of the Firm* (Cyert and March, 1963) opened up the study of individual organizations to computational approaches. Because these behaviors tended to be more rule following and heuristic than optimizing, it was different in character from microsimulation. Work that grew out of this tradition includes the widely used garbage can model of organizational choice (Cohen, March and Olsen, 1972).[9]

The dominant use of early digital computing was for solving equations numerically, e.g., to support nuclear weapons research (Edwards, 1996, Mirowski, 2001) and weather forecasting (Edwards, 2010). However, other, non-numerical uses soon emerged, such as reproducing brain behavior (Ashby, 1952, 1956, von Neumann, 1958). Of interest as a precursor to *ABM* are *cellular automata* (*CA*), which are simple finite automatons connected on a lattice. These were pioneered by Ulam (1952) and utilized by von Neumann for the creation of self-reproducing systems (von Neumann and Burks,

[9] In the late 1950s strategy games were inserted into the curricula at some business school, including the *GSIA* at Carnegie (Cohen *et al.*, 1960), UCLA (Jackson, 1959), and elsewhere (Cohen and Rhenman, 1961). These were an offshoot of war-gaming and military simulations that were already widely practiced in non-digital form in the first half of the 20th C. These tools and techniques were augmented by digital computers in the 1950s (Shubik, 1960a). They are a good example of qualitative, largely non-numerical computation in which no equations are solved. More recently, *ABMs* have found wide use by the military. For example, force-on-force models were traditionally treated as systems of differential equations, e.g., the Lanchester equations (Lanchester, 1916, Engel, 1954), but modern combat models are increasingly agent-based. Each soldier, each weapon system, and even each unit of ammunition present is treated as an object. Important early work on the *ABM* approach to combat models includes Ilachinski's *ISAAC* model and *EINSTEIN* software toolkit (2004).

1966).[10] *CAs* became known to the scientific public in the 1970s through John Conway's *Game of Life* (Gardner, 1970).[11] More directly relevant to economics, Peter Albin (1975) used *CAs* to model economic development and others studied urban dynamics with them (Tobler, 1970, 1979, Couclelis, 1985, 1989).[12]

We have suggested above that *ABMs* can be viewed as a form of non-numerical computing in which there are typically few if any equations governing agent interactions that are solved in the traditional sense. While individual agents often use mathematical expressions in their own reasoning, macro (multi-agent) phenomena are produced in *ABMs* by directly aggregating individual actions. This makes it natural to use *ABMs* to study emergent outcomes.

A simple example is the late Thomas Schelling's model from the 1960s of residential segregation. This is perhaps the best-known *ABM* in economics.[13] The idea that a modest number of whites moving away from small pockets of black immigration might 'tip' neighborhoods in major American cities is due to Grodzins (1958), but Schelling demonstrated that high levels of segregation can result from the bottom-up, decentralized home location decisions of individual households, even if none of them have particularly segregationist preferences (Schelling, 1969b, 1971a, b, 1972b).[14] Figure 1 shows the progressive segregation of a 50 x 50 spatial grid when agents are happy having as few as three of their immediate 8 neighbors with the same color in a population with equal numbers of reds and blues.[15] Unhappy agents move to any site where they would be happy. Over time high levels of segregation emerge.

The panel on the lower right is a depiction of segregation in Buffalo, N.Y. based on data from the 2010 Census. Note the gross similarity. It is important to keep in mind that no agent prefers a highly segregated *aggregate* outcome to a more integrated configuration in the model, but in a world of distributed, decentralized action there are simply many more social configurations that look like the extreme outcomes on display in figure 1 than more integrationist ones, and so it is not clear how such outcomes might

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10 This research investigated minimum specifications needed to have a machine reproduce itself logically.

11 For more on the origins of the *Game of Life* see the biography of Conway by Roberts (2015). By looking only at the rules of the *Game of Life* it is very hard to see what will emerge once the rules are applied (Faith, 1998).

12 *CA*s played an important role in the rise of 'artificial life' (*ALife*) in the late 1980s (Langton, 1989, Langton *et al*., 1992, Langton, 1994). *ALife* models typically feature some highly idealized representation of an ecological or social process using a population of simple computational entities in order to study phenomena at a higher organizational level (Hillebrand and Stender, 1994). Reynolds' (1987) demonstration that realistic-looking bird flocks could be generated from a few simple rules of individual behavior is a quintessential example.

13 Mirowski (2001) disagrees with the assertion that Schelling's model is the first *ABM* (fn. 48, p. 369) based on the belief that it is essentially a *CA* (Mirowski, private communication, 29 August 2020).

14 Interestingly, Schelling's early efforts were *not* computational. Rather, he began working in one dimension but was convinced by Herbert Scarf that the exercise might be more clear in two dimensions (Schelling, 2006). Moving two kinds of coins around on a chessboard, with coins of the same kind preferring to be next to one another, he showed that widespread segregation characterized long run configurations of the model. Later, while visiting the RAND Corporation, he had the model rendered computationally (Casti, 1994) but felt not much new was learned from this exercise. Later he worked with a student to code a version of the model himself and wrote a long, unpublished essay entitled "On Letting a Computer Help with the Work" (Schelling, 1972a), which makes clear he understood the value of computational renditions of his model (Hegselmann, 2012).

15 Schelling's model is sometimes called a *CA* but this is not strictly correct as in most version of it the agents are mobile *beyond* their local neighborhoods. *CAs* are usually defined by strictly local interactions.

be prevented. Schelling's model has been elaborated in various ways, both to render it more realistic (Ingram, Kain and Ginn, 1972, Vandell and Harrison, 1978), to give it rigorous foundations (Young, 1998, Zhang, 2001, 2004a, b, Vinkovic and Kirman, 2006, Benard and Wiler, 2007, Pancs and Vriend, 2007, Dall'Asta, Castellano and Marsili, 2008, Gerhold *et al*., 2008, Zhang, 2011, Brandt *et al*., 2012, Barmpalias, Elwes and Lewis-Pye, 2014), to generalize it (Bruch and Mare, 2006, Benard and Wiler, 2007), and to render it on realistic geographies (Crooks, 2010).[16] But its real power derives from its simplicity.[17]

<Figure 1 about here>

By the 1970s the folk theorem of game theory indicated that a wide variety of strategies could be supported as equilibria in repeated games, providing little guidance for which strategies one might expect to encounter in real-world play of such games. This situation served as the basis for an innovative computational experiment run by Robert Axelrod at the University of Michigan in the early 1980s. He solicited strategies to play the prisoner's dilemma game in round-robin fashion and received many submissions, from simple to complex. In the resulting tournament he discovered that a relatively simple strategy, submitted by Anatol Rappaport—so-called tit-for-tat—did very well against more elaborate strategies, winning the tournament. He ran a second tournament with new strategies and found that tit-for-tat won again. He described his findings in a now classic book, *The Evolution of Cooperation* (Axelrod, 1984). The strategies used in his tournaments consisted of a series of small-scale *ABMs*. This was an early use of heterogeneous computational agents in a competitive environment, which was very novel at the time.

By the 1980s conventional *AI* had succeeded in building deep representations of highly restrictive domains (e.g., chess), but had largely failed to create anything like general-purpose intelligence. Distributed artificial intelligence (*DAI*) began from the perspective that agents can learn from one another (Gasser and Huhns, 1989). Very soon the individual *AIs* in *DAI* models were being given purposive behavior via utility functions, preferences, goals, and so on, and in a matter of a few years the field was transformed into multi-agent systems (*MAS*), with research monographs (Maes, 1990, Wooldridge and Jennings, 1995b, O'Hare and Jennings, 1996, Weiss, 1999) and textbooks (Ferber, 1999, d'Inverno and Luck, 2001, Liu, 2001, Wooldridge, 2002) soon

[16] Schelling's first descriptions of the segregation model were terse (1969b), unpublished (1969a), or somewhat informal (1971b), the latter appearing in a Washington-based policy journal. Meeting some resistance from economists, he described the model at length in volume 1, number 2 of the then new *J. Math. Sociology* (Schelling, 1971a). Remarkably, in the previous (very first) issue of that journal there appeared a paper having ambitions comparable to Schelling's, albeit considerably broader. It was entitled "The checkerboard model of social interaction" and authored by James Sakoda, a pioneer in computational sociology (Sakoda, 1971). The model developed by Sakoda admits Schelling's as a special case (Hegselmann, 2017), and derives from his dissertation (Sakoda, 1949). The latter is an extraordinary document, little cited but essentially a generation ahead of its time in seeing the possibilities of building computational models of a variety of social phenomena using *CA*-like specifications.

[17] Contemporaneous with Schelling, Gordon Tullock and Colin Campbell (1970) experimented with simple models of voting and committee behavior in situations too complex to be solved analytically. They rendered their models computationally, as described in detail by Wallick (2012), essentially another proto-*ABM*.

appearing, including ones blending *MAS* concerns with game theory and economics (Parsons, Gmytrasiewicz and Wooldridge, 2002, Shoham and Layton-Brown, 2009).[18,19]

In early work on traffic it was conventional to model vehicular and pedestrian traffic as if it were fluid flow in a conduit (Transportation Research Board, 1961). Such models involved computational fluid dynamics, which was initially done on mainframe computers and later on vector supercomputers. In the mid 1990s researchers at Los Alamos National Lab (*LANL*) took an agent-based approach to the subject, giving driving rules to individual vehicles and studying traffic jams and related aggregate phenomena as emergent (Nagel and Rasmussen, 1994, Nagel and Paczuski, 1995, Nagel and Barrett, 1997). This proved to be a more flexible and useful approach, as it was possible to incorporate *GIS* map layers, road grids, and even real-time traffic data directly into models. Soon the *TRANSIMS* code (Barrett *et al*., 1995, Nagel, Beckman and Barrett, 1998) was being instantiated at city-scale, creating high-fidelity models of traffic flow involving millions of vehicles (Barrett and Beckman, 1995, Beckman, 1997). Traffic has been one of the great success stories of *ABMs* and today it is rare to find traffic models built any other way. Later, pedestrian movement was similarly revolutionized by *ABM* (Helbing, Farkas and Vicsek, 2000, Farkas, Helbing and Vicsek, 2002).[20]

Agent-based computational models in ecology began appearing sporadically before 1990 and more systematically afterward.[21] In ecology it is conventional to call this 'individual-based modeling' (*IBM*).[22] The driving forces behind the development of *IBM* grew out of a dissatisfaction with models based on continuous populations with dynamics represented through ordinary and partial differential equations (Grimm and Railsback, 2005). Formalisms in which individual animals are explicitly modeled may have closer

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18 An important technical contribution of computer science to the agent modeling paradigm was object-oriented programming (*OOP*), which came of age with the *SmallTalk* and *Objective-C* languages in the early to mid-1980s. *OOP* refers to the encapsulation into software objects consisting of data elements (aka instance variables) and methods for manipulation of those data, resulting in self-contained and often reusable code. Agents are naturally implemented as objects, and their interactions are succinctly captured by object methods. Today, *OOP* is standard for implementing *ABM* and *MAS* (Wooldridge, 2002).

19 All of this was happening as the Internet came to fruition and the idea of local, networked devices was replacing older notions of centralized computing facilities. At this time certain computer scientists began viewing groups of heterogeneous computing resources as *ecologies* (Huberman, 1987), invoking biological and economic formalisms for understanding how resources might be shared across devices, within networks. Curious ideas like market-based control (Clearwater, 1996) and market-oriented programming (Wellman, 1996) were put forward by crossing over ideas from economics and computer science, clearly indicating a certain appetite among computer scientists for importing extant ideas from the social sciences (Huberman and Hogg, 1994). See Das (2016) for a survey of the literature in this field, with a particular focus on finance.

20 *ABM* has also played a role in the *NextGen Air Transportation System* (e.g., Calderón-Meza, 2011).

21 A recent *ABM* textbook, while focused broadly, is written by ecologists (Railsback and Grimm, 2019).

22 *IBM*, instead of *ABM*, was proposed as the name for the entire field back in the early days of the Santa Fe Institute when the SWARM agent modeling framework was first being created. It will come as no surprise that when 'individual-based modeling' was proposed by some as a good descriptor it was vetoed by Chris Langton since the acronym for it is identical to the standard name of a large U.S. corporation active in the computing field, which was thought to be at least confusing if not more sinister. Another name for the field that did not stick was 'actor-based modeling'. This is because in sociology 'stochastic actor theory' already existed as an active research program, focusing mostly on social network models.

fidelity to the true spatial variability present in the field (Fahse, Wissel and Grimm, 1998).[23,24]

Another area in which *ABMs* had early success is epidemiology. Traditional models based on ordinary and partial differential equations assume that populations are homogeneous and well-mixed, i.e., that there are no network effects (e.g., Kermack and McKendrick, 1927). In the wake of 9/11 the U.S. National Institutes of Health (*NIH*) created the *MIDAS* project (Models of Infectious Disease Agent Study) to catalyze the development of better models of infectious disease spread and response. *ABMs* were a primary focus of *MIDAS*. As a result of this project there now exist a large number of *ABMs* relevant to a variety of diseases, including influenza, smallpox, *SARS*, *MERS*, Ebola, West Nile virus, and Zika, at both the national and international levels (e.g., Halloran *et al*., 2002, Eubank *et al*., 2004, Longini Jr. *et al*., 2005, Carley *et al*., 2006, Gemann *et al*., 2006). Epidemic *ABMs* have also been applied to a variety of pathological behaviors that have social origins, including smoking (Wallace, Geller and Ogawa, 2015), drug addiction (Agar and Wilson, 2002, Hoffer, Bobashev and Morris, 2009, Heard, Bobashev and Morris, 2014), and obesity (Hammond, 2009). Most recently, *ABM*s have found important use modeling the effects of the SARS-CoV-2/COVID-19 pandemic (Ferguson *et al*., 2020), where they are the tool of choice since comparable events have not occurred in the recent past and therefore there is little data on which to base statistical, econometric, or related models.

A. *ABM* in Economics

In this section we look at the literature on agents in economics from the past 30 years and describe some of the most influential work.[25]

1. Microeconomics and markets

The most prevalent area of economics for the application of *ABM*s has been microeconomics and models of markets in particular. As an example, consider the most basic model in all of microeconomics, the supply and demand for a single homogenous good in one market. Typically, the behavior of people in such a market is represented by a downward sloping demand curve for buyers, with quantities and prices on the horizontal and vertical axes, and a supply curve for sellers, usually upward sloping. This is depicted graphically in all microeconomics texts and studied algebraically by the time students reach intermediate micro. The figure—normally drawn as two straight lines—is often accompanied by the assertion that markets operate at the intersection point. In

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[23] Two early and influential *IBMs* were the *JABOWA* model of forest dynamics (Botkin, Janak and Wallis, 1972) and a model of fish cohort growth due to DeAngelis, Cox and Coutant (1980). Grimm and Railsback claim that neither of these pioneering efforts saw *IBMs* as a general-purpose approach, i.e., as a paradigm. Rather, Huston, DeAngelis and Post (1988) articulated a more unified vision for the role of *IBMs* in ecology as Hogeweg and Hesper (1990) did later.

[24] Progress in the 1980s-90s in *CAs* (Wolfram, 1983, 1984, 1986, Toffoli and Margolus, 1987, Gutowitz, 1990, 1991) led to applications in biology (Ermentrout and Edelstein-Keshet, 1993), including neurobiology, developmental biology, population biology, even cancer oncology. Since then *CAs* have given way to *ABMs* in many areas (e.g., Schlesinger and Parisi, 2001, Zhang *et al*., 2009, Chapa *et al*., 2013, Norton and Popel, 2014, Wang *et al*., 2015).

[25] Holland and Miller (1991) give an early statement of the potential of *ABMs* in economics. For a view of *ABMs* as artificial economies see Lane (1993a, b). The *Handbook* edited by Tesfatsion and Judd (2006) is thorough but dated.

general attainment of market clearing is known to be a hard problem.[26] [27] Early experiments (Smith, 1962) showed that it may or may not be a good assumption, depending on the circumstances (Bergstrom and Miller, 1997).

It is possible to use an *ABM* to model the way the agents behave in Smith's experiments. No individual needs to know very much about how the market functions:

1. buyers, who have heterogeneous internal valuations, will together create a downward sloping demand curve (Becker, 1962) , while
2. sellers, with heterogeneous costs, will produce an upward-sloping supply curve,
3. so if buyers pay less than their internal values and sellers try to cover costs,
4. there can result relatively high market efficiency (Gode and Sunder, 1993).

There are many examples that model local markets (cf. Palmer *et al.*, 1994, Epstein and Axtell, 1996, Cliff and Bruten, 1997b). For a demonstration, see Professor Mark McBride's website (http://www.memcbride.net/models/2014/7/11/zi-trading). This *ABM* features a user definable number of buyers and sellers whose internal valuations can be specified and from which supply and demand curves can be plotted. The supply and demand curves that emerge are much less regular than those found in microeconomics textbooks. After hitting 'Go', trades take place and the quantity exchanged is comparable to the point estimate of the supply and demand curves, as shown in figure 2, although rarely exactly the same. This leads students to realize that the point estimate of the textbook story is an approximation. By rerunning the model multiple times students see that (1) the same price and quantity never repeat, and (2) there is sufficient variability from run to run to show that the point prediction is just a central tendency and not the only thing that happens. Students can then ask how this changes as the size of the market increases, or as elasticities change.

ABMs have the ability to enrich student understanding by treating economic phenomena as emerging from the bottom up as a direct result of the actions and interactions of purposeful individuals. This approach to economics turns student attention away from mathematical difficulties that may or may not be relevant for real markets to how people behave in more realistic market contexts. By utilizing the computer in ways that are richer than merely solving equations—by displaying model output visually, computing statistics dynamically, and permitting students to modify how models work—a whole new way of teaching and learning economics is opened up.

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[26] Technically, Brouwer and related fixed point problems are contained in the complexity class *PPAD* (Papadimitriou, 1994), and it is not known if there exist polynomial algorithms for the computation of market-clearing prices and allocations, as the number of commodities increases; see Moore and Mertens (2011) for a pedagogical description of this complexity class.

[27] One line of research that is *not* very relevant to *ABM*, but which is sometimes confused with economic *ABMs*, concerns the computation of economic equilibria. Beginning with Scarf (1973, 1982) there is a large literature on efficient solution of fixed point problems in economics. This gave rise to certain specialized mathematical programming algorithms (e.g., the Eaves, Lemke, and Merrill algorithms (Todd, 1976)), in the wake of which so-called computable general equilibrium (*CGE*) models were born (Scarf and Shoven, 1984, Shoven and Whalley, 1992). Models of this type were widely adopted for policy and other applied work in economics and can yet be found in large institutional settings, e.g., the World Bank. Commercial software eventually emerged for the solution of *CGE* models (e.g., *GAMS*) and textbooks on the subject appeared (Thompson and Thore, 1992). Although such models can be used to represent the behavior of individual firms and other economic agents, they typically deal with only a few agents, or else represent an economy in overall supply and demand terms (i.e. no agents), solving for equilibria numerically. They are thus more top down than bottom up and do not focus on agent interactions. Such models are quite different from *ABMs* as the solution techniques are purely numerical and not representative of an economic process. The same is true for the solution of input-output models (Morgenstern and Thompson, 1976).

<Figure 2 about here>

A wide variety of *ABMs* have been built to study markets without a centralized auctioneer (Leijonhufvud, 1967).[29] This work includes Albin and Foley (1992, 1998), who looked at distributed, decentralized bilateral trading with local price formation and contrasted their results with Walrasian outcomes. They focused on the effects of price dispersion when no auctioneer is present, including welfare effects associated with the production of *horizontal inequality*.[30] The *Sugarscape* model of Epstein and Axtell (1996) extended Albin and Foley with heterogeneous agents, endogenous interactions, changing preferences, and so on. Wilhite (2001) investigated this same class of models for agents connected in various network topologies. Vriend (1995) looked at the self-organization of markets using a classifier system and later asked how such models relate both to the 'invisible hand' (Kochugovindan and Vriend, 1998) as well as to Austrian market process theory (Vriend, 2002). Axtell (2005) studied the computational complexity of such markets and proved that exchange at local prices yields allocations in polynomial time—linearly in the number of agents and quadratically in the number of commodities—in contrast to the *PPAD* complexity of computing Brouwer fixed points (Papadimitriou, 1994). In essence, such decentralized exchange processes act like a giant distributed computation of Pareto optimal allocations, with final prices representing the marginal rates of substitution that all agents converge to in equilibrium.[31] All of the above results pertain to pure exchange economies. Gintis (2007) built an *ABM* economy with production and studied its convergence to general equilibrium, finding many of the same phenomena on display in pure exchange, e.g., initial price dispersion, welfare effects, and so on.

Kirman and Vriend (2000, 2001) took a more empirical approach in a model of the Marseille fish market, where they discovered that buyer-seller loyalty plays a large role and is often more important than price in purchase decisions. Vriend (2004) reviews a variety of distinct market forms that have been studied with *ABMs*.

Continuous double auctions (*CDAs*) have been investigated using a tournament approach (Friedman and Rust, 1993, 1994), analogous to Axelrod's prisoner's dilemma tournament, with individual participants submitting computational strategies. No single strategy consistently outperforms others, while many sophisticated strategies seem to fare poorly. *CDAs* are commonly used in real-world markets, for electric power, Treasury securities, and many others, and there are now *ABMs* of such markets (e.g., Nicolaisen, Petrov and Tesfatsion, 2000, Koesrindartoto, 2004, respectively). Other kinds of auctions have been studied with *ABM*s (e.g., Hailu and Schilizzi, 2004, 2005, Hailu and Thoyer, 2006, Hailu and Thoyer, 2007), while computer scientists have provided software tools for configuring many different auction types (Wurman, Wellman and Walsh, 1998).

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[29] Analytical models of this type include Rader (1968), Feldman (1973) and Bell (1997), who derive conditions under which decentralized exchange yields Pareto optimal allocations, results analogous to the conventional welfare theorems of general equilibrium.

[30] This refers to differences in welfare that arise between twins—agents having the same preferences and endowments in pure exchange—and was first investigated by Foley (1994). Horizontal inequality cannot arise in standard Walrasian equilibria because of the so-called *equal treatment property* (Green, 1972).

[31] While Cheng and Wellman (1998) have shown how to compute Walrasian equilibria in distributed fashion, their algorithm does not appear to represent any economic process that takes place in real economies.

Another microeconomic topic studied with *ABM* is bipartite matching, which has a wide variety of real-world applications, including marriage, college admissions, and assignment of medical residents. The well-known Gale-Shapley (1962) algorithm produces stable solutions via the so-called 'deferred acceptance' mechanism in a wide variety of circumstances (Roth and Sotomayor, 1990). Unfortunately, it also yields extremes of welfare, where proposers get maximal payoffs while acceptors receive far less (Knuth, 1976). Gale-Shapley matching is a centralized mechanism as no pair can be considered finally matched until all agents are paired. This is unrealistic in many circumstances such as marriage, where decisions must be made incrementally. This has led to a search for good decentralized matching models, e.g. Henrickson (2002) and Fuku, Namatame and Kaizoji (2006). Axtell and Kimbrough (2008) found simple distributed matching mechanisms exist that produce only a very small number of unstable pairs. They argue that the existence of such pairs is unlikely to lead to the unraveling of matches due to the low probability that unstably matched individuals encounter one another; related work includes (Anshelevich, Das and Naamad, 2013).

2. Game theory

*ABM*s have found broad use in game theoretic investigations, often by adding some feature to the specification of the game that is either not easy to handle analytically, e.g., networks, or else is simply not standard, e.g., agent memory. Following Axelrod's work mentioned previously, there has been a large amount of research on the prisoners dilemma game using *ABM*. Evolutionary biologists, for example, have used computational agents to investigate the hypothesis that the ability for local populations to form in space might favor cooperation. Nowak and May (1992) fixed the position of simple agents in a two-dimensional space and let them play the prisoner's dilemma with their neighbors using parallel updating. They discovered the formation of beautiful, dynamic, transient patterns that enabled relatively high levels of cooperation. However, (Huberman and Glance, 1993) demonstrated that the Nowak and May results were artifacts of their synchronous updating mechanism, and that any relaxation of it whatsoever broke all the beautiful patterns and caused the cooperative results to unravel into high levels of defection. This demonstrated that there are important devils in the details of how such *ABM*s are constructed. At a higher level, it demonstrates that the details of the social process can play an important role, e.g. whether information is revealed in a synchronous or asynchronous manner may dramatically effect outcomes. More recently, strategies associated with endogenous group formation and agents being able to select the group in which they play the game have been investigated with *ABM*. (Aktipis, 2004, 2006, 2008, 2011).

The problem of the origins of cooperation is just one example of the problem of the emergence of social norms, conventions and institutions, which has received considerable attention from economists and other social scientists, as well as from philosophers and computer scientists.[32] *ABM* has been used to study the situation where there are multiple equilibria. Are agents able to coordinate on a "good"equilibrium, and if

[32] For work by economists see (Kandori, Mailath and Rob, 1993, Young, 1998, Burke, Fournier and Prasad, 2006); for other social scientists see (Coleman, 1964); philosophers (Lewis, 1969, Bicchieri, Jeffrey and Skyrms, 1997, Bicchieri, 2006) and computer scientists (Rosenschein and Zlotkin, 1994, Walker and Wooldridge, 1995, Shoham and Tennenholtz, 1997, Ossowski, 1999).

so, how is this achieved? Many of the game theoretic models are formally ergodic, and there are asymptotic results that show that only the good equilibria survive. However, studies using *ABM* have shown that such models may display 'broken ergodicity', in the sense that they are ergodic only on time scales that are long in comparison to human lifetimes, say, and may get stuck far from the good equilibria for very long times (Axtell and Epstein, 1999, Axtell, Epstein and Young, 2001, Epstein, 2001, Hales, 2002, Eisenbroich and Gilbert, 2014).

Coalition formation has been studied by both economists and computer scientists using agents. The former tend to be concerned with constraints on the generation of realistic-looking groups (De Vany, 1993a, b, 1996a, b, c) while the latter are often concerned with the complexity of producing groups having certain properties (Shehory and Kraus, 1993, Klusch and Shehory, 1996a, b, Sandholm *et al*., 1998, Chalkiadakis, Markakis and Boutilier, 2007). The number of possible coalitions for any realistically sized population is given by the Bell numbers (Knuth, 2005: 61-86) and is so vast that models of coalition evolution are not plausible as mechanisms for the creation of anything like optimal groups of agents, since the number of coalitions that can be sampled is tiny. Yet optimal coalitions are the primary focus of the cooperative game theory literature (e.g., Ray and Vohra, 1999, Ray, 2007). Real-world coalitions formed via evolutionary mechanisms make the kinds of coalitions studied by *ABM* more realistic than optimal ones.

Arthur introduced the so-called El Farol or 'bar attendance' problem (Arthur, 1991, 1994) as a paradigm for inductive learning in contrast to rational behavior. In his model there is a population of agents all of whom have the same preference for attending a club that evening. If the club is either too crowded or too empty it is not fun for any of the attendees. Arthur demonstrated that with enough heterogeneity in forecast functions the population can evolve toward good solutions so that the bar has very nearly the right number of people attending each week. It is a paradigm for heterogeneous agents arriving at mixed strategy Nash equilibrium despite none of them trying to compute such a thing. This model has generated a large secondary literature (Bell and Sethares, 2001, Bell, Sethares and Bucklew, 2003) and in finance has come to be known as the 'minority game' (e.g., Challet and Zhang, 1998, Jefferies, Hart and Johnson, 2001, Johnson, Jefferies and Hui, 2003).

An important motivation for many game theoretic *ABMs* has been to relax rationality and other conventional specifications (Moss, 2001a), in the spirit of table 1 above. There are *ABM*s using game theory set-ups that add networks (Vega-Redondo, 2007), *k*-level cognition (Latek, Kaminski and Axtell, 2009), and so on. Many *MAS* researchers seek optimal solutions for engineering purposes rather than behavioral realism (Shoham and Layton-Brown, 2009, Tambe, 2011).[33]

3. Industrial organization, firms, and organizational behavior

There have been many applications of *ABM* to industrial organization, the theory of the firm, and organizations. Some of these models relax conventional specifications (e.g., rationality) yet recover conventional results. Others are able to rationalize large

[33] An emerging field in computer science known as *algorithmic game theory* (Nisan *et al*., 2007) focuses on computational issues. Similarly, *computational social choice theory* has grown up as a field within computer science (Brandt *et al*., 2016). Neither of these areas has close connections to *ABM*.

swaths of micro-data that are increasingly available. It is also the case that *ABM* has been broadly applied in the context of firm operations, from logistics to marketing and organizational performance. We briefly review these areas here.

Standard oligopoly models use firms that rationally select prices and/or quantities in competitive environments. An early *ABM* in this area was Marks (1992) who used an evolutionary specification of behavior to breed better strategies.[34] Kimbrough and co-authors attempted to reproduce most of conventional oligopoly theory, e.g. Cournot and Bertrand competition, using simple agents via *ABM* (Kimbrough and Murphy, 2009, Haas, Kimbrough and van Dinther, 2013, Kimbrough and Murphy, 2013). These agents do not have deep internal models of how their local economies work. Instead they probe their economic environment for performance gradients and adjust their behavior accordingly, moving in the direction of higher profits. These simple models do a good job of reproducing most of the content of the rational theory and extend it in various ways.

The economic theory of the firm as it grew up from Coase through Williamson has had important empirical dimensions but was never grounded in firm-level micro-data. The discovery that the sizes of the largest firms closely follow a Pareto distribution led to Simon's (1955) proportional growth model. Starting from an arbitrary distribution of firm sizes, he demonstrated that if, with high probability, growth increments are added to firms in proportion to their sizes, and occasionally a new firm is created, this leads to a Yule distribution of firm sizes, similar asymptotically to the Pareto distribution. This model and its many extensions—see de Wit (2005) for an overview—closely reproduce firm sizes but has little economic content. Lucas (1978) formulated a model with microeconomic structure, which explains Pareto distributions of firm sizes as resulting from Pareto distributions of managerial talent (but the latter is difficult to measure). Later work of this type uses different mechanisms (e.g., technological change) and also addresses firm age (e.g., Luttmer, 2011, 2012).

Today there is ever-increasing availability of firm level data from national statistical agencies, making it possible to formulate and test theories for *many* structural properties of the economy, including firm sizes, productivities, ages, growth rates, financial characteristics, inter-firm networks, and geographical locations. A conventional strategy is to investigate these data using econometrics (Davis, Haltiwanger and Schuh, 1996). This has been useful to illustrate some of the relationships in the data.

An alternative approach is to build *ABM*s that can explain the origins of the regularities observed in the data. Axtell (1999, 2002, 2016, 2018) reports on a family of *ABM*s consisting of worker agents who form productive teams.[35] Each period in the model agents decide how much effort to deliver to their team and assess whether there are better opportunities elsewhere in other teams, based on past payoffs received from working in their team. Over time, following particular effort adjustment and job search behaviors, the agents arrange themselves into a population of teams having distributions of sizes, ages, growth rates, productivities, locations, and so on. With reasonable choices of parameters the overall population reproduces many of the underlying properties of U.S. firms, including a Zipf distribution of firm sizes, heavy-tailed firm productivities,

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[34] Technically, he used a genetic algorithm.

[35] Related work on firms includes Luna (2000) and Aoyama *et al*. (2010).

Weibull age distributions, heavy-tailed growth rates and distributions of financial variables, and agglomerations of firms in space.

This also illustrates that matching the scale of a model to the data can be important. Initial attempts to build such models were done at reduced scale, involving only tens or hundreds of thousands of agents (Axtell, 1999, 2002), whereas the US economy has hundreds of millions of workers. Reduced-scale models achieved only partial success in explaining the data. Later versions of the model were made at *full-scale*, matching the number of workers in the US. The full-scale models match a wider set of properties than the reduced scale models. This is necessary for properties such as firm growth rates that are essentially fluctuations, and scale nonlinearly with the size of the economy being modeled. Using the full scale model it is possible to apply the same statistical and econometric tests to both the data and the model outputs and demonstrate that there is a good match. This work has also revealed that it is possible to generate realistic economic fluctuations from *endogenous* microeconomic dynamics, without recourse to external shocks.

Agent models of firm operations is a very active area of research, including work on supply chains (e.g., Lee, Padmanaabhan and Whang, 1997), marketing (e.g., Rand and Rust, 2011), customer behavior (e.g., Said, Bouron and Drogoul, 2002), diffusion (e.g., Garcia, 2005), e-commerce (e.g., Glushko, Tenenbaum and Meltzer, 1999), and manufacturing logistics (e.g., Leitao, 2009). A significant fraction of this literature spills over into management science and is too large to be succinctly summarized here. Among well-known *ABMs* is the supply chain model of (Parunak, Savit and Riolo, 1998), notable for contrasting *ABM* results with a more conventional equation-oriented model. They find that *ABM*s are more appropriate when supply chains have local, distributed information and are dominated by discrete decisions.

The enormous literature describing product diffusion (Rogers, 1995, Valente, 1995, 1996) has long had a bottom up perspective, while corresponding mathematical formalizations tend to be more aggregate (Bass, 1969). There is a growing *ABM* presence in this area that has been reviewed (Kiesling *et al*., 2012). Rahmandad and Sterman (2008) contrast agent models of diffusion with mathematical approaches. Much of the work on diffusion is closely related to models of opinion dynamics (e.g., Goldenberg, Libai and Muller, 2001, Deffaunt *et al*., 2002, Hegselman and Krause, 2002), which is itself an active area of research at present, especially vis-à-vis social media. Closely related is the application of *ABM* to marketing (Rand and Rust, 2011), another area of rapid growth, especially given the behavioral realism possible with agent models. Such models are capable of representing marketing programs in more detail than simple mathematical models, even giving guidance on things like product placement in stores when *ABM*s take shelf geometry and customer movement into account.

Organization theory was an early area of application of *ABM* methodology and computational organization theory is increasingly agent-based, as evidenced by the edited volumes of Prietula, Carley and Gasser (1998) and Lomi and Larsen (2001) and the survey of Carley (2002).[36] This literature covers a wide range of topics, including information flow within organizations (e.g., email networks (Klimt and Yang, 2004, Keila and Skillicorn, 2005)), hierarchy and power relations, compensation, work effort

[36] An important but now dated statement of this research program is Carley and Prietula (1994).

and monitoring issues, learning curves, efficiency, and the trade-off between exploration and exploitation. For example, worker turnover has been investigated with *ABM* by Dal Forno and Merlone (2004), Phelan (2004) has studied promotion policies, and Harrison and Carroll (2006) simulate organizational culture and demography. Among their many findings are that rapid growth and high turnover can actually enhance the stability of organizational cultures through large influxes of new employees who are susceptible to socialization.

4. Labor economics

There has been considerable work studying labor markets using *ABM*, including Tesfatsion (1998, 2001, 2002, 2003), Fagiolo, Dosi and Gabriele (2004), Richiardi (2004), Neugart (2004), 2008) and others. The typical motivation for these studies is either to generate aggregate labor market performance measures from the bottom up, such as Beveridge curves and matching functions, or to relax one or more of the conventional assumptions in standard labor economics, such as homogeneous workers, uniform reservation wages, and rational decision-making. Notable about several of these studies is the explicit focus on policy issues, such as the economic effects of the size and duration of unemployment payments. The expressiveness of *ABM* is a comparative advantage here since such policies often have features that are hard to represent mathematically and are more easily stated in computer code.

The role of social networks for job referral has been studied for some time (Granovetter, 1973, 1995) and the effects of such networks on economic outcomes, via segregation, the production of inequality, and so on, has been investigated both analytically (Calvó-Armengol and Jackson, 2004, 2007, 2009) and with *ABM* (Tassier and Menczer, 2001, 2008). While idealized networks may facilitate analytical solutions, realistic networks usually mean turning to *ABM* as Jackson (2008: 406-7) has suggested:

> "*[O]ne difficulty [when modeling network formation] is that complex networks and/or patterns of behavior can emerge from simple specifications, especially when even minimal heterogeneities (e.g., in geography, age, costs, or preferences) are introduced…[ABM] techniques can be used to analyze systems in which equilibrium or dynamics cannot be determined analytically. They are useful as tools to illustrate systems or for exploratory analyses that help in formulating hypotheses and conjectures. Such techniques are also useful in empirical analyses for generating distributions of behavior that emerge under a model, which can then be compared to or fitted to observed data.*

Real-world networks may have millions of nodes and/or edges and, in addition to conventional network science tools, *ABM* can be a useful methodology for understanding them.

Longitudinal employer-employee matched data for whole countries have progressively become available. These data permit the construction of networks between firms formed by workers who follow employment opportunities from firm-to-firm. These have been termed *labor flow networks* (*LFN*) since they describe how workers move between firms. *ABM*s have been built to reproduce these networks (Guerrero and Axtell, 2013) and to study how shocks to firms accrete into aggregate employment (Guerrero and López, 2015, Axtell, Guerrero and López, 2019). *LFN*s are an alternative to the aggregate matching function of standard labor economics, providing micro-foundations for wage dispersion and other empirical features of labor markets. Specifically, *LFN* topologies having Pareto-distributed degree distributions, as is the case empirically, cause disproportionately large changes in aggregate unemployment under high labor supply elasticity.

5. Macroeconomics, money and policy

Since the early days of *ABM* researchers have been interested in building models relevant to macroeconomics. Early work includes Allen and Carroll (2001), who studied consumption behavior in a population of imitators and compared their results to standard buffer savings models. Bullard and Duffy (2001) modeled how agents learn about macro volatility. Howitt and Clower (2000) studied the emergence of money in a model that featured many goods and stores selling those goods, with barter arrangements. They established properties that a commodity should possess in order to serve as money. An earlier *ABM* on the same topic is Marimon, McGrattan and Sargent (1990).

These initial efforts were hampered by limited computing power. Then, some fifteen years ago there arose the idea of bringing high-performance computing to bear on macro using large-scale agent models. The *EurACE* model (Deissenberg, van der Hoog and Dawid, 2008, Cincotti, Raberto and Teglio, 2010) was the first example of this endeavor. It yielded a model featuring some ten thousand consumers and firms, generating a variety of macroeconomic phenomena. Eventually the model was made policy relevant and exercised to study policy alternatives (Dawid *et al*., 2012).

At about the same time, and as the Financial Crisis began to unfold, ideas of ‘emergent macro’ and ‘macro from the bottom up’ were invoked to study ‘financial fragility’. These models featured populations of consumers, employed in firms, who borrow from banks to operate. The linkages between firms and banks can lead to credit crises and recessions. This literature includes Delli Gatti (2008, 2011). Given the success of *ABM* financial markets, the so-called *CRISIS* project proposed blending agent-based finance and macro models in order to study the events surrounding the crisis—how it unfolded and how to ameliorate its effects. This effort combined macro and financial sector models in order to produce credible bottom up dynamics of lenders (banks), households, investors, regulators, and consumers. It produced several interesting *ABMs*, including high quite detailed models of banks, bankruptcy resolution, and so on (see Poledna and Thurner, (2016)).

Over the last decade macroeconomic *ABM*s (*MABM*s) have made steady progress, both in the underlying economic and financial processes they represent as well as in their empirical grounding. An up to date review of this area is Dawid and Gallegati (2018), who compare the design, structure, and output of some eight *MABM*s. Essentially all of these include household and firm agents and several include banks and financial sectors. Most of these models have a role for technology and feature capital goods sectors. Each of them has policy making through central banks. These *MABM*s tend to each be calibrated in its own way, as empirical estimation of such models is in its infancy today. The output of such models compared to data from real economies includes skewed size distributions, unemployment rates and durations, Beveridge curves, price change frequencies, growth rates, and so on. Most of these *MABM*s are capable of generating endogenous fluctuations/business cycles, in which case additional model output comparisons include investment being more volatile than output, consumption less volatile, mark-up and wages countercyclical while investment, output and employment are procyclical. Several of these models have been used for policy analysis (Dawid and

Gallegati, 2018: 120-134). Much work remains to be done with *MABM*s and, as appetite among policy-makers for more realistic models grows alongside increased computing power, this area will continue to evolve.

6. Environmental economics

Many problems in natural resource and environmental economics involve features from the right column of table 1, such as spatial processes, networks, and disequilibrium dynamics. *ABM* has been widely applied in this area of economics.

Common pool resource management was brought to prominence by Ostrom (Ostrom, 1990, 1994, 1999), who advocated *ABM* methods late in her life. The ability of groups of people to manage their own exploitation of scarce resources begs for realistic models. For renewable resources *ABM* has been used for some time (Antona *et al.*, 1998, Rouchier *et al.*, 2001). For fisheries a variety of *ABMs* have appeared (Bousquet, Cambier and Morand, 1994, Bousquet, 1996, Bastardie, Nielsen and Miethe, 2013, Bailey *et al.*, 2019), some of which represent fish populations as *IBM*s. Overviews of this literature include Bousquet and Le Page (2004) and Janssen (2002).

The standard models of climate change economics (Nordhaus, 1993b, a) are built around older macroeconomic models, and include strong assumptions such as a single representative agent. So-called integrated assessment models (*IAM*s) of climate change, which are always computational in nature, were historically not agent-based (Dowlatabadi and Morgan, 1993b, a), but are increasingly becoming so. (Downing, Moss and Pahl-Wostl, 2001, Moss, Pahl-Wostl and Downing, 2001, Gerst *et al.*, 2013, Lamperti *et al.*, 2018). Large-scale models have yet to appear, the case for which is made by Farmer *et al.* (2015). Rai and Henry (2016) review *ABM*s of consumer energy choices.

An interesting variant of *ABM* that has grown up largely within environmental applications is known as *participatory modeling* (Ramanath and Gilbert, 2004). In keeping with the bottom up spirit of *ABM*, when stakeholders can be engaged in the modeling process they can be given a role in the model and are then permitted to act in lieu of an artificial agent. This approach has found success in a variety of natural resource environments (Martin *et al.*, 2004, Castella, Trung and Boissau, 2005, Siebenhüner and Barth, 2005).[37]

7. Political economy, development economics, public policy, and related areas

Agent computing has been used to study a wide variety of topics in political economy, broadly construed, from the transition to agriculture (Bowles and Choi, 2013) to taxation (Mittone and Patelli, 2000), public choice (Wallick, 2012), public economics (Kollman, Miller and Page, 1997b), and regional economic issues (Zhang, 2003, Waters,

[37] Land use and cover change are important topics in agricultural economics and related areas and have proven fertile ground for *ABM* because they demand the representation of spatial processes. There is a large literature on such applications, summarized by Parker *et al.* (2003) and then by Matthews *et al.* (2007), which we do not summarize here. A closely related topic is markets for land. Because the value of land depends importantly on its spatial proximity to other economic goods and services, spatial models are again a key motivation for *ABM*s. There is a large and growing literature on this topic (Filatova, Parker and van der Veen, 2007, Filatova, 2009, Filatova, Parker and van der Veen, 2009, Magliocca *et al.*, 2011, Filatova *et al.*, 2013). The usefulness of agents for agricultural economics was pointed out by Berger (2001) who also suggested that policy analysis could be readily accomplished using *ABM*s.

2019).[38] There are now several *ABM*s relevant to development economics, including models relevant to recent conflicts (e.g., Latek, Rizi and Geller, 2013).

As mentioned above, models of taxation have long been the province of microsimulation. The fine level of detail in these models proved both a strength and a weakness. On one hand, detailed micro-data are necessary to make accurate assessments of changes in the tax code. On the other hand, households do not interact in conventional microsimulation models, and a very large number of parameters are needed to march the models forward in time. The data that would be needed to calibrate these models often does not exist. Therefore, there has been a slow but steady migration of tax models to *ABM*, both as a way to study interactive behaviors not easily studied with microsimulation (e.g., use of tax preparers), but also because many of the life behaviors that require parameters are more naturally represented in agent models (e.g., demographic events like births, marriage or divorce). It is also the case that agent models facilitate the representation of boundedly rational taxpayer behavior, important in models of compliance (Antunes *et al*., 2007, Korobow, Johnson and Axtell, 2007, Bloomquist, 2010, Hokamp and Pickhardt, 2010). Models featuring social networks of taxpayers, with various kinds of information flows, are naturally studied with agents (Bloomquist, 2012, Andrei, Comer and Koehler, 2014). While such models make predictions for specific agents, the results are normally interpreted as more meaningful at the population level.

For some time it has been noted by several researchers that the expressiveness of *ABM* combined with the methodology's ability to interact with stakeholders (e.g., participatory modeling (Downing, Moss and Pahl-Wostl, 2001, Castella, Trung and Boissau, 2005)) and communicate to decision-makers make it potentially very useful for policy (Lempert, 2002, Moss, 2002, Gulden, 2004). Against this perspective, Durlauf has argued that the complexity approach to economics, overall, and *ABM* methodology, specifically, "does not fundamentally affect policy evaluation" (Durlauf, 2012: 68).

B. *ABM* in Finance

One of the most active areas of *ABM* is in finance. From agent-based stock markets featuring software traders to banking regulation and financial crisis modeling, there now exists a large and growing literature.[39]

1. Clustered volatility and fat tails

The origin of changes in price is among the most fundamental questions in finance. As originally noted by Mandelbrot (1963), prices tend to move in "bursts". There are times when prices are volatile and times when they are relatively stable, a phenomenon that is called *clustered volatility*. *ABM*s provide insight into the causes of clustered volatility, both yielding conceptual insight and providing better practical tools for forecasting volatility.

In a rational expectations equilibrium (*REE*) agents fully process all available information and immediately incorporate it into prices, so that the only possible reason

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[38] Older, non-agent-based computational models, having more of a numerical flavor, are also used in this area (Kollman, Miller and Page, 2003).

[39] For an excellent early overview of this literature see LeBaron (2006). The financial crisis stimulated several calls to expand work in this area (Buchanan, 2009, Farmer and Foley, 2009, Battiston *et al*., 2016). For an *ABM* model of currency crises see (Arifovic and Masson, 2004).

for prices to change is the arrival of new information. This has been used to justify the efficient markets hypothesis, which says that because prices fully reflect all available information it is not possible to make excess profits by processing publicly available information. If markets are efficient, then price changes are in one to one correspondence with new information, and the clustered volatility of prices merely reflects the clustered volatility of information arrival.

To test this hypothesis, Cutler, Poterba and Summers (1989) studied the 100 largest daily moves in the S&P 500 index between 1946 and 1987. They recorded the *New York Times*' (*NYT*) explanation on the day after each move and made a subjective judgment as to whether the explanation could plausibly be called "real news". Their results were striking: Only about a third of the largest price movements were associated with new information arriving from outside of the market. The other two thirds were not. For example, the largest daily price movement during this period occurred on October 19, 1987, when prices dropped more than 20% in one day. If this indeed reflected a rational expectations equilibrium in an efficient market, it would suggest that the value of the U.S. economy decreased in a single day by an amount corresponding to roughly a decade of typical economic growth. Under *REE* one would expect news of profound importance. In contrast, the *NYT* explanation was "worry over dollar decline and rate deficit" and "Fear of US not supporting dollar". Both express collective emotions rather than facts, and both are internal to the market. In contrast, the largest move that was classified as real news occurred on May 28, 1962, when the *NYT* reported "Kennedy forces rollback of steel price hike" (this was only the fifth largest move). Studies on finer timescales comparing large price movements to news feeds show an even smaller ratio of price movements given by news vs. those that occur without any news (Joulin *et al*., 2008). It seems that markets often "make their own news". This is incompatible with rational expectations and efficient markets.

The origins of clustered volatility were studied via an agent-based model that has come to be called the *Santa Fe stock market model*. Arthur *et al*. (1997) simulated a market where investors had a choice between investing in a stock or a bond. The stock paid dividends of variable size while the bond paid a fixed interest rate, which was lower on average. Each artificial trader had an "artificial brain", which was based on a machine learning algorithm called a classifier system.[40] Their artificial brains allowed them to form their own expectations and make decisions based on past experience. They had two sources of information to choose from. They could pay attention to fundamentals (the dividends paid by the stock). Alternatively, they could look at the recent behavior of prices and base their decisions on technical indicators, such as whether prices were trending. Or they could use a combination of the two, though since the computational resources and data available to each agent were finite, their ability to perform each task was limited accordingly.

It was also possible to over-ride the algorithms of the agents and give them beliefs corresponding to a perfect understanding of the dividend process. When this was done the market obeyed the rational expectations equilibrium; the stock price stayed near the

[40] In a classifier system each agent is given a set of rules, where each rule consists of an input condition and an output if that condition is met. Successful rules reproduce, albeit imperfectly, with modifications to the rules via random mutations and recombination, and unsuccessful rules are removed from the population, so that the system can learn over time.

correct valuation and price fluctuations closely tracked the random draws of the dividend process. However, when the artificial traders used their machine learning algorithms the model showed realistic looking clustered volatility. This was true even though the autocorrelations of price returns were minimal, indicating a market that was efficient with respect to linear time series prediction. Arthur *et al*. (1997) showed that this was caused by fluctuations in the market ecology. That is, traders tended to specialize as either trend followers or fundamentalists. When conditions favored one over the other, the number of traders and the capital they deployed tended to shift between the two strategies. When trend followers became more active, destabilizing the market, volatility tended to be higher. Similar results for a simpler agent-based model setting were independently obtained by Brock and Hommes (1997, 1998).

These early *ABM*s matched the data only in a qualitative sense, but later *ABM*s were able to quantitatively reproduce the empirical statistical properties of real markets. This included matching the autocorrelation of price returns, the autocorrelation of volatility and the marginal distribution of price changes.[41]

Thurner, Farmer and Geanakoplos (2012) showed that leverage can also cause clustered volatility. In an *ABM* with only fundamentalists, when leverage is banned there is no clustered volatility; however it appears as soon as leverage is allowed. When leverage is used at realistic levels there is a good empirical match to the tails of price distributions. Poledna *et al*. (2014) extended this model and showed that it provides a much better empirical match to the decay of volatility following spikes than standard GARCH models (Engle, 2001). Poledna *et al*. (2014) also compared several different policies for regulating leverage, and showed that systematic effects lead to counterintuitive results. For example, in the high leverage domain it is better to use fixed leverage than to use variable leverage based on market conditions, such as Value at Risk (which was recommended under Basel II).

Agent-based models can also be used to forecast volatility. For example, the parameters of the Franke and Westerhoff (2012) model, which is similar in spirit to the Santa Fe and Brock and Hommes models, can be matched to empirical data, and provides a substantially better fit than standard *GARCH* volatility models (Barde, 2016).

Overall, *ABM*s have played a useful role in showing how bounded rationality results in a realistic empirical match to actual markets with respect to the properties of both price changes and volatility. As such, they stand in sharp constrast to the lack of satisfactory explanations of these phenomena in analytical models. With improved estimation methods, has led to useful tools for practical problems such as volatility forecasting.[42]

2. Application of *ABM*s to NASDAQ decimalization

These early successes with financial market *ABMs* led to their practical use in predicting the consequences of regulatory changes. At the end of the twentieth century

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[41] These models include Lux and co-authors (1998, 1999, 2000), Levy, Levy and Solomon (2000), LeBaron (2001a, b, c, d, 2002), Farmer and Joshi (2002) and Cont and co-workers (Ghoulmie, Cont and Nadal, 2005, Cont, 2006).

[42] *ABMs* of more specialized markets have also appeared, such as those of Arifovic and co-workers who modeled foreign exchange markets using agents who learn via genetic algorithms (Arifovic, 1996, 2001). See also work on flash crashes (Paddrik *et al*., 2012), high-frequency trading (Leal *et al*., 2016) and transaction taxes (Fricke and Lux, 2015).

the Securities and Exchange Commission (*SEC*) ordered the *NASDAQ* market to move from trading in terms of eighths and sixteenths of a dollar to pennies, a change that was called "decimalization". The *NASDAQ* had just been sued with accusations of collusion and its management was keen to not jeopardize market performance with regulatory changes. It therefore commissioned a high-fidelity *ABM* of its market, which was calibrated using its own proprietary data. This included many institutional details that had not previously been built into agent-based financial markets. The resulting model of the 'Small Order Execution System' (*SOES*) faithfully rendered the protocols for dealing with the diverse types of orders that were used. Over the better part of a year an *ABM* of the *NASDAQ* was created and its performance was calibrated to actual market behavior. (Darley *et al*., 2001). It was an evolutionary model and made several predictions as to how decimalization would affect market function. Most of the model's predictions were subsequently borne out after decimalization took effect (Darley and Outkin, 2007). This use of *ABM* to address the effects of alternative regulations has since been duplicated in evaluations of alternative circuit breakers for financial markets (Yeh and Yang, 2010).

3. The square root law of market impact and *ABM*

In the last decade there has been a proliferation of financial market *ABM*s for understanding continuous double auctions as they are actually used in contemporary financial markets. An application of particular interest has been to explain empirical regularities of financial markets. Perhaps the most striking example is the square root law of market impact, whose explanation sheds new light on the relationship between supply and demand. The story of how this came to be understood provides a good example of the usefulness of making a series of *ABM*s to understand a problem, ranging from simple to more realistic. It also shows how *ABM*s can be used to help develop better theoretical models.

Market impact is the relationship between the initiation of new trades and changes in prices. A new order to buy that results in an immediate trade tends to be associated with an increase in price, and a new order to sell tends to be associated with a decrease in price. Under normal conditions, the average change in price for a buy order is well approximated by a square root function of the form

$$\Delta p = K\sigma \left(\frac{Q}{V}\right)^{1/2}$$

where Δp is the price change, Q is the size of the order, V is the trading volume, σ is the volatility, and K is a constant of order one, that varies from market to market. Sell orders follow the same relationship, but with a negative sign. The volatility and volume are measured using the same timescale. The price change, Δp, can have both a temporary and permanent component. Due to the fact that this relationship is noisy and is an average over many trades, it is difficult to measure the permanent component. Nonetheless, on time scales of up to a day or so, this relationship has very strong empirical support.[43]

The square root law is surprising for many reasons. If excess demand is a smooth function, then based on an expansion in a Taylor series one would expect market impact

[43] Some deviations have been observed but these are small and may be due to measurement problem in aggregating orders; see Bouchaud *et al*. (2018) for a summary of the empirical evidence.

to be linear. In contrast, the square root function has an infinite slope at the origin, which means that very small trades have a surprisingly large effect on prices, and similarly, large trades have a surprisingly small effect. This means that one cannot add the impact of successive orders together. One would have naively thought that market impact should depend on the market capitalization of the asset, but this does is not the case – instead, it depends on trading volume, which does not necessarily vary proportionally to market cap. Yet another surprising feature is that there is no dependence on time – an order can be executed quickly or slowly, yet this doesn't matter.[44] Finally, this is surprising because this relationship is universal, in the sense that under orderly market conditions this functional form provides a good approximation in many different types of markets, including equities, foreign exchange, futures, options, commodities, and even bitcoin.

Theoretical explanations of the square root law are based on markets that operate via the continuous double auction. Such markets have an order book containing pre-existing orders. When a new order is submitted, if it crosses with the best price in the order book, there is an immediate transaction at the limit price of the pre-existing order(s); if not, the new order accumulates in the order book. Existing orders in the order book may also be cancelled at any time.[45]

The square root law corresponds to a situation in which the volume of orders at a given price increases linearly as one moves away from the current best price. To see why, assume prices and quantities are continuous and consider a new buy order of size Q. If pre-existing limit orders have density $\rho(p)$ then the new price, $p + \Delta p$, to which the new order will penetrate the order book satisfies the condition

$$\int_{p}^{p+\Delta p} \rho(x)dx = Q.$$

If $\rho(p) \propto p$ then market impact follows a square root law.

Early models of the continuous double auction assumed that orders are submitted and cancelled at random (with submission at random prices and random times).[46] A combination of models based on simple *ABM*s, together with the application of techniques from statistical mechanics made it clear that this does not generally result in a linear order density, $\rho(p)$, and thus random order submission cannot explain the square root law (Smith *et al.*, 2003). Furthermore, the simple *ABM* used in this model demonstrated that the resulting price diffusion process for random order submission does not follow normal diffusion; instead, its variance grows more slowly than the square root of time. This implies that prices are mean reverting, i.e. that upward price movements tend to be followed by downward price movements, creating the opportunity for arbitrage using a very simple strategy.

The situation is further complicated by the fact that market participants do not place large orders all at once. Instead, they engage in order splitting, breaking large

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44 The independence of time is probably due to the fact that market participants understand that they cannot execute orders too quickly, and thus never attempt to do so. Nonetheless, there is still a wide range in the speed of execution and there is little or no dependence on market impact.

45 There are many possible rules for determining which orders are executed and at what price. A common specification is price-time priority, meaning that transactions occur with the oldest pre-existing limit order with the best price.

46 For example, Cohen, Conroy and Maier (1985), Domowitz and Wang (1994), and Bak, Paczuski and Shubik (1997).

orders into small pieces and submitting them incrementally. (Large funds sometimes split orders over timescales that span months.) Thus, only a small fraction of the actual supply and demand exists in the limit order book at any given time. To properly understand market impact it becomes necessary to think in terms of a virtual order book, which contains not only the limit orders that are currently sitting the book, but also the orders that participants intend to place at a later time.

Donier *et al*. (2015) showed that a linear virtual order book profile is a necessary condition for normal diffusion. They and their collaborators constructed more complicated *ABM*s with agents who use more sophisticated order placement strategies and demonstrated that these lead to linear order book profiles and market impact functions that follow the square root law (see Bouchaud et al. 2015). This makes it clear that the square root law can be understood in terms of a combination of the dynamic properties of the continuous double auction and the fact that market participants eliminate arbitrage opportunities.

4. Systemic risk modeling using *ABM*s

Systemic risk in financial markets is one of the areas where *ABM*s have made important conceptual contributions, and where the state of the art is getting close to concrete empirical applications. This field also illustrates the close connection between *ABM*s and network models. Systemic risk occurs when the decisions of individuals, which might be prudent if considered in isolation, combine to create risks at the level of the whole system that may be qualitatively different from the simple combination of their individual risks. By its very nature systemic risk is an emergent phenomenon that comes about due to the nonlinear interaction of individual agents, making it natural to think of it as a complex system and study it using agent-based modeling.

The Great Financial Crisis of 2008 dramatically raised our awareness of systemic risk. The seminal papers of Nier *et al*. (2007) and Gai and Kapadia (2010) imported ideas from network theory and epidemiology, respectively, showing how interbank lending leads to financial contagion: If a given bank fails, it both defaults on its loan obligations and stops lending to other banks. Using very simple models, they showed how this can cause other banks to fail, setting off avalanches of cascading failures that can dramatically amplify shocks to the financial system. These papers triggered a large body of work investigating the circumstances under which such failures are likely to occur, as well as generalizations to include other channels of contagion. Perhaps the most important other channel of contagion is overlapping portfolio risk: If institutions *A* and *B* hold the same asset, if *A* comes under stress and sells the asset, this drives its price down, depressing *B*'s values and causing it to sell, further depressing prices, and so on. Analysis in terms of simple *ABM*s led to counter-intuitive conclusions. While diversification is good for individual institutions in the absence of contagion, if many institutions diversify into the same assets so that they become overcrowded, the resulting nonlinear feedback can cause substantial systemic risk, making diversification detrimental. The models that have been used to study this phenomenon extend network models by adding the effect of sales and defaults on prices and balance sheets in a dynamic context. For a review see Aymanns *et al*. (2018). In related work, Lux (2015) investigated the emergence of a core-periphery network structure among banks.

Leverage cycles are another important example of systemic risk where *ABM* has added insight. Leverage refers to buying assets with borrowed funds. If many investors borrow they may inflate the price of an asset, while external triggers may force them to sell in unison, causing a crash. The idea of a leverage cycle was first articulated by Minsky (1982, 1986, 1992) and first studied using an equilibrium model by Geanakoplos (Fostel and Geanakoplos, 2008, Geanakoplos, 2010). As already mentioned, Thurner, Farmer and Geanakoplos (2012) constructed an *ABM* that showed how leverage causes clustered volatility and heavy tails in price changes and causes crashes in a distribution of different sizes. This provides a good example of the complementary use of equilibrium and ABM models.

In a simple *ABM*, Aymanns *et al*. (2016) showed how the use of a standard risk management protocol called Value at Risk (*VaR*) can give rise to an endogenous leverage cycle. In their model there are two representative agents, an investment bank who uses leverage and a value investor who does not. There are also two assets, a risky asset and a riskless asset. Under Value at Risk investors adjust their leverage based on their forecast for volatility – when the volatility forecast is high they decrease leverage, and vice versa. This model assumes simple behavioral rules: Value investors buy the risky asset when it is underpriced, and investment banks buy or sell based on changes in *VaR*. During periods where volatility is low, the investment bank steadily increases its leverage by buying more of the risky asset, driving its price up. Eventually the leverage becomes so high that the dynamics become unstable: A small drop in price causes the investment bank to sell, which amplifies the price decline, causing more selling, and so on, causing a large crash. The cycle then repeats itself, as shown in figure 3.

<Figure 3 about here>

One of the remarkable results of this model is that the leverage cycle is endogenous–there is a cycle even in the absence of any external noise. This is due to the fact that the dynamics are chaotic, meaning that small perturbations are amplified exponentially through time. The model is simple enough to be calibrated using *a priori* values for its parameters, and gives rise to a cycle with a period of roughly ten years, providing a reasonable match to the run-up to the Crisis of 2008. Note that as parameters are varied, such as the relative size of the investment bank and the aggressiveness of its use of leverage, the leverage cycle suddenly appears at full amplitude, so the presence or absence of such a cycle depends sensitively on parameters. A study of alternative policies indicates how the leverage cycle can be eliminated or damped by making leverage less responsive to volatility.

Another good example where *ABM*s have been used to explore alternative policies is given by Poledna and Thurner (2016). They propose taxing transactions based on their marginal increase in systemic risk and test their policy using an *ABM* that emerged from the CRISIS project. The imposition of the tax causes a self-organized restructuring of the financial network that almost eliminates systemic risk, without any side effects. This is in contrast to the widely proposed unconditional transaction tax, also called the Tobin tax, which has little or no effect in damping systemic risk while causing a substantial decrease in market liquidity.

At present *ABM*s for systemic risk are in the process of moving from qualitative models, such as those discussed above, to quantitative models that can be used to test policies and yield good numerical values for their parameters. The European Central Bank and the Bank of England, for example, have created *ABM*s that can explicitly simulate the behavior of all systemically important European financial institutions (Covi, Montagna and Torri, 2019, Farmer *et al.*, 2020). These models have been designed to take advantage of the fact that central banks have detailed access to the balance sheets of key institutions, which can be used to initialize such models. This offers the potential to dramatically improve the accuracy of financial stress tests so that they properly take into account systemic risks as well as individual risks.

5. *ABM*s of housing markets

There is now a substantial literature of agent-based models of housing markets (e.g., Gilbert, Hawksworth and Swinney, 2009, Erlingsson *et al.*, 2014, Kouwenberg and Zwinkels, 2015, Magliocca, McConnell and Walls, 2015, Ge, 2017). Together with Peter Howitt, John Geanakoplos, and a number of our students we have built such a model for the housing market bubble in the Washington, D.C. metro area c. 2000-2010 (Geanakoplos *et al.*, 2012). This model uses administratively complete data on housing stock from county records, data on mortgages from CoreLogic, data on households from various sources, and attempts to match the universe of real estate transactions during this period with data acquired from the local Multiple Listing Service (*MLS*). This *ABM* is somewhat unusual insofar as it can be run, in principle, at 1-to-1 scale with the actual regional economy under study. Giving each household intelligible rules of behavior for home-buying and selling, applying for a mortgage, paying taxes, refinancing, and so on, we have found that it is possible to closely calibrate the model to the actual events, matching a variety of time series both qualitatively and quantitatively, as shown in figure 4. In addition to the overall price bubble, we have been able to do a reasonable job on the inventories of homes for sale, original listing versus actual sale price, days-on-market, loan-to-value of new mortgages, and so on. All of these quantities changed structurally over the course of the bubble, with inventories and days-on-market shrinking during the price run-up and then greatly expanding as the bubble burst. There are other aggregate variables that our model was not able to reproduce closely, such as the overall home ownership rate. This was almost surely due to having very limited data on the rental market. A model based on ours for the London real estate market was developed in a collaboration between researchers at Oxford and the Bank of England; results from this model were presented to the UK Financial Policy Committee in consideration of changes in lending requirements (Baptista *et al.*, 2017). Subsequently, models of this type have been developed for Sydney, Australia (Glavatskiy *et al.*, 2021), Italy (Catapano *et al.*, 2021), and Hungary (Nikolett, 2018).

<Figure 4 about here>

6. Theoretical frameworks for *ABM*

Agent-based modeling is in a sense a departure from theory. In fact *ABM*s can be very useful auxiliary tools for developing theories, including those based on methods that are quite different from those traditionally used in economics. As already mentioned, work in finance provides several examples where techniques from statistical mechanics have been used to interpret and explain *ABM* simulations (see e.g. Smith *et al.* (2003) and Donier *et al.* (2015)).

Another useful theoretical framework comes from ecology. Trading strategies in financial markets are highly specialized and fall into distinct types. Because trading influences prices and vice versa, trading strategies implicitly interact with each other, and profitable trading strategies can be viewed as "feeding on" other strategies. As shown by (Farmer, 2002) this can be used to interpret the results of *ABM*s, and can be used to understand how different types of trading strategies influence each other and how they give rise to excess volatility depending on market conditions. This provides a way to understand how the introduction of new strategies influences the market, and to understand the effect of overcrowding. Market ecology provides a useful framework for going beyond traditional theories based on market efficiency and equilibrium (see also Lo (2004)).

Despite the many accomplishments of *ABM* in finance, much work remains to be done. To date, almost all *ABM*s involve one risky and one riskless asset, with work on multi-asset markets in its infancy (e.g., Oldham, 2017). *ABM*s have yet to be developed that tackle issues such as crowded trades (e.g., Khandani and Lo, 2011).

C. *ABM* in Related Fields

There has been systematic use of *ABM* in *quantitative* branches of other social sciences. An overview of the span of agent computing across the social sciences is Berry, Kiel and Elliott (2002), although somewhat dated now. *ABM* methodology is an integral part of the emerging field of computational social science (Chen *et al*., 2014), along with data-intensive methodologies like machine learning (Lazer *et al*., 2009, Watts, 2013). Here we review important *ABM* work in several disciplines, though we are brief due to the availability of recent disciplinary-specific surveys.

Within political science early use of *CA* includes models of international relations (Cusack and Stoll, 1990). Work of Axelrod is very much in the *ABM* tradition (Axelrod, 1995, 1997a) while that of his student Cederman has more of a *CA* flavor insofar as it describes the behavior of country agents on landscapes (Cederman, 1997, 2001b, 2002, 2003). A dated but useful overview of *ABM* in political science is Cederman (2001a). An overview of computational methods, less focused on *ABM*, is de Marchi (2005).

The behavior of political parties as they seek to attract voters has been studied at book length by Laver and Sergenti (2011), the emergence of parties by Schreiber (2014); older work includes Kollman, Miller and Page (1992). An interesting contrast between analytical models and *ABM* is that the latter are characterized by perpetual adaptation and adjustment, not equilibrium. There has been a variety of work on voting systems, redistricting, gerrymandering, and so on using *ABM*, (Kollman, Miller and Page, 1997a, Bendor *et al*., 2011).

In sociology good overviews of the use of *ABM* are Squazzoni (2012) and Bruch and Atwell (2015), the latter being more empirically focused. Some have called for unifying the discipline with *ABM* (Gintis and Helbing, 2013). An older overview is Macy and Willer (2002). Methodological advocacy for so-called 'analytical sociology' (Hedstrom and Swedberg, 1998, Hedstrom, 2005, Manzo, 2014) is very supportive of *ABM*. Collective action has been studied with *ABM* (Macy and Castelfranchi, 1998, Macy and Flache, 2002). Biggs looked at 19th C strikes in Chicago as propagating between factories like a forest fire (Biggs, 2005, Andrews and Biggs, 2006). Agent learning across multiple games through spillovers has been interpreted as a kind of emergent *cultural behavior* by Bednar, Page and co-authors (2007, 2010, 2012).

Agent-based demography is an active area of research (Billari and Prskawetz, 2003). Local norms of fertility exist and have been modeled with *ABM* (Kohler, 2001) along with the role of local social interactions generally (Fürnkranz-Prskawetz, 2010).

ABM is an active area of research in geography; see Heppenstall *et al*. (2012) for an overview and Crooks *et al*. (2019) for a more recent and comprehensive survey of applications. The complex systems perspective on urban dynamics (Batty, 2005, White, Engelen and Uljee, 2015) uses both *ABM* and *CA* approaches.

Within anthropology and archaeology there are a number of researchers using *ABM* to simulate societies by modeling individual behaviors; see Kuznar (2006) for an overview. These models can be quite data intensive, often with environmental and archaeological data extending over hundreds or thousands of years. Early examples include (Lansing, 1991) , Kohler and Gumerman (2000), and Axtell *et al*. (2002). A more recent survey is Cegielski and Rogers (2016). Broadly speaking, *ABM*s in these areas attempt to reproduce historical trajectories of populations based on physical and cultural records, representing agricultural practices, hunting and gathering activities, tool-making behaviors, mating rules, and so on. When good data are available it is often possible to create high fidelity reproductions of societal trajectories.

III. Current *ABM* Practice

The number of researchers who employ agent computing in their work has grown exponentially over the past decade. Today thousands of papers per year are published annually across the *ABM*, *MAS,* and *IBM* communities with no sign that growth is tapering off.[47] Indeed, as software for creating agent models matures, as researchers become more computer-savvy, and as hardware capabilities expand, it has become easier to build and experiment with agent models. Here we will look at the contemporary literature on agents in economics and finance and describe the main features characterizing the work, focusing in particular on the ways in which agents may be able to enrich analytical and empirical research in economics and finance.

A. Heterogeneous Agents

There is a large literature pointing out the limitations of representative agent models, e.g., Kirman (1992). The notion of using a representative entity in economic models is ostensibly due to Marshall (1920) who invoked the 'representative firm' as an analytical expedient. Although representative agent models dominated economics through much of the twentieth century, incorporating agent heterogeneity is an important topic in macroeconomics today (e.g., Guvenen, 2011).

The ability to move away from representative agents is an important advantage of *ABM*, and there has been an emphasis on this from the beginning of the field.

[48] In traditional *DSGE* models it is necessary to write down and solve first order conditions for the agents in the model. As a result, incorporating heterogeneity requires choosing a functional form, which usually assumes a distribution over an infinite

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47 The penetration of *ABM* into the economics, finance, and the social sciences today looks a lot like the diffusion of game theoretic ideas into these fields over the past 70 years—initial appearance, followed by a decade or more of low adoption, then an acceleration leading to take off and exponential growth. Experimental and behavioral economics have gone through their own explosive growth in between the rise of game theory and the emergence of *ABM*, as discussed in detail in section IV below. Interestingly, bibliometric analyses of these distinct communities suggest that there is very little overlap between them (Niazi and Hussain, 2011) while some have called for more (Wellman, 2015).

48 Volume IV in the *Handbook of Computational Economics* series (edited by Hommes and LeBaron (2018)) focuses on heterogeneous agents and is a good source for work in this area.

population of agents. For *ABM* there are no such restrictions: One can have any number of agents, and there is generally no need to assume that their heterogeneity matches any particular parameterized form.

Heterogeneity can be both exogenous and endogenous. With *ABMs* the many dimensions of exogenous population heterogeneity can be represented to any degree justified by data. Software for creating synthetic populations grounded in data has begun to appear (Adiga *et al*., 2015). This can be used to provide an appropriate initial condition; once this is done, the action of the model will usually generate further heterogeneity endogenously. For example, income inequality may be imposed as an initial condition, and as the economy operates it may exaggerate or diminish inequality.

Indeed, early work with *ABM*s often focused on simple models capable of generating skewed income or wealth distributions (Epstein and Axtell, 1996, Hommes, 2002), some of this work falling into the category of *econophysics* when agents have particle-like characteristics and their fortunes follow simple stochastic processes (Levy, Levy and Solomon, 2000, Chatterjee, Yarlagadda and Chakrabarti, 2005, Yakovenko and Rosser, 2009). More recently there has grown up a literature that uses *ABM* to study the amplification of inequality in richer economic settings. The role of social networks in referral hiring is well-known and how such networks can lead to stratification and wage inequality has been investigated using *ABM* (Gemkow and Neugart, 2011, Dawid and Gemkow, 2014). Effects of labor market reforms on income inequality have been looked at with a heterogeneous agent *ABM* by Dosi *et al*. (2018). The role of consumer credit and other kinds of leverage have been studied (Russo, Riccetti and Gallegati, 2016), especially in the context of the Financial Crisis of 2008-9 (Cardaci, 2018, Papadopoulos, 2019). The role of technology in exacerbating inequality through skill differentials has been an active area of *ABM* research recently (Carvalho and Di Guilmi, 2020, Mellacher and Scheuer, 2021, Terranova and Turco, 2022). Also appearing recently are multi-country *ABM*s that attempt to reproduce empirically-observed divergences in growth rates (Dawid, Harting and Neugart, 2018, Dosi, Roventini and Russo, 2019) and balance of payments dynamics within a currency union (Cardaci and Saraceno, 2019).

A number of heterogeneous agent *ABM*s explore gender inequalities (Grow and Van Bavel, 2015, Bullinaria, 2018). Unequal access to skills and jobs through social and spatial networks has been examined with *ABM*s featuring heterogeneous agents (Cardona, 2014, Antinyan, Horváth and Jia, 2019, Tomasiello, Giannotti and Feitosa, 2020). The role of segregation and income inequality in access to food has been examined using *ABM*s (Auchincloss *et al*., 2011, Blok *et al*., 2015). {{Cochran, 2019 #5244@@author-year}} analyze the emergence of inequitable norms and conventions in quite general settings.

B. Limited Information, Bounded Rationality

Another very common motivation for *ABMs* is the desire by researchers to move beyond rational agents. For many years rationality specifications have been under widespread revision from behavioral economics (1978a, b, Kahneman and Tversky, 1979, Tversky and Kahneman, 1981, Slovic, Fischoff and Lichtenstein, 1982, Tversky and Kahneman, 1986, 1997a, b, c, d, Gigerenzer, Todd and ABCResearchGroup, 1999, Gigerenzer, 2000, Gigerenzer and Selten, 2001). However, rationality persists as the default in much of economic theory, presumably largely for reasons of analytical tractability. (Experimental results on how people really behave typically do not have simple mathematical structure (Simon, 1998)). Because *ABMs* do not face this same constraint, it is usually straightforward to incorporate behavioral specifications directly into computational models.

In fact, the tables are turned for *ABM* – rationality is typically difficult to incorporate in complicated environments. Computer scientists working with *MAS* have investigated this problem in some detail (Sandholm, 1999, Shoham and Layton-Brown, 2009, Parkes and Wellman, 2015). As a practical matter, full-blown rationality is often very difficult to implement in agent models, due to computational intractability (Papadimitriou and Yannakakis, 1994, Daskalakis, Goldberg and Papadimitriou, 2006, 2009). At the aggregate level the computational job of a Walrasian auctioneer or a Hurwiczian mechanism designer is provably among the hardest problems in all of computer science (Hirsch, Papadimitriou and Vavasis, 1989, Papadimitriou, 1994, Conitzer and Sandholm, 2002). For a review of the complexity of computing economic equilibria see Roughgarden (2010).[49]

Agent behaviors based on bounded rationality, in contrast, tend to be use simple rules and local information, and so usually require very few computational resources. Thus for *ABM* bounded rationality is motivated by convenience as well as empirical realism. While rationality makes analytical models easier to work with than behavioral alternatives, the reverse seems to be true for *ABM*.

A further motivation for *ABMs* lies in their ability to match the economic processes of a real economy. Real economies are decentralized in deep and important ways (Hayek, 1945, 1964), making information not just diffuse but also tacit (Polanyi, 1948). There are wide swaths of knowledge having to do with production, distribution, pricing, and so on, that are not available to most agents. This kind of dispersed information is hard to represent analytically in a satisfactory way. In a typical *ABM* agents glean information from some combination of their local environment and gross aggregate data. They make decisions on the basis of dynamic, idiosyncratic information that constitutes their knowledge. Under what conditions does it make sense to acquire more knowledge, or to jettison old information in pursuit of better performance or outcomes? These questions can be addressed with *ABM*s (e.g., Nissen and Levitt, 2004).

Sargent (1999) argued that there is a danger of getting lost in the 'wilderness of bounded rationality'. What we have today in the *ABM* literature is a spectrum of approaches to agent sophistication, involving different forms of bounded rationality. The types of boundedly rational agents that are commonly employed in *ABMs* are briefly described next, ranging from simple to sophisticated.[50]

1. Simple (myopic/reactive/adaptive) agents

More than 50 years ago Becker (1962) demonstrated that randomly behaving buyers could create downward sloping demand curves. While completely random behavior might be an interesting lower bound, Later work in this tradition has focused on simple agents who behave randomly in some sense but who are *purposive*, i.e., they have

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[49] A related problem with rationality has to do with the difficulty of predicting opponent behavior (Foster and Young, 2001), although if playing anonymously in a large population the problem is easier (Kearns and Mansour, 2002).

[50] Chen has written a history of agent types in use in *ABM*, thus permitting brevity here (Chen, 2012). The standard textbook in *AI* (Russell and Norvig, 2010) also takes an agent-centered approach and includes several kinds of agents that have, historically, had little application in economics, e.g., logical agents.

some facility for judging the welfare consequences of their actions and modifying their behavior accordingly. Such adaptive agents typically do not have detailed internal models of their environment. Rather, they are myopic and are commonly referred to as *reactive* agents in the *MAS* computer science literature (Weiss, 1999). In economics and finance such agents are often called 'zero-intelligence' (*ZI*) agents (Gode and Sunder, 1993, 1997), although this name can be a source of confusion since it suggests behavior that is completely random, which is not typically what *ZI* agents do. For instance, in simple market environments *ZI* sellers will try to find buyers in order to cover the cost of the goods they are selling, while *ZI* buyers will not pay more for a good than they believe it is worth. But the determination of exchange prices between the buyer bid and the seller ask is often modeled as being random in some sense.

In many cases simple adaptive agents can produce high performance, particularly in the vicinity of an equilibrium (Lucas, 1986), making their study relevant to more traditional rational solutions. However, it is also known that there are a variety of contexts in which very simple agents may not perform well. For example *ZI* agents do not do well when supply and demand curves have unusual shapes, although rather slight modifications to their behavioral specification can produce much better performance, e.g. zero-intelligence plus (*ZIP*) traders (Cliff and Bruten, 1997a, b). *ZI* agents are closely related to 'probe and adjust' agents (Kimbrough and Murphy, 2009, Huttegger, Skyrms and Zollman, 2012), and the work on limit order books discussed in Section II.B.3 is closely related to this tradition.

Another class of simple, myopic yet purposive agents appears in game theory, where they are called 'low rationality' agents. Such agents use strategies such as *best reply*, which means that the agent plays the move that would have been the best response to her opponent's previous move (Young, 1993a, 1998). There also exist 'best reply to best reply' strategies, 'best reply to best reply to best reply' and so on. Example *ABMs* include Axtell, Epstein and Young (2001). Such strategies are often accompanied by noise so that players select random strategies occasionally as well. Best reply to best reply suggests discrete cognitive 'levels'. *K*-level cognition (Camerer, Ho and Chong, 2004) has found use in *ABMs* (Latek, Kaminski and Axtell, 2009).

In summary, simple purposive agents—perhaps the simplest ones of significant interest—behave by, in essence, taking their environment as fixed and reacting in their own immediate self-interest, by adapting their behavior to their immediate circumstances. If they find that additional adaptations might improve their welfare in subsequent periods they do this as well. They effectively follow utility or profit or payoff gradients. Over time they can effectively learn but they do so without a formal model of their environment.

This brings up the broader question of convergence to equilibrium under bounded rationality. Pangallo, Heinrich and Farmer (2019) exhaustively studied this for normal form games when the players use best reply strategies. They showed that when a game is complicated and competitive the players are unlikely to converge to a pure strategy Nash equilibrium. Here *complicated* means that the payoff matrix is large, either because the game has many moves or many players. *Competitive* means that the incentives of the players are not lined up, i.e. that the payoffs of one player are anticorrelated with the payoffs of the other player (zero sum games being the extreme example). When the game is both complicated and competitive, the players tend to converge to cycles rather than fixed points.

They also studied seven different commonly used learning algorithms, such as reinforcement learning, fictitious play, experience weighted attraction, and level-K learning, and showed that their behavior was broadly similar, and that it is closely related to that under best reply: When best reply converges to a fixed-point equilibrium, these algorithms also tend to converge to a fixed-point equilibrium, and when it does not converge, they tend not to converge either. When these strategies fail to converge their trajectories are typically chaotic, meaning that they generate perpetual dynamics that never settles into an equilibrium. This work suggests that in situations that are complicated and competitive, convergence to equilibrium is unlikely, calling into question some situations where equilibrium is assumed from the outset, and motivating the use of agent-based modeling.

2. Agents who learn (formally)

When agents have models of their environment they are capable of formal learning. There are at least three broad classes of learning discussed in the voluminous literature on this topic. *Individual* learning is typically treated as a single agent gleaning data from its environment and updating its model of the environment. It has roots in decision theory, as a game against Nature. *Social* learning concerns multi-agent situations in which individuals build models either of the population of other agents or of individual agents. This kind of learning can involve strategic dimensions while such considerations are normally absent from individual learning. Finally, *group* learning refers to how and what individuals learn in order to behave as a group for the good of the group. This is more common in biology than the social sciences—think fish schools (Couzin *et al.*, 2011, Miller *et al.*, 2013, Kao *et al.*, 2014) or flocks of starlings (Hemelrijk and Hildenbrandt, 2011)—but occasionally appears in *MAS* computer science.

There is much work today on individual learning in behavioral and experimental economics as well as in the *ABM* and *MAS* research communities. In each area there is a wide variety of learning formalisms in use. Reinforcement learning, cue learning, probably approximately correct (*PAC*) learning and other schemes common to *MAS* have been reviewed (Shoham, Powers and Grenager, 2004, Panait and Luke, 2005). In experimental economics Erev-Roth learning (1995) and experience-weighted attraction (*EWA*) learning (Camerer, Ho and Chong, 2002) are well-known and have been surveyed (Camerer, 2003). Excellent reviews of *ABM* learning by Brenner (1999, 2006) make it unnecessary for us to rehash this literature further here.

Some early work in *ABM* learning focused on evolutionary algorithms, including genetic algorithms (Arifovic and Eaton, 1995, Bullard and Duffy, 1999, Dawid, 1999). However, lacking a strong basis as individual learning, such methods are often interpreted as population-level (social) learning, or perhaps simply as a way to incorporate richer notions of optimization into *ABM*. For example, Lindgren's (1992) use of evolutionary learning leads to interesting cycles in prisoner dilemma games, including the endogenous growth of strategy complexity. Learning at multiple levels may amplify the complexity of economic and financial phenomena.[51]

[51] It is interesting to compare approaches taken by economists to those of computer scientists when it comes to learning. More than a decade ago in a special issue of the journal *Artificial Intelligence* (Vohra and Wellman, 2007),

Twenty years ago it was common to use neural networks inside agents (LeBaron, 2001a), although this is less common today. Deep learning (LeCun, Bengio and Hinton, 2015, Schmidhuber, 2015) has begun to be used in *ABM*s but it is too early to know the implications.

3. Behavioral agents

ABMs in which agent behavior is made to reproduce the results of experiments are growing in number. There are *ABM*s in which agents behave in accord with prospect theory (Kahneman and Tversky, 1979, 1992), including the previously mentioned EurACE model, others where agents engage in hyperbolic discounting (Chen and Gostoli, 2014), and yet others in which agents possess one or more behavioral biases in their decision calculus. An *ABM* employing behavioral agents in the context of elections is the work of Bendor *et al*. (2011).

There is a relatively long history of building the behavioral specifications of *ABMs* from experimental data, as surveyed in the now somewhat dated review by Duffy (2006), assertions to the contrary notwithstanding (Wunder, Suri and Watts, 2013). For instance, Hommes and Lux (2013) used learning-to-forecast laboratory experiments with human subjects (Hommes *et al*., 2007) to create a model of heterogeneous interacting agents capable of explaining macro phenomena. Similarly, Bao, Hommes and Makarewicz (2017) use data from a laboratory experiment on bubble formation to calibrate an *ABM*. Cotla (2016) has built *ABMs* to reproduce laboratory experiments, then perturbed the experimental set-up and the *ABM* in non-trivial ways, running the computational model in advance of the actual experiment to forecast the likely outcome, then comparing the result with human subjects directly to the computational results, and finding good agreement.

Many of the behavioral specifications that have come out of laboratory experiments are not readily tractable analytically so computation may be the natural way to proceed (Simon, 1998). *ABM* is also good for 'scaling-up' laboratory results to realistic population sizes, to look at side effects, unintended consequences, etc. Surely there will be more use of behavioral and experimental results in *ABMs* going forward.

4. Other kinds of agents

There are at least three broad classes of agents that have found significant use in *ABM*s in computer science and some of the other behavioral and social sciences, but which have yet to find their way into economics and finance. First, agents outfitted with explicit cognitive mechanisms are widely used in cognitive science; a book length review is Sun (2006).[52] Rich cognitive models face challenges of intelligibility. When a social

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researchers from both disciplines wrote about the topic. The economists were mainly interested in learning schemes that led to Nash equilibria (Erev and Roth, 2007, Fudenberg and Levine, 2007, Young, 2007) while the computer scientists (Sandholm, 2007, Shoham, Powers and Grenager, 2007, Stone, 2007) asked where computationally plausible learning rules led.

[52] Cognitive science originated roughly contemporaneously with the behavioral revolution in economics (Newell and Simon, 1972). Cognitive theories implemented as cognitive architectures (Anderson, 1983), include *SOAR* (Rosenbloom *et al*., 1985, Laird, Rosenbloom and Newell, 1986), used in a variety of high-fidelity simulation environments—such as military and civilian air traffic—typically involving one or just a few humans, *ACT-R* (Anderson, Matessa and Lebiere, 1997), *CLARION* (Sun, 2006), and several focused on social behavior (Dautenhahn, 1999) and more suitable for *MAS* and *ABMs*. Although technically not a cognitive architecture the Beliefs-Desire-Intention (*BDI*) architecture is a non-human model of cognition that has been very popular in the *MAS* community (Rao

phenomenon arises in models featuring many agents, all with deep cognition, it is not clear whether it should be thought of as a consequence of the cognitive model, of the social interactions, or both. This same difficulty haunts all of the empirical social sciences, so in confronting it *ABMs* are simply recapitulating the real world. A conceptual issue that arises with cognitive models involves foresight. Building agents that make predictions about their social world is a difficult task and people use heuristics to deal with this problem (Gigerenzer, Todd and ABCResearchGroup, 1999). Gross simplifications, such as giving all the agents the same expectations, or specifying that they all have certain propensities, is one solution. It is difficult to outfit artificial agents with foresight for precisely the same reason that it is hard to make predictions in the real world, and precisely because it is hard we should expect agents to take varied approaches to the problem, adding further complexity through heterogeneity (Izumi, 2001, 2002). It is as if a veil of complexity hangs over all attempts by individual agents to forecast the future, at least for models having non-trivial levels of social complexity. To put this another way, given that one reason we turn toward *ABM* is because of the complexity of the economy, it may make sense for some of the agents in our models to have *ABM*s running in their heads. In principle such computations are possible, although rarely executed in practice.

The second kind of agent occasionally encountered beyond economic *ABM*s are those with emotions. The role of emotions in decision making has been long noted (Hume, 1896 [1739]) and has been studied with regard to economic decisions in particular (Frank, 1988). A variety of models for the role of emotions have appeared within the *MAS* literature (Elliott, 1992, Bates, 1994, Velásquez, 1997, Fix, von Scheve and Moldt, 2006, Rodríguez and Ramon, 2014). Sometimes these are grounded in data while other times they are more notional. In any case, this is an active area of research.

A third class of agent not yet much utilized in economic *ABM*s are deontic agents. While it is almost universally true that models in economics and finance make use of utilitarian agents, i.e., who pursue their own self-interests and are *purposive*—there are other ways to motivate agents. A small but growing body of research, investigates individuals who take account of norms, recognize duties and obligations, "see to it that…," while being capable of subordinating their self-interest to group goals. It turns out that there are important relations between utilitarian and deontic agents under certain conditions (Horty, 2001). Research on deontically-motivated agents has uncovered a set of modal logics known as *KD45* that have many nice properties vis-à-vis human behavior (Lomuscio and Sergot, 2002). A closely related area is doxastic logic, or reasoning about beliefs. Early surveys of these topics include Wooldridge and Jennings (1995a) and van der Hoek and Wooldridge (2007). These are active research areas, recently reviewed by {{Calegari, 2021 #5243@@author-year}}. Relatedly, Danielson (1992, 1996) considers moral agents while {{Cointe, 2016 #5245@@author-year}} assess ethical judgment in multi-agent systems.

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and Georgeff, 1995), less so in *ABM*. None of these models have deep biological grounding. A relatively new kind of cognitive model has components that resemble biologically structures. These are called biologically-inspired cognitive architectures (*BICA*), and a number of them have appeared; see Goertzel *et al*. (2010) for a review. So far they have been little used in *ABM*.

C. Direct Agent Interactions

In many economic models agents do not interact directly with one another but rather decide how to behave using aggregate economic quantities like prices, interest rates, and wage levels. That is, there are few examples of models where agents glean information directly from their peers, communicate in any meaningful way with anyone, or consummate economic exchange directly with each other. It is a kind of methodological individualism without individuals! This abstraction does little violence in static equilibrium settings, since the *mechanisms* by which fixed points are assumed to be achieved are not studied. That is, *substantive* rationality abstracts from the details of who trades with whom, or where information comes from. However, when one attempts to generate economic phenomena using *ABM*, the notion of direct agent-agent interactions comes quite naturally. In this section we focus on the varieties of such interactions, first with respect to their topology and then the manner in which agents are activated to interact.

1. Networks

In the last 15 years there has been a great flowering of the science of networks (Watts, 1999, Barabasi, 2002, Newman, 2010) with social (Jackson, 2008) and physical networks (e.g., Atalay *et al.*, 2011) earning their distinct places within economics. Networks are not part of the standard neoclassical picture, as suggested in table 1 above, as the default assumption in economics is that agents are 'well-mixed'—each sees the same prices, interest rates, wages, and so on. Relaxing the completely connected character of neoclassical agents with a network of interactions was attempted early on by Föllmer (1974) and discussed in detail by Kirman (1997). *ACE* models usually take networks into account in one way or another. A reasonably recent monograph reviewing the literature on agents and networks (Namatame and Chen, 2016) relieves us from having to review this large literature here. An important motivation for agent computing and social networks is that realistically complex networks are often difficult to work with analytically, making recourse to *ABM* natural (Jackson, 2008: 406-7).

2. Agent interaction regimes

A facet of direct agent-agent interactions beyond networks involves agent activation. It is conventional practice in *ACE* models to permit only one—or at most a few—agents to be active at any one time. In part this stipulation derives from the serial nature of the computer hardware on which agent models are typically executed. But it also stems from the desire to not engage in perfectly synchronous updating, as is common in cellular automata. Perfect synchrony should be avoided because (1) it can lead to the production of meaningless artifacts in model output, as we saw in the Nowak and May versus Glance and Huberman affair, and (2) the social world is clearly asynchronous. [53]

With all agents running on a single thread the question arises as to which should move first, which second, and so on. The order of execution can be randomized in various ways, and one might hope that the overall results of a model would not depend on

[53] One way to model complete asynchrony, and a high degree of agent autonomy, is to put each agent on its own thread of execution. This is rarely done today, due to the difficulties of writing highly parallel code. Rather, nearly all *ABMs* (>99%) are single-threaded, making agents only partially autonomous. Although it can be proved to be the case in some special circumstances (Chen and Micali, 2013).

such microscopic details, but that is generally *not* the case. While some *ABMs* use an endogenous internal model state to activate agents, such as when a profit opportunity is sensed, most do not. Rather, most models, in effect, generate a schedule of agent activations, usually stochastically. There are three ways this is commonly done (Axtell, 2001). First, in a population of A agents, *uniform activation* is the process by which all A are activated once, sequentially, in effect defining a unit of model time or a period. Over p periods each agent is activated p times. In order to insure that no artifacts are produced because the i^{th} agent always moves before the $(i+1)^{st}$, the order has to be partially randomized regularly. Usually this can be done efficiently.

Contrast this with a second activation regime, called *random activation*, in which a period is still defined as A agents being activated, but at each instant within the period the agent selected to be active is random. This means that during any particular period some agents may get turned ON more than once while others do not get activated at all. In essence, the activity of any particular agent in a period now has some variability where it did not for uniform activation. This subtle difference is known to matter in some *ABMs* (Axtell *et al*., 1996, Lawson and Park, 2000).

The third most common agent activation regime is *Poisson clock activation*. In this scheme each agent is given a series of times—a schedule—when it will be active by drawing from an exponential distribution, meaning that the times between activations are independent. The activation times of the whole population are then sorted into a *master schedule*, used to determine which agent moves next. This type of activation produces A activations/period *on average*, and within each period some agents can be more active than others, as in random activation. This regime has been used by game theorists, since it has tractable analytics (e.g., Lagunoff and Matsui, 1997).

Other activation schemes are possible (Comer, 2014) but the three above are the most common ones used in *ABM* today. They are compared in table 3. Today we still have little understanding of how to select between these distinct activation regimes. In the best of all possible worlds we would have empirical data on the character of agent activity, but such data are virtually non-existent today.

Such activation schemes are imposed on the agents exogenously. It is also possible to have *endogenous* activation in which quasi-autonomous agents decide themselves when to be active and then put in a request (to a central authority or to the operating system) to be activated. Such activation schemes are used in certain market models in which agents decide when to enter bids.

Activation Regime	***# active/period***	***activations/agent/period***
Uniform	A	1
Random	A	1 on average
Poisson clock	$\bar{A}$	1 on average

Table 3*: Comparison of common activation regimes for a population of* A *agents*

All of the above are highly democratic activation schemes. Considering agent activation to be a scarce resource, in the sense that there are only so many *CPU* clock cycles to be allocated to all agent activations, some (e.g., Page, 1997) have wondered what might happen if agents could 'buy' or otherwise acquire additional activation cycles. By analogy, when a firm hires a worker in the real world it is essentially purchasing 8 hours/day of effort that it would not otherwise have at its disposal. While it is fair to say this idea for valuing agent activation has not found its way into many models to date, it seems like a fertile idea, deserving closer study.

D. *ABM* Markets: Beyond the Walrasian Model

Markets are workhorses in *ACE* models. These come in all shapes and sizes, from *CDAs* to bilateral trading, with 2 to 2 million goods, either divisible or indivisible, and so on. But what is perhaps surprising is that it is rare for such market specifications to hew very closely to the conventional Walrasian picture of markets. One reason for this is the computational intractability of such market mechanisms, i.e., the computational complexity of Brouwer fixed points (technically in complexity class *PPAD*), as mentioned above. While there exist algorithms to approximate these, they have little to do with markets operating from the bottom up (Rust, 1997).

Given this state of affairs, the kinds of markets that appear in *ABM* often have messy features like local prices, miscoordination, and inefficiency, at least initially (Moss, 2001b). But as *ABMs* evolve through time they often reduce these problematical features: prices become more homogeneous as marginal rates of substitution (*MRSs*) get aligned, agents progressively coordinate their behavior, and inefficiencies decline. But it is rare to find anything like the perfect, noiseless, crystalline world of Walrasian prices and allocations.

A consequence of having local prices in many *ABMs* is that large volumes of trades take place at prices that are different from the Walrasian ones that could, in principle, be computed by a social planner having perfect information. Such trades have welfare effects, meaning that the utility levels produced in such decentralized, distributed markets are generally *not* Pareto optimal. At first blush this departure from the welfare theorems may appear to limit the value *ABMs* for theoretical or other work. However, the spontaneous appearance of price heterogeneity can be thought of as a *feature*, not a *bug*, for price dispersion is common in real-world markets and economies (e.g., Abbott III, 1992). Prices can fluctuate over time and vary over space, both in the real world and in *ABM* models of it. This is fine as long as price dispersion in the model reflects that found in the real world.

E. Institutions, Emergent

A common complaint against general equilibrium theorizing is that it is 'institution-free' (e.g., Leijonhufvud, 1993). The essence of this critique is that real-world economies feature a variety of institutions for either formally or informally coordinating activity. In a fully satisfactory economic model a wide spectrum of such institutions would arise, or at least be present, acting to at least partially coordinate economic activity.

Today we lack an understanding of which rules of agent behavior are sufficient to produce realistic-looking multi-agent institutions. However, there are several agent models in which certain intermediate level, multi-agent conventions and norms arise (e.g., Kandori, Mailith and Robb (1993), Young (1993a, b), Bicchieri and co-authors (1993, 1997), Shoham and Tennenholtz (1997)). For example, in models of agricultural crop sharing social conventions associated with contracting are the usual way harvests get divided (Young and Burke, 2001). The previously mentioned book by Cederman (1997) investigates the emergence of state actors. While there is still no fully satisfactory theory of such emergent institutions, some progress has been made (Young, 1998).

Work with *ABM*s on the emergence of institutions includes Smaldino and Lubell (2011, 2014) who look at coalition formation in the context of agents who play an *ecology* of games, not unlike previously mentioned work of Bednar and Page (2007) and Pangallo, Heinrich and Farmer (2019). Another example is Axtell's (1999, 2002, 2016) firm formation model. While multi-agent firms are permitted to form in this model, the overall size and structure of the population of firms can be thought of as emerging from the interactions of the agents. Under specific conditions very large firms arise, having millions of workers, and once these are produced, they alter the landscape of employment opportunities that are available to other agents in the population. That is, once this structure has emerged from the bottom up, it has important ramifications from the top down for subsequent epochs of the economy. In finance, questions related to which kinds of securities populate markets have been studied by Noe, Rebello, and Wang (2003, 2006). Using *ABM* they find that certain market environments support the formation of complex securities but heterogeneous, learning agents may have difficulty coordinating on their pricing, so simpler types of financing can persist.

In order for an institution to be considered emergent it is necessary to describe a mechanism that produces it. Some have claimed that the quintessential example of emergence in economics is Adam Smith's invisible hand, and the corresponding welfare theorems of general equilibrium (e.g., Durlauf, 2012). But today we still do not know how to generate Walrasian prices from the bottom up, so we do not understand the mechanisms for emergence.[54, 55]

F. Economies as Many-Level Systems

It is conventional for economists to consider economies as multi-level systems, to treat the agent level as different from the aggregate level, as shown in figure 5. In moving between levels one must be careful not to succumb to the dual fallacies of composition and division. It is a truism in economics that knowing how the micro (agent) level works may not give us much insight about the operation of the macro-level, an explicit acknowledgement of the *fallacy of composition*. But explaining aggregates in terms of individuals is all too common, as when particular stock market fluctuations are anthropomorphized as reflecting the 'mood' of investors or of the market overall. Likewise, attempting to draw conclusions about the agent level from aggregate data is equally problematical, a *fallacy of division*, closely related to the ecological inference problem in econometrics. Agent computing offers a way to explore the multi-level character of economies by permitting higher order structures to emerge from lower level interactions.[56]

<Figure 5 about here>

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[54] For Nozick (1994), invisible hand explanations required credible underlying social processes.

[55] For instance, Crockett, Spear and Sunder (2008) have studied how individuals might learn general equilibrium prices by repeatedly facing approximately the same market conditions from day-to-day and learning what prices to pay (locally) that yield individually budget-balanced conditions throughout the economy, i.e., for all agents. But even in this highly restricted environment they find non-convergence for sufficiently large numbers of agents and goods.

[56] Interestingly, within the *MAS* research community, populated as it is mostly by computer scientists and engineers, there is strong difference of opinion as to the value of *emergence* in agent models. This was clearly on display in the inaugural edition of a new journal in which some of the editors thought that focus on emergent behavior was warranted and part thought it was not (Jennings, Sycara and Wooldridge, 1998).

For example, consider again the emergence of firms from the decisions of individual workers (Axtell, 2016). In general equilibrium theory it is normal to consider firms as occupying the same ontological level as consumers. But it is useful to consider them as 'above' the agent level, since they are composed of multiple agents. They occupy a meso level, with the aggregate level, involving the entire population of firms, above. This three level picture of an economy is shown in figure 6. Considering the aggregate state to be z while the meso-level is y. An *ABM* operates solely at the lowest level x, and its code represents the function f for marching the model forward in time. How one might derive the functions g and h for faithfully representing the higher levels is an important question for fields like industrial organization and macroeconomics. These functions are implicit in the *ABM* in the sense that at each instant in time we can simultaneously observe x, y, and z. When the many levels of regulation and governance are taken into account real economies may be 4, 5, or 6 level systems.

<Figure 6 about here>

G. Social Steady-States With or Without Agent-Level Equilibrium

When agents are adaptive and their environment is changing, or when agents are learning how to alter their behavior to take advantage of their circumstances, behavior at the agent level may change. Changes in behavior may indicate that the agent level is out of equilibrium. As argued by Arthur (2006), this situation is very common in *ABM*. In fact, in wide classes of economic and financial *ABMs* it is normal to observe sizable fractions of agents changing their behavior regularly. Perpetual adaptation and adjustment is the norm in *ABM*, which may or may not lead to systemic changes at the aggregate level. Such changes may resemble mixed strategy Nash equilibria, as we saw in the El Farol model, but they need not.

Equilibrium at the agent level, e.g., Nash or Walras, certainly implies equilibrium at the aggregate level, i.e., it is *sufficient* (Farmer and Geanakoplos, 2009). However, it is not *necessary*, for it is possible to have aggregate stationarity even without equilibrium at the agent level. In practice, most *ABM*s exhibit flux at the agent level yet stable patterns and statistics at the aggregate level. Mathematically it would be useful to have solution concepts that permit agent dynamics and population patterns of this type. While important research on this topic has appeared (Aoki, 1998, 2001), much work remains.

One way to think about all this is that macro-steady-states are emergent phenomena. What can we say about them ahead of time? Can we deduce things about their size or stability? We are reductionists and these emergent configurations are produced by the actions of the agents. But we are also pragmatic anti-reductionists (Simon, 1996 [1969], Faith, 1998), and just as it is hard to determine what will emerge in the 'Game of Life' simply by looking at the rules, experience shows that it is very hard to forecast the character of emergent steady-states in the economy.[57]

[57] Occasionally one encounters the claim that all *ABM*s are simply large albeit finite Markov processes—in fact, Markov chains—but even if this were strictly true it would provide little leverage on determining emergent properties.

H. Empirical Grounding of Agent Economies

There are a wide variety of ways that *ABM*s attempt to represent the real world, and several distinct approaches to making such models reflect reality.[58] Given the multi-level character of all *ABMs*, a specific model may be empirically-relevant at one or more levels. For example, Friedman (1953) famously argued that a model could be useful at the aggregate level while being behaviorally wrong at the agent level, what Simon dubbed "the principle of unreality" (Simon, 1963). In this section we will briefly review distinct approaches to building progressively more realistic *ABM*s, roughly following the typology laid out in Axtell and Epstein (1994).

ABMs always need behavioral specifications. Where do these come from? Possibly the model builder has sufficient knowledge of the domain to create reasonable behavioral rules in software, at least up to some unknown parameters (to be estimated). Possibly domain experts can be queried for rules, parameters, or both, a process known in decision theory as *expert elicitation* (Morgan, Henrion and Small, 1990), although care must be taken to avoid certain pitfalls (Morgan, 2014). There are also techniques to infer rules directly from data (Thagard, 1988).

1. Agent models *qualitatively* reproduce aggregate patterns

The next level for an *ABM* to pass muster is whether it is capable of producing outputs that qualitatively resemble aggregate data, i.e., that can match stylized facts. There are typically gross patterns in aggregate data that an *ABM* should match in order to be considered successful. The way this is done with *IBMs* has been nicely surveyed (Grimm *et al*., 2005) and many of the same considerations apply in the social sciences. Returning to artificial stock markets, for example, models that cannot reproduce qualitative aggregate phenomena like clustered volatility, heavy-tailed return distributions, and log prices time series with little autocorrelation should be viewed with skepticism.

In practice there seem to be a wide range of approaches for specifying parameters in models of this type. Sometimes they can be approximated from experience, sometimes set from logical, dimensional, or model-specific regularity considerations (e.g., suppliers will not lower their posted prices below their costs), and sometimes they are simply invented and tested through experimentation to determine what is needed to produce the kinds of aggregate patterns desired.

2. Agent models *quantitatively* reproduce aggregate data

When aggregate patterns are quantifiable then more formal calibration and estimation techniques can be employed. Perhaps the most common approach used in *ABM* for specifying parameters is search of a model's parameter space in order to minimize the difference between model output and the aggregate data. When the

[58] The phrase 'verification and validation', abbreviated V&V, is common in operations research for questions concerning the veracity of computational models. Verification refers to whether a model is logically sound, e.g., free of bugs, and executing in accord with an independent specification of the model—basically, that it is doing what it is supposed to be doing. This is typically a low bar and checked heuristically. (While formal methods exist in computer science that can be brought to bear on such questions (Wooldridge *et al*., 2002, Belardinelli *et al*., 2018), they are usually impractical for non-trivial *ABM*s.) Validation is a relatively uncommon term in the social sciences, for in asking whether a model is a valid depiction of a social process, the answer is rarely yes or no. Given that these terms are not widely used in economics and finance we shall not discuss the empirically-grounding *ABM*s as V&V.

parameter space is not prohibitively large then conventional estimation techniques can be used (Heard, 2014, Heard *et al*., 2015). Computational techniques created for analytically intractable models, such as 'estimation by simulation' (McFadden and Ruud, 1994) can often be adapted to *ABM*. Such formal estimation procedures are commonly used in financial market *ABM* (e.g., Alfarano, Lux and Wagner, 2005, 2006, 2007). Spatial *ABMs* can also be estimated using such approaches (Hooten and Wikle, 2010). When the parameter space is large it becomes necessary to search heuristically (Michalewicz and Fogel, 2000, Luke, 2013), e.g., via evolutionary algorithms (e.g., Terano, 2007). A recent review of empirical validation methods for *ABM*s is Lux and Zwinkels (2018).

3. Agent models *quantitatively* reproduce micro-data

Many of the same techniques can be used when the kind of data that are available are at the individual level. Considerations related to microeconometrics are now in play (Cameron and Trivedi, 2005), such as the Manski critique (Manski, 1993, 1995, 1997). In essence, if data are not gathered to preserve independence and other properties it will not be possible to distinguish selection effects.

One way around some of these problems is to acquire individual-level data from experiments. In sections III.B.2 and III.B.3 we have discussed the use of experimental data to specify agent models (Duffy, 2006, Wunder, Suri and Watts, 2013, Cotla, 2016). This approach has been utilized in finance settings as well (e.g., Hommes, 2011). It may turn out that there are fundamental limits to what can be predicted at the individual level with any method (Salganik *et al*., 2020).

I. *ABMs* for Policy

In section II.B.2 we described the use of *ABM* by *NASDAQ* management to assess the effects of decimalization on the operation of their market in advance of its implementation as policy was described. In the future it would seem reasonable for policy-makers to avail themselves of *ABM* technology in order to test in advance which kinds of regulations make the most sense, whether for producing greater social welfare or simply to avoid noxious side effects of untested policies (Helbing, 2012). For instance, it may turn out that certain kinds of policies are sensitive to things like agent heterogeneity, that *ABM*s are good at representing (e.g., Arifovic, Bullard and Kostyshyna, 2012). Alternatively, evolutionary or other open-ended approaches for policy synthesis could be employed, possibly yielding surprising, even counter-intuitive policy ideas, such as the 'faster is slower' phenomenon described in Gershenson and Helbing (2015). While the penetration of *ABM* for such purposes into governance institutions in economics and finance has been modest to-date, in other fields policy decisions based on *ABM* have become standard (e.g., in epidemic control, Gemann *et al*., 2006, Gomes *et al*., 2014, Ferguson *et al*., 2020).

IV. Future Opportunities and Challenges for *ABM*

In this section we discuss areas where *ABM* is poised to make progress and others where there appear to be significant roadblocks and further research is needed.

A. Opportunity: Micro-Data Integration (Including Social Network Data)

We discussed multiple *ABMs* in which micro-data were essential to the operation of the model. When such data are available, whether from administrative records, customer information, *GIS* layers, or elsewhere, *ABM* can serve as a platform for integrating them.

Social network data represent a case in point. Twenty-five years ago neither *ABMs* nor social network analysis (*SNA*) were in the vocabulary of economists. A great deal has changed since the early 1990s. It is our experience that many people working at the research frontier tend to think either in *ABM* or *SNA* terms but few do both. Thus, independent software ecosystems have grown up around these two distinct approaches and overall, there is currently very little integration. Surely this will change. A decade ago there was very little integration between *GIS* software and *ABMs*, but now many of the major agent-based software systems are able to read shape files, making true spatial modeling possible.

B. Opportunity: Moving To Large-Scale and Full-Scale Models

An unusual property of *ABM*s is that typically the actual programs are quite small, involving only a few hundred lines of code for prototype models or perhaps a few thousand lines for models written in native code. Nonetheless, very large amounts of memory can be filled by the agent population.[59] It is normally a relatively simple matter to expand an agent model to fill available memory by simply instantiating more agents. Larger numbers of agents can generate qualitatively different behavior—Anderson's (1972) 'more is different' idea—and this may be important in establishing the empirical relevance of an agent model. Larger populations also produce more robust statistics.

Indeed, this last point has a special interpretation when it comes to *ABM* economies. For many empirical quantities in economic data are distributed with heavy-tails. Example of this include Pareto distributed wealth, firm sizes, and city sizes. These data are so skew that for small numbers of agents it is hard to assess the character of the distributions that arise. With firm sizes there is a big difference in going from 10,000 agents, in which the biggest firm might be size 100 (1% of the population in the largest firm), up to 1,000,000 agents and a size 10,000 firm. Clearly an economy with a biggest firm of size 100 is quite different from one in which it is 100x larger.

C. Challenge: Software Tools that are Easier to Use

Existing platforms and frameworks for building *ABMs* are summarized in an appendix to this article. But are these really the right tools for the economics profession today? There is a good opportunity to create economics-specific *ABM* tools and technologies that facilitate the creation of the kinds of models that economists care about. Examples include purposive agents, profit-seeking (*CES* or Dixit-Stiglitz) firms, central banks that have the Taylor rule in their behavioral repertoire, and commercial banks that lend only to businesses. In the same way that specific statistical software packages grew up to service the needs of econometricians, we need new *ABM* packages that are geared toward economists who would like to construct models without needing to become experts in software engineering. Such software tools are on the horizon. It us now up to us to shape those in ways suitable for both research and teaching.

[59] A more technical way to say this is that *ABM*s have a small compile-time, potentially large run-time character.

D. Challenge: Parallel Execution

There are two reasons why it is highly desirable to use many computational units in parallel. The first is the practical question of execution speed: *ABM*s typically require substantial computation for testing and estimation. If Moore's law slows down, as it is predicted to do soon and may already be doing, parallel execution provides the most promising avenue for speeding things up. However, getting significant speedup through parallel processing is a difficult problem. (The exception is so called "embarrassing parallelism", which corresponds to running models independently; this is sufficient for some purposes, such as exploring a model's parameter space, but is inadequate for many others).

The other reason why parallel processing is important is because this is how the economy operates. We must be careful when we use single-threaded code, which does everything serially, to mimic a world in which actions occur in parallel. The real social and economic worlds *are* parallel and asynchronous and it is an open question as to whether there are things that can happen in such worlds that are either difficult or impossible to faithfully represent using the kinds of parallel computing tools available today.

E. Opportunity and Challenge: Is *ABM* a Kind of Nanoeconomics?

More than 30 years ago the late Kenneth Arrow, in commenting on remarks of certain economic historians that seemed to be at a level of analysis lower than conventional microeconomics, wondered if a new kind of economics was needed to properly address those concerns, what he termed 'nanoeconomics' (Arrow, 1987: 734). We have seen above that there may be certain aspects of *ABM* that go below standard microeconomics, such as the order of execution in single-threaded models, parallel agent execution, and related issues having to do with how and when agent states are updated. Such considerations manifest themselves to some extent in the theory of oligopoly, with first-mover advantage and so on, and to some extent in certain kinds of games, but are not present in other branches of micro. Largely this is because microeconomic models are solved for equilibria, without worrying about the paths along which the agents might move toward such equilibria.

This leads one to wonder if one upshot of *ABM*, in which the details of agent-agent interaction matter, may require moving to a more fine-grained nanoeconomic level. The usual micro-macro distinction in economics refers to the level of abstraction in terms of agents—traditionally macro abstracted from individuals. But with micro-foundations of macro now the norm, this distinction has been muddied. Macro today is primarily concerned with aggregate economic variables, produced by individuals. *Nanoeconomics* might contrast to microeconomics in similar fashion. We seek nanoeconomic foundations for micro in the sense that the detailed interaction histories—the nano-level—determine the micro-level. In the end, for nanoeconomics to become a field it would be necessary for it to find a home in other branches of economics. How might new new trade models unfold when one exporter moves first? How can a developing country catch-up to a neighbor when the neighbor has modernized first? These are the kinds of questions economists working in trade and development might care about and today there is no very satisfactory way to pose them because microeconomics does not operate at levels that would seem to be needed to resolve them.

F. Opportunity: New Kinds of *ABMs* for Economics and Finance

Some 25 years ago the science journalist Mitchell Waldrop thought researchers at the Santa Fe Institute were on the verge of producing "economies under glass" in software (Waldrop, 1992). This vision has not been realized. To do so would require new kinds of models. For instance, imagine a model of a developing country in which all citizens and firms are represented, that can be used to digitally experiment with alternative government policies. In computer science such high fidelity digital environments are sometimes called 'mirror worlds' (Gelernter, 1992) or 'digital twins' (Grieves and Vickers, 2017). Sometime within the next few decades it should become possible to produce high fidelity, digital models of economies in which every household, every consumer, every firm, every worker, and every policy-maker are represented in some way. It is our belief that technologies like *ABM* will play an important role in such developments.

G. Challenge: The Curse of Dimensionality

Models in economic theory tend to be minimal of necessity in order to be analytically tractable. But we have seen that such tractability is not a concern with *ABM*. Rather, tractability is a rationale for moving from mathematical models to *ABMs*. Thus, while a microeconomic model on some topic may have just a few parameters, many of which can be explored systematically, it is often the case that a corresponding *ABM* has more parameters. Many of the software platforms and frameworks have special tools for studying the effect of parameters on model output, such as *NetLogo's* BehaviorSpace facility, so simply having more parameters is not automatically a problem.

However, as the size of the parameter space grows the cost of exploring it rises exponentially. This is called the curse of dimensionality, first adumbrated by Bellman (1957) in the context of dynamic programming (*DP*). This problem is by no means unique to *ABMs*, but rather occurs in many domains of science and in many different types of models. There are examples of very minimal *ABMs* having only a few parameters, e.g., segregation (Schelling, 1971a), *Sugarscape* (Epstein and Axtell, 1996), or Axelrod's (1997b) culture model. There are other examples of *ABMs* having a great many parameters, many of which can be tied down with data, including power market models (Nicolaisen, Petrov and Tesfatsion, 2000, Tesfatsion, 2020). To reiterate an earlier point, *ABM*s range from very simple to quite complex—there is nothing about *ABM* per se that mandates that the models need to be complicated.

H. Challenge: The Forecasting Problem for *ABMs*

In conventional macroeconomic models the ability to create forecasts is an essential feature that makes them useful to policy-makers. Such models are estimated with current data and then initialized to make forecasts or to study the effects of alternative policies. For an *ABM* macro model to be used for time series forecasting, an important problem arises that has no analog in normal macroeconomics.

Conventional macro-models are constructed in terms of aggregate *variables* that coincide with acquired data about the economy. This is convenient because the resulting data can be directly used to initialize these models. But *ABM*s are dynamical systems that model the world at the level of individual agents, such as households and firms. In order to run the model it is necessary to initialize the states of all of the individual agents in a way that is also compatible with aggregate measurements. Doing this properly requires complete micro-data on individuals, which is typically not available.

Absent such data it becomes necessary to invent plausible states for each individual agent so that the aggregate states of the model match the measured aggregate data. However the individual states in the model also need to compatible with each other and with the inherent dynamics of the model. If the states are not compatible in this sense, the model will generate transient behaviors that will result in poor forecasts. Given that model forecasts are never perfect, this is a recurrent problem—as time passes the forecasts of the model inevitably deviate from the measured aggregate data, and the initialization process must be repeated again and again. Finding good methods for doing this is an open problem that must be solved if we are to use ABMs for time series forecasting.[60]

I. Challenge and Opportunity: How to Create *ABM* Community Models?

With the rise of the digital computer over the past 70+ years, scientific disciplines have institutionalized its use in different ways. While computing resources were initially operated in centralized fashion within most colleges, universities, think tanks, government laboratories, and other research institutions, the personal computer revolution of the 1980s largely decentralized computing, putting it close to the ultimate users. However, in a few fields things evolved somewhat differently.

Weather modeling is one of the important legacies of John von Neumann's early efforts with digital computing (Edwards, 2010). In the wake of progress, work on weather at Princeton became institutionalized at the Geophysical Fluid Dynamics Lab (*GFDL*), supported by strong Federal government funding to keep the nascent numerical weather modeling enterprise alive. One rationale for this funding was that accurate weather forecasts were viewed as a military asset. Over decades there grew up at Princeton, and later at the National Center for Atmospheric Research (*NCAR*) in Boulder, Colorado, a suite of *community models* relevant to weather, climate, and other atmospheric and oceanic projects. These models today continue to be sustained by federal research funding—the National Oceanic and Atmospheric Administration (*NOAA*) has a line in its budget for *GFDL* and *NCAR* is a Federally-funded research and development center (*FFRDC*) operated by *NSF*—to the tune of \$40M and \$200M per year, respectively. Much of this money is spent on high-performance computing (*HPC*). Combining these numbers with the \$1.1B spent by the National Weather Service (*NWS*), also under *NOAA*, gives some sense of the kind of support it takes to mount a scientific enterprise at continental scale. By comparison, the *entire* budget of *NSF*'s Social and Economic Sciences (*SES*) Division, within the larger Social, Behavioral & Economic Sciences (*SBE*) Directorate, is about \$100M annually, the Economics Program receiving about \$30M in a recent fiscal year.

Creating a branch of economic modeling that is a community activity will likely take a generation, as it did with numerical weather and climate models. Figure 7 portrays the evolution. If high performance computing technologies are going to come to the economics profession, we should learn from the model for husbanding computational resources and building scientific constituencies that atmospheric and oceanic scientists have built.

[60] Other aspects of forecasting social systems have been addressed by Hofman, Sharma and Watts (2017).

<Figure 7 about here>

J. Opportunity: *ABM* as a Teaching Tool

As Economics has matured as a discipline and instructional content has become standardized, the textbooks have grown ever thicker and we ask students to read more and more, either explicitly, as stipulated on syllabi, or implicitly, given the heft of the texts. While reading is a traditional path toward learning, not all students are traditional learners. Indeed, reading 500 or more pages over the course of a semester is not something that is readily accomplished by all students. While various modern forms of educational materials, from instructional slides to ebooks and apps, are available for many economics courses, what is largely absent from contemporary curricula is the ability for students, both undergraduate and graduate, to interact with and experiment on virtual economies. While initial attempts to provide somewhat interactive materials exist, for example in the guise of computable general equilibrium (*CGE*) models (Thompson and Thore, 1992) or dynamic stochastic general equilibrium (*DSGE*) ones (Adjemian *et al*., 2018), these have not found wide use for instructional purposes.

A potential opportunity for *ABM* is in the classroom where specific ideas could be illustrated with a working model and then students could actively engage with software to perform 'what if' and related analyses. The zero-intelligence NetLogo code of McBride, discussed above, is an example of what can be done for supply and demand ideas. Given that the main abstraction of *ABM* is the individual agent, and that this comports closely to the methodological individualism of the entire field, most areas of economics and finance could be illuminated for students in this fashion, through the creation of *ABM*s on specific topics. It is also the case that whole miniature economies could be created precisely for purposes of self-instruction and experimentation by students. Such 'synthetic economies' need not have high-fidelity with any real economy, and thus not be large-scale in nature, but rather be representative of typical economies so that when students vary parameters and test hypotheses they receive, in return, model output that is qualitatively similar to how actual economies operate. Perhaps someday soon economics textbooks will have links to working *ABM*s illustrating key points.

V. Conclusion: *ABMs* as an Emerging Methodology for Economists

Computational economies composed of software agents represent a new paradigm for economic research. Their role in substantive model building is yet nascent with some clear successes, as in finance. Nonetheless, many areas remain relatively untouched by this new technique.

A. *ABM* as a Modern Computational Methodology for Economics and Finance

By now it is hopefully clear that the strengths of the multi-agent systems methodology for economic model building involve its expressiveness (through behavioral rules not rationality assertions), its ease of implementation technologically (via software objects), its extensibility (as the user community learns how to share and extend models), its ability to let macroscopic structures emerge (no need for pre-specification of what will happen), and its agnosticism toward agent-level equilibrium. These many features of the methodology conspire to produce great potential for relaxing some of the strong assumptions that are required for making standard mathematical models in economics.

Perhaps these many features of *ABM* methodology can serve all branches of the economic profession, from the behavioral economist's desire to better represent human behavior, to the applied economist's efforts to more realistically specify models and test them with data, to the theorist's focus on basic insight from abstract, stylized models.

B. *ABM* as Analogous to Earlier Methodological Evolutions in Economics

The recent development of *ABM* and its application to problems in economics and finance is broadly comparable to developments in other areas of economics, some closely related.

The appearance of von Neumann's and Morgenstern's monograph (1944) opened up a whole new field in economics for immediate exploitation in the post-WWII era.[61] A bevy of important game theorists came through the Princeton Mathematics Department at this time, including John Nash, Lloyd Shapley, Martin Shubik and later Robert Aumann and Alan Kirman. Initially, game theory appeared to be far from economics proper, and most of these people took jobs in math departments. It was not until the 1980s that game theory really took off within economics, primarily because models of antitrust behavior led to its widespread application by the Department of Justice. By the 1990s game theorists were actively recruited by economics departments, representing more than a 40 year lag from the birth of the discipline to its widespread adoption. This is for a methodology that was as mathematical as the field it was trying to penetrate.

The situation in behavioral and experimental economics is perhaps like that in game theory, but with a 15-20 year lag. The initial results in these fields began appearing in the late 1950s, but it was not until the 1990s that behavioral economics began to be accepted in mainstream economics departments, with hiring really taking off only in the 2000s, again some 40+ years after inception.

Figure 8 is a plot of the number of published papers over time that make overt use of game theory as a main methodology (green), experimental techniques (magenta) and *ABM* (blue). (The numbers for game theory and *ABM* include all fields, not just economics.) The figure on the left is in linear coordinates and an exponential takeoff for each is apparent. The figure on the right is in *log* coordinates on the vertical axis and the roughly straight lines indicate exponential growth. Note that the takeoff phase for *ABM* has occurred with smaller lag from inception than in either of these other areas. Clearly *ABMs* resemble these other areas with a lag. Given these patterns it would seem reasonable to expect continued penetration of agent techniques into economics.

<Figure 8 about here>

C. Computational Progress in Other Areas of Science

We are living through a computational revolution. We have already compared advances in computational economics to those that are occurring in nearly every other branch of science. Earlier in this section we made explicit comparisons to numerical weather and climate computing. Among the most ambitious computational models to date are general circulation models (*GCM*s) used by climate scientists to forecast the likely effects of Earth's climate of human-produced emissions of (fossil) carbon, primarily from

[61] The Morgenstern diaries recount that some early mathematical economists were nonplussed by game theory in the 1940s and '50s.

combustion. These models are written at Earth-scale and include atmospheric, oceanic, and terrestrial zone, each disaggregated into millions of discrete bins. Figure 9 gives results from the Coupled Model Intercomparision Project (*CMIP*). Each colored circle is a model and lower values of the I^2 metric are better. *CMIP*-1 goes back more than 20 years, *CMIP*-2 about 15 years, and *CMIP*-3 about 10 years. Substantial progress in climate modeling has occurred over time. Better hardware and deeper understanding of the science involved drives this progress, while the community structure of the models facilitates it.

<Figure 9 about here>

D. Computational Economics and the Economics of Computation

A generalized version of Moore's law amounts to the statement that microprocessor hardware capabilities grow exponentially with time. This is largely independent of how performance is measured, whether in *CPU* frequency, floating point operations/second (*FLOPS*), transistor or memory density, hard disk capacity, or external communication rates. In most of these dimensions, performance has doubled each 18-24 months over the past two-three decades, although along some dimensions progress has stalled, leading to multi-core *CPUs*. These developments are depicted in figure 10. A microeconomic way of describing these developments is that the cost of a unit of computation has fallen exponentially fast for several generations. As the price of computing falls economists will use more of it, unless it is an inferior good!

We have no doubt that as computing technology continues to improve researchers in economics and finance will make progressively more and more use of it, for numerical economics, *ABM*, *SNA*, machine learning, high-frequency trading, and so on. It may not be long before we can manage our computational models from our mobile phones. We are well on our way toward what we might reasonably call computationally-enabled economics (Axtell, 2008). Indeed, the biggest challenge that researchers would seem to face today may be, 'What can we possibly achieve, on the modest hardware we have at our disposal, that will be of interest to future generations of scholars who have 10x or 100x or 1000x more?' The risk of computational economics today is not that it is being adopted too quickly, potentially over-running conventional, analytical approaches, but rather that it is being adopted so slowly that as the price continues to fall we will be deluged with it without knowing fully how to use it. To a first approximation the cost of updating a single agent in an *ABM*—the marginal cost of agent computing—is falling toward zero. Will this lead to changes in the economics profession the way the Internet, with its near zero marginal cost of distributing email, news, magazines, books, images, money, control, security, surgery, even life-and-death communications, has changed all of our lives in the past two decades, for both better and worse? Time will tell.

<Figure 10 about here>

Among the many flavors of computational economics, *ABM* is poised to fully utilize our rapidly expanding computational power and data resources. Greater compute power means that natural scientists can model larger systems or build models of specific phenomena at higher spatial and/or temporal resolutions. Numerical economics, along with its close cousins computational econometrics, computable general equilibrium modeling, and even microsimulation, typically do *not* fully utilize all the parts of the machine, i.e., they may use the greater processing power of *CPUs*, even *GPUs*, but typically do not utilize that enormous amounts of *RAM* that can be addressed today, or they use large-scale storage but make no use of the tremendous visualization capabilities that modern workstations often possess. Agent computing techniques, on the other hand, permit the complete utilization of all extant hardware—*CPUs*, *GPUs*, *RAM*, disks, displays with millions of color pixels and palettes, fast networks, all cores, etc.[62]

One of the main results of financial economics concerns portfolio diversification. Under standard Markowitz portfolio criteria, assets with low correlation to the rest of the portfolio are assigned positive weights as long as their mean expected return is positive. Similar reasoning should apply to the research portfolio of economics. *ABM* certainly satisfies the criterion of low correlation to standard methods. We hope that we have demonstrated here that it has a substantial positive expected mean return, and should thus play a larger role in economic research in the future.[63]

Between ever-increasing computer power to execute agent-based models, the new availability of micro-data to parameterize such models, and ongoing advances in behavioral and experimental economics to provide rules of behavior for the agents in *ABM*, the time is ripe for the field of economics to embrace agent computing as another tool in its quiver as it tries to solve hard problems associated with complex economies.

62 This point has been made at length elsewhere (Axtell, 2008).

63 This point is reinforced by sociological studies that show that a diversity of viewpoints results in better solutions (Page, 2007).

Appendix 1: Meanings of Acronyms Mentioned

ABE: agent-based economics
ABM: agent-based modeling or agent-based model
ACE: agent-based computational economics
AI: artificial intelligence
ALife: artificial life
BR: bounded rationality
CA: cellular automaton or automata
CAS: complex adaptive system
CDA: continuous double auction
CES: constant elasticity of substitution
CGE: computable general equilibrium model
CRISIS: European Union funded project to model the Financial Crisis with *ABM*
DAI: distributed artificial intelligence
DSGE: dynamic stochastic general equilibrium model of macroeconomics
EU: European Union
EWA: experience-weighted attraction, an empirically-grounded learning algorithm
FFRDC: Federally-funded research and development corporation
GARCH: generalized autoregressive conditional heteroskedasticity
GFDL: Geophysical Fluid Dynamics Laboratory at Princeton University
GIS: geographic information systems
GSIA: Graduate School of Industrial Administration at the Carnegie Institute of Technology; today: Tepper School of Business at Carnegie-Mellon University
IBM: individual-based model
LANL: Los Alamos National Laboratory
LFN: labor flow network
MAS: multi-agent systems
MERS: Middle East Respiratory Sickness
MIDAS: Models of Infectious Disease Agent Study at *NIH*
MLS: Multiple listing service, a real estate firm and data aggregator
MRS: marginal rate of substitution of one good for another
NASDAQ: National Association of Securities Dealers Automated Quotations
NCAR: National Center for Atmospheric Research
NIH: National Institutes of Health
NOAA: National Oceanic and Atmospheric Administration
NSF: National Science Foundation
NWS: National Weather Service
OFR: Office of Financial Research within the Department of Treasury
OR: operations research
REE: rational expectations equilibrium/equilibria
SARS: severe acute respiratory syndrome
SBE: Social, Behavioral & Economic Sciences Directorate at *NSF*
SD: system dynamics
SEC: Security and Exchange Commission
SES: Social and Economic Sciences Division at *NSF*
SNA: social network analysis

SOES: Small Order Execution System on the *NASDAQ*
UCAR: University Consortium for Atmospheric Research
V&V: verification and validation
VaR: value at risk
WMAD: Walras-McKenzie-Arrow-Debreu model of general equilibrium
ZI: zero-intelligence, trading agents who act purposively but without an internal model
ZIP: zero-intelligence plus trading agents

Appendix 2: Computer Terms, Languages, and Systems Discussed

ABM: agent-based model or agent-based modeling
ACT-R: computational cognitive architecture
AgentSheets: simple, user-friendly *ABM* software environment
ASCII: American Standard Code for Information Interchange, for character encoding
BASIC: early programming language, little used today
BDI: belief-desires-intentions representation of agent behavior, popular in *MAS*
C: early low-level programming language, still in wide use today
C++: object-oriented version of *C*
C#: object-oriented programming language from Microsoft
CLARION: computational cognitive architecture
CMIP: Coupled Model Intercomparison Project
CORMAS: *ABM* software commonly used for natural resource models
CPU: central processing unit
DP: dynamic programming, pioneered by Richard Bellman in the 1950scu
DSGE: dynamic stochastic general equilibrium model of macroeconomics
EINSTEIN: combat modeling toolkit
EPISIMS: epidemic simulation code derived from TRANSIMS at Los Alamos
EurACE: agent-based macroeconomic model in use in Europe for research and policy
FLAME: *ABM* software environment for running models on *GPU*s
FLOPS: floating point operations per second
FORTRAN: early programming language, still in use today for scientific computing
GAMS: General Algebraic Modeling Systems
GEMS: General Electric modeling and simulation language
GPSS: general purpose simulation system
GPU: graphics processing unit
HPC: high-performance computing
ISAAC: Irreducible, Semi-Autonomous Adaptive Combat model, early military *ABM*
JABOWA: early forest simulation system in *IBM* ecology
Java: *OOP* language originally created by Sun Microsystems, currently owned by Oracle
MABM: macroeconomic *ABM*
MASON: *ABM* software framework in Java from George Mason University
Mathematica: commercial mathematics software and programming package
MATLAB: commercial software package
MESA: *ABM* software framework in Python
NetLogo: popular *ABM* environment requiring modest programming background
NP: complexity class of problems solvable nondeterministically in polynomial time
Objective-C: early object-oriented programming language, still in use at Apple
ODD: protocol for reporting *ABM*s
OOP: object-oriented programming
P: complexity class of problems solvable in polynomial time
PAC: probably approximately correct learning, a learning algorithm
Pascal: programming language created at ETH Zurich in the 1970s, litte used today
PPA/*PPAD*: complexity classes between *P* and *NP*; polynomial parity argument on either undirected or directed graphs
RAM: random access memory

RePast and RePast HPC: open source *ABM* software framework in Java, C++, and C# from Argonne National Laboratory
RNG: random number generator
SimScript: early simulation language, still in use today
SIMULA: the first OOP language and a family of simulation languages
SmallTalk: early object-oriented programming language, in little use today
SOAR: early computational cognitive architecture
StarLogo: early programming language for beginners from MIT
Sugarscape: early *ABM* in which agents forage for resources and engage in exchange
SWARM: early agent-based modeling language
TRANSIMS: transportation simulation code created at Los Alamos

Appendix 3: Implementation of *ABM*s

Creating an *ABM* involves some amount of computer programming, so a researcher's ability to effectively utilize this new approach is often proportional to one's computing skills. But no very specific computational background is required, since *ABM*s can be created in a wide variety of ways. While courses in algorithms and data structures are helpful, the most important skill to possess for creating an *ABM* is strong command of some specific programming language, such as Java, Python, C/C++, C#, and so on. By far the most common question people have who are new to *ABM* is 'What software should I use to build my model?' This question has many facets and picking the wrong software for a project can be disastrous. Here we provide some guidelines based on current technology. Happily, there are good comparisons of existing software packages—Kravari and Bassiliades (2015), supplementing older ones of Gilbert and Bankes (2002) and Dibble (2006)—meaning we can be brief, editorializing a bit based on our experience.

There are essentially four distinct ways to create an *ABM*, (1) code in a native programming language like *Java* or *Python*, (2) write your model in a mathematical or statistical environment like *MatLab*, *R*, or *Mathematica*, (3) code your model using a software framework for *ABM* like *RePast*, *MASON*, *FLAME*, or *MESA*, or (4) create your model in a high-level, *ABM*-specific software environment like *NetLogo* or *HashAI*. Each of these systems has advantages and disadvantages, so selection involves trade-offs. Specifically, the *lower* the number on our list the faster your model will probably run, eventually, once it is successfully coded and debugged. However, the coding and debugging time declines as the number on our list gets *higher*. For example, native *Java* code is going to run much faster than *NetLogo* code but it might take you significantly longer (2-10x) to get a non-trivial model up and running in *Java* than in *NetLogo*.
Empirically, most *ABM*s used for research in economics are built in *NetLogo*, *MASON*, or *RePast*. These are each mature systems with large user bases, reasonable documentation, and performance good enough to use for research. In finance it is probably the case that more than half of all *ABMs* are created in *MatLab*. This is because that system is designed to high-performance numerical computation and is especially suitable for solving equations—agents in finance *ABM*s often have to solve portfolio optimization, arbitrage, and other mathematical problems in determining how to behave. We summarize the characteristics and performance of these four systems in table 4 where we also include *Mathematica*, not because it is widely used for *ABM* but because many economists use it. Software systems less often used for research *ABM* include SWARM (Minar *et al*., 1996, Terna, 1998, Luna and Stefansson, 2000, Stefansson, 2000), CORMAS (LePage *et al*., 2000), and AgentSheets (Repenning, Ioannidou and Zola, 2000), and we will not say more about these here. For each of the entries in the table we provide a few points of description and one or more references to the literature. *NetLogo* (Wilensky and Rand, 2015) combines a programming language (having hybrid *OOP* and functional features) with an highly configurable development and analysis environment. It is excellent for rapid-prototyping but too slow for large models. *MASON* (Luke *et al*., 2005) is based on Java and requires users to code in that language. It has excellent analysis and visualization interfaces. *RePast* (North, Collier and Vos, 2006) users code their model in either *Java* or *C#*. It has many features in common with *MASON*. *MatLab* has object extensions but they were added relatively recently and are often not used by

people writing *ABM* code. Its performance is good. Objects are not a natural part of *Mathematica* but the functional aspects of its programing language means that *ABM*s can be written *very* compactly. For instance, Gaylord and D'Andria (1998) have programmed the Schelling model in 5 lines of *Mathematica* code! However, it tends to be slower than the others in execution of *ABM*.

Software	***OOP?***	***Programming***	***Compiled?***	***Animations?***	***Speed***	***Max agents***
NetLogo	yes	own language	no	yes	poor	10,000?
MASON	yes	*Java*	byte code	yes	good	1,000,000
RePast	yes	*Java*, *C#*	byte code	yes	good	1,000,000
MatLab	partial	own language	can be	yes	good	100,000
Mathematica	partial	own language	can be	slow	poor	1,000?

***Table 4**: Comparison of software environments for creating* ABM

A newer approach to *ABM* deserving of brief mention is through programming the video boards that are part of all modern microcomputers. These so-called graphics processing units (*GPUs*) have greatly improved their performance with progress in video game technology. D'Souza, Lysenko and Rahmani (2007) programmed the *Sugarscape* model to run 1,000,000 agents at 25 frames/second while only a few hundred agents could run at that speed when the model was first created (Epstein and Axtell, 1996). *FLAME* is an *ABM* programming environment designed specifically for *GPUs* (Kiran *et al*., 2010).

Figures:

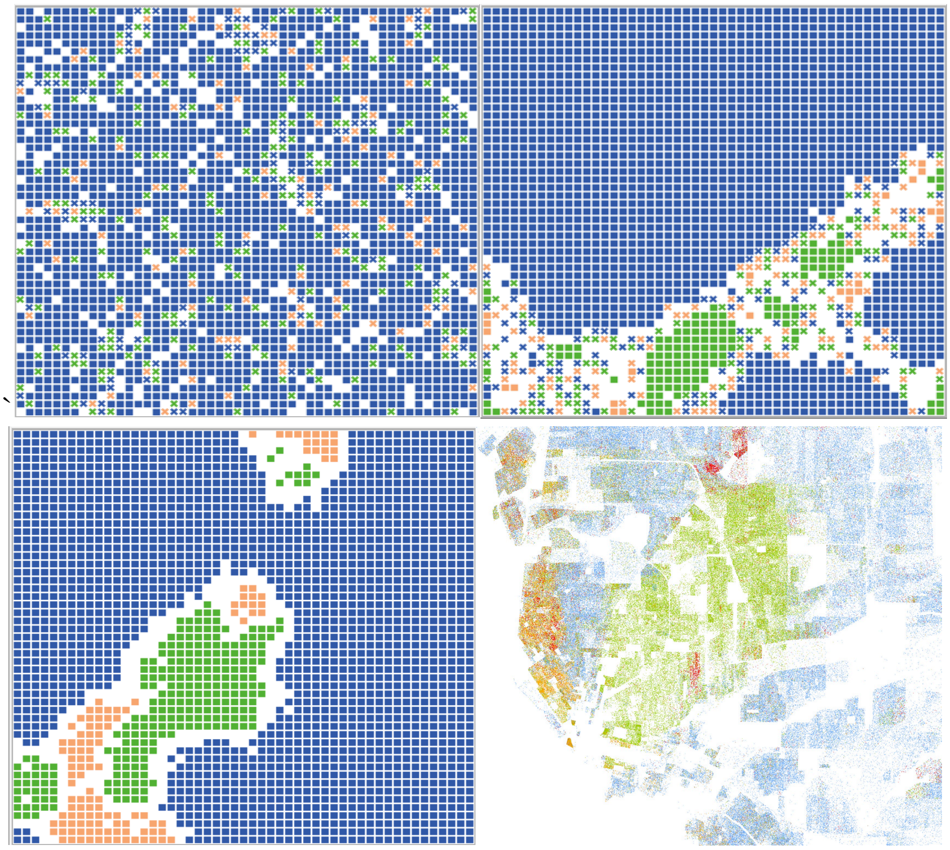

Figure 1: *Onset of segregation in a realization of a Schelling model of residential choice from random initial conditions (upper left) through changing conditions (upper right) to segregated neighborhoods (lower left), and actual segregation in Buffalo, N.Y. c. 2010, with each person represented by one pixel*

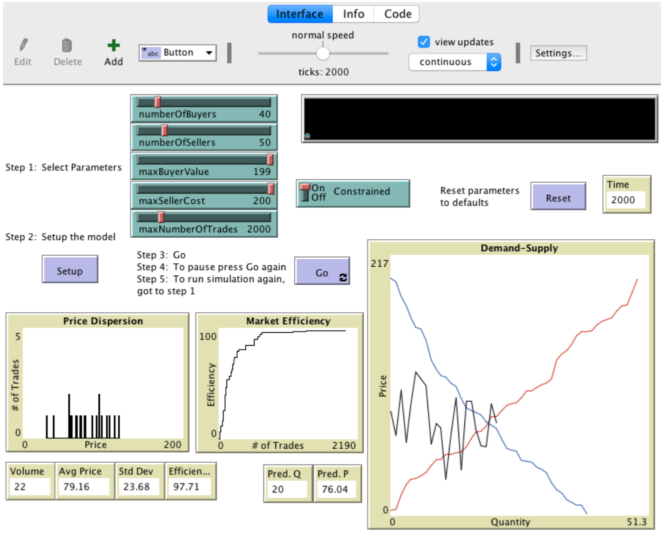

***Figure 2**: Screen capture from the zero intelligence trader model of M. McBride, in NetLogo software; the red line is the supply curve, the blue line demand, and the black line represents prices paid; other figures show overall market efficiency and price dispersion; each realization of the model yields a different result*

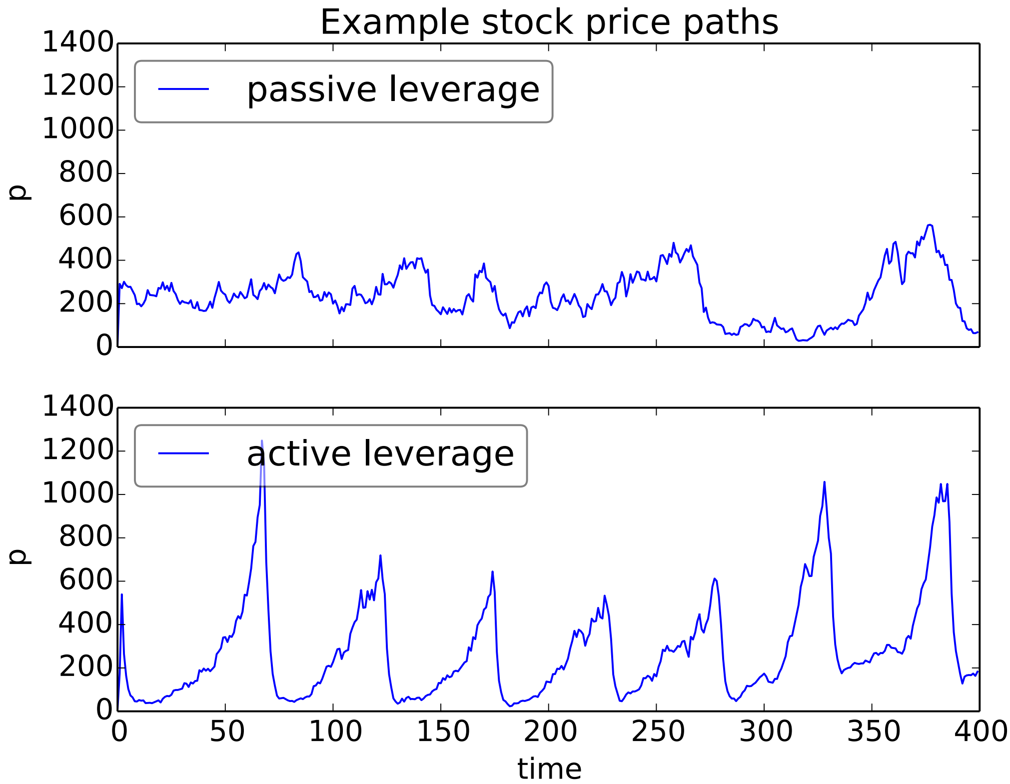

Figure 3: *Example paths for the price of the risky asset from Aymanns et al. (2016). In the top panel, banks behave like households, i.e. they do not adjust their balance sheets following a change in leverage. Prices roughly follow a random walk – volatility is driven by the exogenous noise fed into the system. In the bottom panel, banks actively manage their leverage attempting to achieve a risk-dependent target leverage, similar to broker-dealers. Prices now show endogenous, stochastic, irregular cycles.*

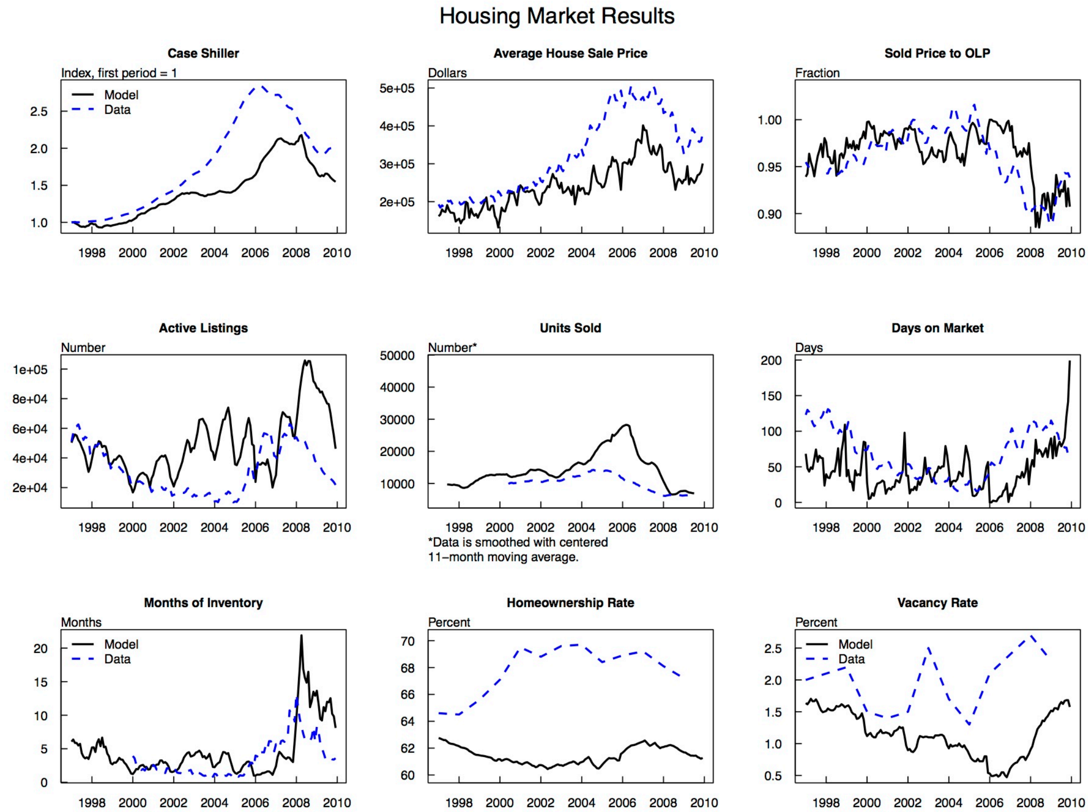

***Figure 4**: Output from an agent-based housing market bubble model, with data shown as dashed lines and model output solid black lines*

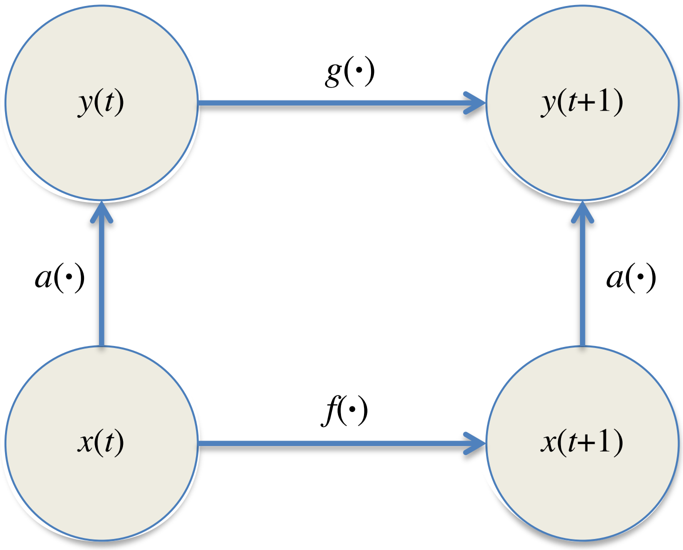

Figure 5*: A two level economy with* x(t) *the micro-level,* y(t) *the macro level, with aggregation functions* a() *mapping states from micro to macro*

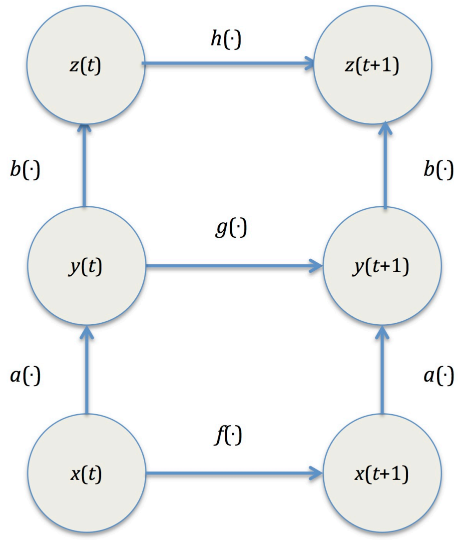

Figure 6*: A three level economy with* x(t) *the micro-level,* y(t) *the meso-level, and* z(t) *the macro level, with aggregation functions* a(.) *and* b(.) *mapping states from micro to meso and meso to macro, respectively*

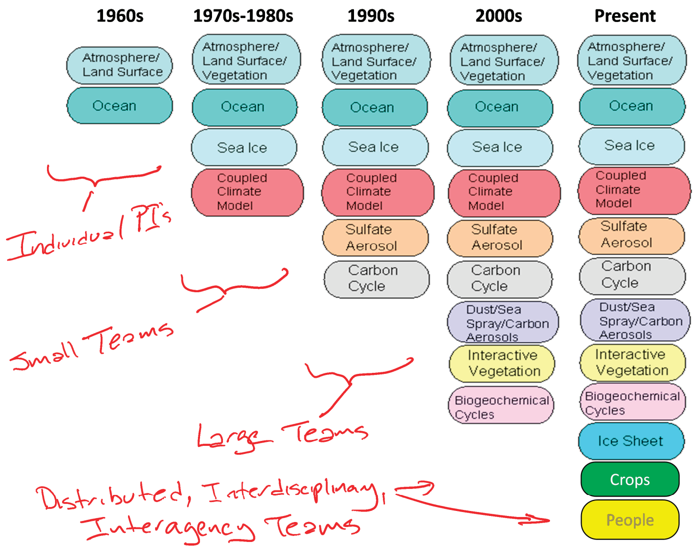

Figure 7*: Growth of community models in weather and climate research; source: Higdon et al. (2016)*

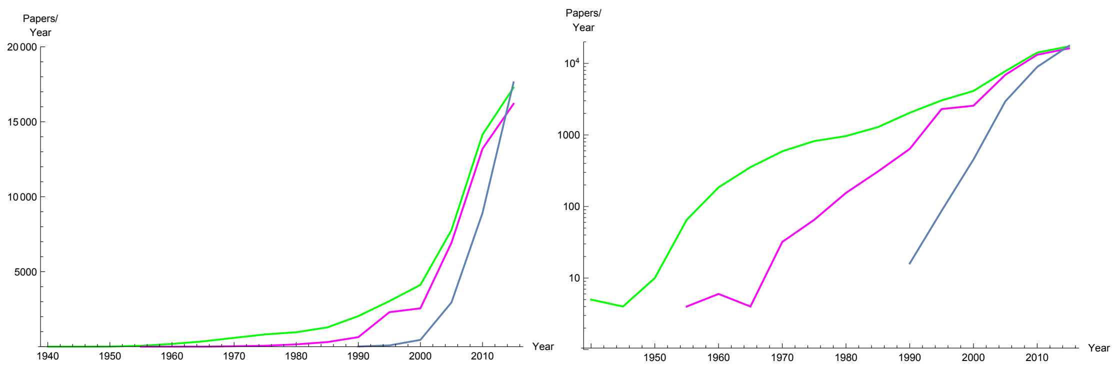

Figure 8*: Number of papers appearing each year that explicitly identify as using game theory (green), experimental economics (magenta), and* ABM *(blue) methods, (a) linear and (b) log coordinates*

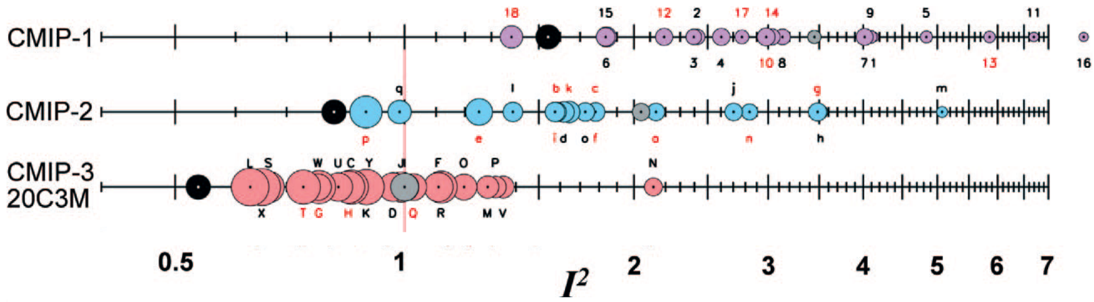

Figure 9*: Increasing accuracy in climate modeling; CMIP-1 is from the 1990s, CMIP-2 dates from the early 2000s, while CMIP-3 c. 2007; each colored circle is one model; importantly, the multi-model mean outperforms any single model; source: Higdon et al. (2016)*

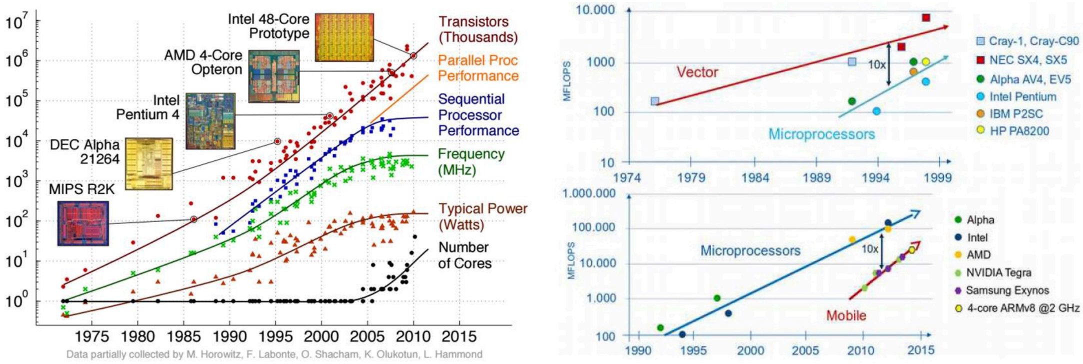

Figure 10*: Evolution of computer hardware with total transistors following Moore's law, with the performance of individual processors plateauing and greater overall performance achieved with more cores (right); convergence of commodity computing to high-performance computing over time*

References

Abbott III, Thomas A. 1992. "Price Dispersion in U.S. Manufacturing: Implications for the Aggregation of Products and Firms," In Center for Economic Studies Discussion Paper. Washington, D.C.

Adiga, Abhijin *et al.* 2015. "Generating a synthetic population of the United States," In Network Dynamics and Simulation Science Laboratory Technical Report.

Adjemian, Stéphane *et al.* 2018. "Dynare: Reference Manual Version 4," In, 202. Paris, France: CEPREMAP.

Agar, Michael and Dwight Wilson. 2002. "Drugmart: Heroin Epidemics as Complex Systems." *Complexity*, 7(5), pp. 44-52.

Aktipis, C. Athena. 2004. "Know when to walk away: contingent movement and the evolution of cooperation." *Journal of Theoretical Biology*, 231(2), pp. 249-60.

____. 2006. "Recognition Memory and the Evolution of Coopration: How Simple Strategies Succeed in an Agent-Based World." *Adaptive Behavior*, 14(3), pp. 239-47.

____. 2008. "When to walk away and when to stay: Cooperation evolves when agents can leave unproductive partners and groups," In Psychology. Philadelphia, Penn.: University of Pennsylvania.

____. 2011. "Is cooperation viable in mobile organisms? Simple Walk Away rule favors the evolution of cooperation in groups." *Evolution and Human Behavior*, 32(4), pp. 263-76.

Albin, Peter S. 1975. **The Analysis of Complex Socioeconomic Systems**. Lexington, Mass.: Lexington Books, D.C. Heath & Company.

Albin, Peter S. and Duncan K. Foley. 1992. "Decentralized, Dispersed Exchange Without an Auctioneer: A Simulation Study." *Journal of Economic Behavior and Organization*, 18(1), pp. 27-51.

____. 1998. **Barriers and Bounds to Rationality: Essays on Economic Complexity and Dynamics in Interactive Systems**. Princeton, N.J.: Princeton University Press.

Alfarano, Simone; Thomas Lux and Friedrich Wagner. 2005. "Estimation of Agent-Based Models: The Case of an Asymmetric Herding Model." *Computational Economics*, 26(1), pp. 19-49.

____. 2006. "Estimation of a simple agent-based model of financial markets: An application to Australian stock and foreign exchange data." *Physica A: Statistical Mechanics and its Applications*, 370(1), pp. 38-42.

____. 2007. "Empirical validation of stochastic models of interacting agents: A "maximally skewed" noise trader model." *European Phys. J. B*, 55(2), pp. 183-87.

Allen, Todd M. and Christopher Dixon Carroll. 2001. "Individual Learning About Consumption." *Macroeconomic Dynamics*, 5(2), pp. 255-71.

Amman, Hans M.; David A. Kendrick and John Rust eds. **Handbook of Computational Economics, Volume I**. New York, N.Y.: North-Holland, 1996.

Anderson, John R. 1983. **The Architecture of Cognition**. Mahwah, N.J.: Lawrence Erlbum Associates, Inc.,.

Anderson, John R.; M. Matessa and C. Lebiere. 1997. "ACT-R: A theory of higher level cognition and its relation to visual attention." *Human Computer Interaction*, 12(4), pp. 439-62.

Anderson, Philip W. 1972. "More is Different." *Science*, 177(4047), pp. 393-96.

Anderson, Philip W.; Kenneth J. Arrow and David Pines eds. **The Economy as an Evolving Complex System**. 1988.

Andrei, Amanda L.; Kevin Comer and Matthew Koehler. 2014. "An agent-based model of network effects on tax compliance and evation." *Journal of Economic Psychology*, 40, pp. 119-33.

Andrews, K.T. and Michael Biggs. 2006. "The dynamics of protest diffusion: Movement organizations, social networks, and news media in the 1960 sit-ins." *American Sociological Review*, 71(5), pp. 752-77.

Anshelevich, Elliot; Sanmay Das and Yonatan Naamad. 2013. "Anarchy, stability, and utopia: Creating better matchings." *Autonomous Agents and Multi-Agent Systems*, 26(1), pp. 120-40.

Antinyan, Armenak; Gergely Horváth and Mofei Jia. 2019. "Social status competition and the impact of income inequality in evolving social networks: An agent-based model." *Journal of Behavioral and Experimental Economics*, 79, pp. 53-69.

Antona, M.; Francois Bousquet; Christophe Le Page; J. Weber; A. Karsenty and P. Guizol. 1998. "Economic Theory of Renewable Resource Management: A Multi-agent System Approach," In Mult-Agent Systems and Agent-Based Simulation. MABS 1998, ed. J. S. Sichman, R. Conte and N. Gilbert. Springer.

Antunes, Luis; Joao Balsa; Ana Respicio and Helder Coelho. 2007. "Tactical Exploration of Tax Complance Decisions in Mult-agent Based Simulation," In Multi-Agent-Based Simulation VII, ed. L. Antunes and K. Takadama. Berlin, Germany: Springer.

Aoki, Masanao. 1998. **New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects**. New York, N.Y.: Cambridge University Press.

____. 2001. **Modeling Aggregate Behavior and Fluctuations in Economics: Stochastic Views of Interacting Agents**. New York, N.Y.: Cambridge University Press.

Aoyama, Hideaki; Yoshi Fujiwara; Yuichi Ikeda; Hiroshi Iyetomi and Wataru Souma. 2010. **Econophysics and Companies: Statistical Lfe and Death in Complex Business Networks**. New York, N.Y.: Cambridge University Press.

Arifovic, Jasmina. 1996. "The Behavior of Exchange Rates in the Genetic Algorithm and Experimental Economies." *Journal of Political Economy*, 104(3), pp. 510-41.

____. 2001. "Evolutionary dynamics of currency substitution." *Journal of Economic Dynamics and Control*, 25, pp. 395-417.
Arifovic, Jasmina; James Bullard and Olena Kostyshyna. 2012. "Social Learning and Monetary Policy Rules." *The Economic Journal*, 123, pp. 38-76.
Arifovic, Jasmina and C. Eaton. 1995. "Coordination via Genetic Learning." *Computational Economics*, 8, pp. 181-203.
Arifovic, Jasmina and Paul Masson. 2004. "Heterogeneity and Evolution of Expectations in a Model of Currency Crisis." *Nonlinear Dynamics, Psychology, and Life Sciences*, 8(2), pp. 231-58.
Arrow, Kenneth J. 1987. "Reflections on the Essays," In Arrow and the Foundation of the Theory of Economic Policy, ed. G. Feiwel. New York, N.Y.: New York University Press.
Arthur, W. Brian. 1991. "Designing Economic Agents That Act Like Human Agents: A Behavioral Approach to Bounded Rationality." *American Economic Review*, 81(2), pp. 353-59.
____. 1994. "Inductive Reasoning and Bounded Rationality." *American Economic Review*, 84(2), pp. 406-11.
____. 2006. "Out-of-equilibrium economics and agent-based modeling," In Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics, ed. K. Judd and L. Tesfatsion. New York, N.Y.: North-Holland.
____. 2015. **Complexity and the Economy**. New York, N.Y.: Oxford University Press.
Arthur, W. Brian; Steven N. Durlauf and David A. Lane eds. **The Economy as an Evolving Complex System II**. Reading, Mass.: Addison-Wesley, 1997.
Arthur, W. Brian; John Henry Holland; Blake LeBaron; Richard Palmer and Paul Tayler. 1997. "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market," In The Economy as an Evolving Complex System II, ed. W. B. Arthur, S. N. Durlauf and D. A. Lane. Reading, Mass.: Addison-Wesley.
Ashby, W.R. 1952. **Design for a Brain**. New York, N.Y.: Wiley.
____. 1956. **An Introduction to Cybernetics**. New York, N.Y.: Wiley.
Atalay, E.; A. Hortascu; James Roberts and Chad Syverson. 2011. "Network structure of production." *Proc Natl Acad Sci USA*, 108(13), pp. 5199-202.
Auchincloss, Amy H.; Rick L. Riolo; Daniel G. Brown; Jeremy Cook and Ana V. Diez Roux. 2011. "An Agent-Based Model of Income Inequalities in Diet in the Context of Residential Segregation." *American Journal of Preventive Medicine*, 40(3), pp. 303-11.
Axelrod, Robert. 1984. **The Evolution of Cooperation**. New York, N.Y.: Basic Books.
____. 1995. "A Model of the Emergence of New Political Actors," In Many-Agent Simulation and Artificial Life, ed. E. Hillebrand and J. Stender. Washington, D.C.: IOS Press.
____. 1997a. **The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration**. Princeton, N.J.: Princeton University Press.
____. 1997b. "The Dissemination of Culture: A Model with Local Convergence and Global Polarization." *Journal of Conflict Resolution*, 41, pp. 203-26.
____. 2003. "Advancing the Art of Simulation in the Social Sciences." *Journal of the Japanese Society for Management Information Systems*, 12(3).
Axtell, Robert. 2008. "The Rise of Computationally Enabled Economics: Introduction to the Special Issue of the "Eastern Economic Journal" on Agent-Based Modeling." *Eastern Economic Journal*, 34(4), pp. 423-28.
Axtell, Robert *et al.* 2016. "Challenges of Integrating Complexity and Evolution into Economics," In Complexity and Evolution: Toward a New Synthesis for Economics, ed. D. S. Wilson and A. Kirman. Cambridge, Mass.: MIT Press.
Axtell, Robert L. 1999. "The Emergence of Firms in a Population of Agents: Local Increasing Returns, Unstable Nash Equilibria, and Power Law Size Distributions," In Working paper. Santa Fe, N.M.: Santa Fe Institute.
____. 2001. "Effects of Interaction Topology and Activation Regime in Several Multi-Agent Systems," In Multi-Agent-Based Simulation, ed. S. Moss and P. Davidsson, 33-48. Heidelberg, Germany: Springer Verlag.
____. 2002. "Non-Cooperative Dynamics of Multi-Agent Teams," In Proceedings of the First International Joint Conference on Autonomous Agents and Multiagent Systems, ed. C. Castelfranchi and W. L. Johnson, 1082-89. Bologna, Italy: ACM Press.
____. 2005. "The Complexity of Exchange." *Economic Journal*, 115, pp. F193-210.
____. 2007. "What economic agents do: How cognition and interaction lead to emergence and complexity." *Review of Austrian Economics*, 20(2-3), pp. 105-22.
____. 2016. "120 Million Agents Self-Organize into 6 Million Firms: A Model of the U.S. Private Sector," In Proceedings of the 15th International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS2016), ed. J. Thangarah, K. Tuyls, C. Jonker and S. Marsella. Singapore: International Foundation for Autonomous Agents and Multiagent Systems.
____. 2017. "Hayek Enriched by Complexity Enriched by Hayek." *Advances in Austrian Economics*, 21, pp. 63-121.
____. 2018. "Endogenous Firm Dynamics and Labor Flows with Heterogeneous Agents," In Handbook of Computational Economics, ed. C. Hommes and B. LeBaron. Amsterdam: North-Holland.
Axtell, Robert L.; Robert Axelrod; Joshua M. Epstein and Michael D. Cohen. 1996. "Aligning Simulation Models: A Case Study and Results." *Computational and Mathematical Organization Theory*, 1(2), pp. 123-41.

Axtell, Robert L. and Joshua M. Epstein. 1994. "Agent-Based Models: Understanding Our Creations." *Bulletin of the Santa Fe Institute*.
____. 1999. "Coordination in Transient Social Networks: An Agent-Based Computational Model of the Timing of Retirement," In Behavioral Dimensions of Retirement Economics, ed. H. J. Aaron, 161-83. Washington, D.C.: The Brookings Institution Press.
Axtell, Robert L. *et al.* 2002. "Population Growth and Collapse in a Multiagent Model of the Kayenta Anasazi in Long House Valley." *Proc Natl Acad Sci USA*, 99(supplement 3), pp. 7275-9.
Axtell, Robert L.; Joshua M. Epstein and H. Peyton Young. 2001. "The Emergence of Classes in a Multi-Agent Bargaining Model," In Social Dynamics, ed. S. N. Durlauf and H. P. Young, 191-211. Cambridge, Mass./Washington, D.C.: MIT Press/Brookings Institution Press.
Axtell, Robert L.; Omar A. Guerrero and Eduardo López. 2019. "Frictional unemployment on labor flow networks." *Journal of Economic Behavior and Organization*, 160, pp. 184-201.
Axtell, Robert L. and Steven O. Kimbrough. 2008. "The High Cost of Stability in Two-Sided Matching: How Much Social Welfare Should be Sacrificed in the Pursuit of Stability," In World Congress on Social Simulation. George Mason University, Fairfax, Virginia, USA.
Aymanns, Christoph; Fabio Caccioli; J. Doyne Farmer and Vincent W.C. Tan. 2016. "Taming the Basel leverage cycle." *Journal of Financial Stability*, 27, pp. 263-77.
Aymanns, Christoph; J. Doyne Farmer; Alissa M. Kleinnijenhuis and Thom Wetzer. 2018. "Models of Financial Stability and Their Application in Stress Tests," In Handbook of Computational Economics, Heterogeneous Agent Modeling, ed. C. Hommes and B. LeBaron, 329-91. Amsterdam, Netherlands: Elsevier, North-Holland.
Bailey, Richard M. *et al.* 2019. "A computational approach to managing coupled human-envioronmental systems: the POSEIDON model of ocean fisheries." *Sustainability Science*, 14(2), pp. 259-75.
Bak, Per. 1996. **How Nature Works: The Science of Self-Organized Criticality**. New York, N.Y.: Copernicus.
Bak, Per; Maya Paczuski and Martin Shubik. 1997. "Price variations in a stock market with many agents." *Physica A*, 246(3-4), pp. 430-53.
Bankes, Steven C. 2002. "Agent-Based Modeling: A Revolution?" *Proc Natl Acad Sci USA*, 99(suppl. 3), pp. 7199-200.
Banks, Jerry and John S. Carson III. 1984. **Discrete Event System Simulation**. Englewood Cliffs, New Jersey: Prentice-Hall.
Bao, Te; Cars Hommes and Tomasz Makarewicz. 2017. "Bubble Formation and (In)Efficient Markets in Learning-to-Forecast and Optmise Experiments." *The Economic Journal*, 127, pp. F581-F609.
Baptista, R.; J. Doyne Farmer; M. Hinterschweiger; K Low; D. Tang and A. Uluc. 2017. "Macroprudential Policy in an Agent-based Model of the Housing Market," In Bank of England Report. London, U.K.
Barabasi, Albert-Laszlo. 2002. **Linked: The New Science of Networks**. Boston: Perseus.
Barde, S. 2016. "Direct comparison of agent-based models of herding in financial markets." *Journal of Economic Dynamics and Control*, 73, pp. 329-53.
Barmpalias, George; Richard Elwes and Andy Lewis-Pye. 2014. "Digital Morphogeneisis vis Schelling Segregation," In IEEE 55th Annual Symposium on Foundations of Computer Science (FOCS). Philadelphia, Penn.: IEEE.
Barrett, Chris and R. Beckman. 1995. "TRANSIMS - Portland Case Study Report, Volume I: Introduction and Overview," In Technical Report. Los Alamos, N.M.: Los Alamos National Laboratory.
Barrett, Chris *et al.* 1995. "An Operational Description of TRANSIMS," In Technical Report. Los Alamos, N.M.: Los Alamos National Laboratory.
Bass, Frank. 1969. "A New Product Growth Model for Consumer Durables." *Management Science*, 15, pp. 215-27.
Bastardie, Francois; J. Rasmus Nielsen and Tanja Miethe. 2013. "DISPLACE: a dynamic, individual-based model for spatial fishing planning and effort displacement—integrating underlying fish population models." *Canadian Journal of Fisheries and Aquatic Sciences*, 71(3), pp. 366-86.
Bates, Joseph. 1994. "The Role of Emotion in Believable Agents." *Communications of the ACM*, 37(7), pp. 122-25.
Battiston, Stafano *et al.* 2016. "Complexity theory and financail regulation." *Science*, 351(6275), pp. 818-19.
Batty, Michael. 2005. **Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based Models, and Fractals**. Cambridge, Mass.: MIT Press.
Becker, Gary S. 1962. "Irrational Behavior and Economic Theory." *Journal of Political Economy*, 70(1), pp. 1-13.
Beckman, R. 1997. "TRANSIMS-Release 1.0 - The Dallas-Ft. Worth Case Study," In Technical Report. Los Alamos, N.M.: Los Alamos National Laboratory.
Bednar, Jenna; Aaron Bramson; Andrea Jones-Rooy and Scott Page. 2010. "Emergent cultural signatures and persistent diversity: A model of conformity and consistency." *Rationality and Society*, 22(4), pp. 407-44.
Bednar, Jenna; Yan Chen; Tracy Xiao Liu and Scott Page. 2012. "Behavioral spillovers and cognitive load in multiple games: An experimental study." *Games and Economic Behavior*, 74, pp. 12-31.
Bednar, Jenna and Scott E. Page. 2007. "Can Game(s) Theory Explain Culture? The Emergence of Cultural Behavior Within Multiple Games." *Rationality and Society*, 19(1), pp. 65-97.
Beinhocker, Eric. 2006. **The Origin of Wealth: How Evolution Creates Novelty, Knowledge, and Growth in the Economy**. Cambridge, Mass.: Harvard Business School Press.

Belardinelli, Francesco; Alessio Lomuscio; Aniello Murano and Sasha Rubin. 2018. "Decidable Verification on Multi-agent Systems with Bounded Private Actions," In Autonomous Agents and Multi-Agent Systems, ed. E. Andre and S. Koenig, 1865-67. Stockholm, Sweden: Association for Computing Machinery.
Bell, Ann Maria. 1997. "Bilateral Trading on a Network: Convergence and Optimality Results," In Department of Economics Working Papers. Nashville, Tennessee: Vanderbilt University.
Bell, Ann Maria and William A. Sethares. 2001. "Avoiding global congestion using decentralized adaptive agents." *IEEE Transactions on Signal Processing*, 49(11), pp. 2873-79.
Bell, Ann Maria; William A. Sethares and J.A. Bucklew. 2003. "Coordination failure and congestion in information networks." *IEEE Transactions on Signal Processing*, 51(3), pp. 875-85.
Bell, Daniel and Irving Kristol eds. **The Crisis in Economic Theory**. New York, N.Y.: Basic Books, 1981.
Bellman, Richard. 1957. **Dynamic Programming**. Princeton, N.J.: Princeton University Press.
Benard, S. and R. Wiler. 2007. "A wealth and status-based model of residential segregation." *Journal of Mathematical Sociology*, 31, pp. 149-74.
Bendor, Jonathan; Daniel Diermeier; David A. Siegel and Michael Ting. 2011. **A Behavioral Theory of Politics**. Princeton, N.J.: Princeton University Press.
Bennett, Robert L. and Barbara R. Bergmann. 1986. **A Microsimulated Transactions Model of the United States Economy**. Baltimore, Maryland: The Johns Hopkins University Press.
Berger, Thomas. 2001. "Agent-based spatial models applied to agriculture: a simlation tool for technology diffusion, resource use changes and policy analysis." *Agricultural Economics*, 25(2-3), pp. 245-60.
Bergmann, Barbara R. 1980. "A Short Description of a Micro-Simulated Macro-Economic Model: The Transactions Model of the U.S. Economy," In Micro Simulation - Models, Methods and Applications, ed. B. R. Bergmann, G. Eliasson and G. H. Orcutt. Stockholm, Sweden: The Industrial Institute for Economic and Social Research.
____. 1990. "Micro-to-Macro Simulation: A Primer With a Labor Market Example." *Journal of Economic Perspectives*, 4(1), pp. 99-116.
Bergmann, Barbara R.; Gunnar Eliasson and Guy H. Orcutt eds. **Micro Simulation - Models, Methods and Applications**. Stockholm, Sweden: The Industrial Institute for Economic and Social Research, 1980.
Bergstrom, Theodore C. and John H. Miller. 1997. **Experiments with Economic Principles: Microeconomics**. New York, N.Y.: McGrow-Hill/Irwin.
Berry, Brian J.L.; L. Douglas Kiel and Euel Elliott. 2002. "Adaptive agents, intelligence, and emergent human organizaiton: Capturing complexity through agent-based modeling." *Proc Natl Acad Sci USA*, 99(3), pp. 7187-88.
Bicchieri, Cristina. 1993. **Rationality and Coordination**. New York, N.Y.: Cambridge University Press.
____. 2006. **The Grammar of Society: The Nature and Dynamics of Social Norms**. New York, N.Y.: Cambridge University Press.
Bicchieri, Cristina; Richard Jeffrey and Brian Skyrms eds. **The Dynamics of Norms**. New York, N.Y.: Cambridge University Press, 1997.
Biggs, Michael. 2005. "Strikes as Forest Fires: Chicago and Paris in the Late Nineteenth Century." *American Journal of Sociology*, 110(6), pp. 1684-714.
Billari, Francesco C. and Alexia Prskawetz eds. **Agent-Based Computational Demography: Using Simulation to Improve Our Understanding of Demographic Behaviour**. New York, N.Y.: Physica-Verlag, 2003.
Binney, James and Scott Tremaine. 2008. **Galactic Dynamics**. Princeton, N.J.: Princeton University Press.
Blok, David J.; Sake J. de Vlas; Roel Bakker and Frank J van Lenthe. 2015. "Reducing Income Inequalities in Food Consumption: Explorations With an Agent-Based Model." *American Journal of Preventive Medicine*, 49(4), pp. 605-13.
Bloomquist, Kim. 2010. "Tax Compliance an an Evolutionary Coordination Game: An Agent-Based Approach." *Public Finance Review*, 39(1), pp. 25-49.
____. 2012. "Agent-Based Simulation of Tax Reporting Compliance," In Department of Computational Social Science. Fairfax, Virginia: George Mason University.
Blume, Lawrence E. and Steven N. Durlauf eds. **The Economy as an Evolving Complex System, III: Current Perspectives and Future Directions**. New York, N.Y.: Oxford University Press, 2005.
Boccara, Nino. 2010. **Modeling Complex Systems**. New York, N.Y.: Springer Science+Business Media.
Bonabeau, Eric. 2002. "Agent-Based Modeling: Methods and Techniques for Simulating Human Systems." *Proc Natl Acad Sci USA*, 99(supplement 3), pp. 7280-87.
Botkin, D.B.; J.F. Janak and J.R. Wallis. 1972. "Some ecological consequences of a computer model of forest growth." *Journal of Ecology*, 60, pp. 849-72.
Bouchaud, J.P.; J. Bonart; J. Donier and M. Gould. 2018. **Trades, Quotes and Prices**. New York, N.Y.: Cambridge University Press.
Bousquet, Francois. 1996. "Fishermen's Society," In Simulating Societies, ed. N. Gilbert and J. Doran. London: UCL Press.
Bousquet, Francois; C. Cambier and P. Morand. 1994. "Distributed Artificial Intelligence and Object-Oriented Modeling of a Fishery." *Mathematical Computation Modeling*, 20(8), pp. 97-107.
Bousquet, Francois and Christophe Le Page. 2004. "Multi-agent simulation and ecosystem mnagement: a review." *Ecological Modeling*, 176, pp. 313-32.

Bowles, Samuel. 2004. **Microeconomics: Behavior, Institutions, and Evolution**. New York, N.Y./Princeton, N.J.: Russell Sage Foundation/Princeton University Press.
Bowles, Samuel and Jung-Kyoo Choi. 2013. "Coevolution of farming and private property during the early Holocene." *Proc Natl Acad Sci USA*, 110(22), pp. 8830-35.
Brandt, C.; N. Immorlica; G. Kamath and R. Kleinberg. 2012. "An Analysis of One-Dimensional Schelling Segregation," In Proceedings of the 44th Annual ACM Symposium on Theory of Computing.
Brandt, Felix; Vincent Conitzer; Ulle Endriss; Ariel D. Procaccia and Jérome Lang eds. **Handbook of Computational Social Choice**. New York, N.Y.: Cambridge University Press, 2016.
Brenner, Thomas ed. **Computational Techniques for Modelling Learning in Economics**. Boston, Mass.: Kluwer Academic Publishers, 1999.
____. 2006. "Agent Learning Representation: Advice on Modelling Economic Learning," In Handbook of Computaitonal Economics, ed. L. Tesfatsion and K. L. Judd, 895-947. New York, N.Y.: North-Holland.
Brock, William and Cars H. Hommes. 1997. "A rational route to randomness." *Econometrica*, 65(5), pp. 1059-95.
____. 1998. "Heterogeneous beliefs and routes to chaos in a simple asset pricing model." *Journal of Economic Dynamics and Control*, 22(8-9), pp. 1235-74.
Brown, J.A.C.; H.S. Houthakker and S.J. Prais. 1953. "Electronic Computation in Economic Statistics." *Journal of the American Statistical Association*, 48(263), pp. 414-28.
Bruch, E. and J. Atwell. 2015. "Agent-based models in empirical social research." *Sociological Methods & Research*, 44, pp. 186-221.
Bruch, Elizabeth E. and Robert D. Mare. 2006. "Neighborhood Choice and Neighborhood Change." *American Journal of Sociology*, 112(3), pp. 667-709.
Buchanan, Mark. 2009. "Meltdown modeling: Could agent-based computer models prevent another financial crisis?" *Nature*, 460, pp. 680-82.
Builder, C. and Steven Bankes. 1991. "Artificial Societies: A Concept for Basic Research on the Societal Impacts of Information Technology," In. Santa Monica, Calif.: RAND Corporation.
Bullard, James and John Duffy. 1999. "Using Genetic Algorithms to Model the Evolution of Heterogeneous Beliefs." *Computational Economics*, 13(1), pp. 41-60.
____. 2001. "Learning and Excess Volatility." *Macroeconomic Dynamics*, 5(2), pp. 272-302.
Bullinaria, John. 2018. "Agent-Based Models of Gender Inequalities in Career Progression." *Journal of Artificial Societies and Social Simulation*, 21(3).
Burke, Mary A.; Gary M. Fournier and Kislaya Prasad. 2006. "The Emergence of Local Norms in Networks." *Complexity*, 11(5), pp. 65-83.
Calderón-Meza, Guillermo. 2011. "Methodology for the Analysis of the Effects of Net-Centric Operatoins in the Presence of Multi-Agent Adaptive Behavior," In Department of Systems Engineering and Operations Research. Fairfax, Virginia: George Mason University.
Calvó-Armengol, Antoni and Matthew O. Jackson. 2004. "The Effects of Social Networks on Employment and Inequality." *American Economic Review*, 94(3), pp. 426-54.
____. 2007. "Networks in labor markets: Wage and employment dynamics and inequality." *Journal of Economic Theory*, 132(1), pp. 27-46.
____. 2009. "Like Father, Like Son: Social Network Externalities and Parent-Child Correlation in Behavior." *American Economic Journal: Microeconomics*, 1(1), pp. 124-50.
Camerer, Colin. 2003. **Behavioral Game Theory**. Princeton, N.J.: Princeton University Press.
Camerer, Colin; T.H. Ho and J.K. Chong. 2002. "Sophisticated experience-weighted attraction learning and strategic teaching in repeated games." *Journal of Economic Theory*, 104(1), pp. 137-88.
____. 2004. "A cognitive hierarchy model of games." *The Quarterly Journal of Economics*, 119(3), pp. 861-98.
Cameron, A. Colin and Pravin K. Trivedi. 2005. **Microeconometrics: Methods and Applications**. New York, N.Y.: Cambridge Universit Press.
Canup, Robin M. 2012. "Forming a Moon with an Earth-like Composition via a Giant Impact." *Science*, 338(6110), pp. 1052-55.
Canup, Robin M. and Erik Asphaug. 2001. "Origin of the Moon in a giant impact near the end of the Earth's formation." *Nature*, 412, pp. 708-12.
Cardaci, Alberto. 2018. "Inequality, household debt and financial stability." *Journal of Economic Behavior and Organization*, 149, pp. 434-58.
Cardaci, Alberto and Francesco Saraceno. 2019. "Inequality and imbalances: A monetary union agenet-based model." *Journal of Evolutionary Economics*, 29, pp. 853-90.
Cardona, Andrés. 2014. "Parental Choices and Children's Skills: An Agent-Based Model of Parental Investment Behavior and Skill Inequality Within and Across Generations." *Journal of Artificial Societies and Social Simulation*, 17(4).
Carley, Kathleen M. 2002. "Computational Organization Science: A New Frontier." *Proc Natl Acad Sci USA*, 99(suppl 3), pp. 7257-62.
Carley, Kathleen M. *et al.* 2006. "BioWar: Scalable agent-based model of bioattacks." *IEEE Transactions on Systems, Man and Cybernetics, Part A*, 36(2), pp. 252-65.

Carley, Kathleen M. and Michael J. Prietula. 1994. **Computational Organization Theory**. Hillsdale, N.J.: Lawrence Erlbaum Associates.
Carvalho, Laura and Corrado Di Guilmi. 2020. "Technological unemployment and income inequality: a stock-flow consistent agent-based approach." *Journal of Evolutionary Economics*, 30, pp. 39-73.
Castella, Jean-Christophe; Tran Ngoc Trung and Stanislas Boissau. 2005. "Participatory Simulation of Land-Use Changes in the Northern Mountains of Vietnam: the Combined Use of an Agent-Based Model, a Role-Playing Game, and a Geographic Information System." *Ecology and Society*, 10(1).
Casti, John L. 1994. **Complexification: Explaining a Paradoxical World through the Science of Surprise**. New York, N.Y.: HarperCollins Publishers.
Catapano, Gennaro; Francesco Franceschi; Michele Loberto and Valentina Michelangeli. 2021. "Macroprudential policy analysis via an agent-based model of the real estate sector," In Banca D'Italia Eurosistema Temi de discussione, 51. Rome, Italy.
Cederman, Lars-Erik. 1997. **Emergent Actors and World Politics: How States and Nations Develop and Dissolve**. Princeton, N.J.: Princeton University Press.
____. 2001a. "Agent-Based Modeling in Political Science." *The Political Methodologist*, 10, pp. 16-22.
____. 2001b. "Modeling the Democratic Peace as a Kantian Selection Process." *Journal of Conflict Resolution*, 45, pp. 470-502.
____. 2002. "Endogenizing Geopolitical Boundaries with Agent-Based Modeling." *Proc Natl Acad Sci USA*, 99(suppl. 3), pp. 7296-303.
____. 2003. "Modeling the Size of Wars: From Billiard Balls to Sandpiles." *American Political Science Review*, 97(1), pp. 135-50.
Cegielski, Wendy H. and J. Daniel Rogers. 2016. "Rethinking the role of Agent-Based Modeling in archaeology." *Journal of Anthropological Archaeology*, 41, pp. 283-98.
Chalkiadakis, Georgios; Evangelos Markakis and Craig Boutilier. 2007. "Coalition Formation under Uncertainty: Bargaining Equilibria and teh Bayesian Core Stability Concept," In Autonomous Agents and Multi-Agent Systems, ed. E. Durfee, M. Yokoo, M. N. Huhns and O. Shehory. Honolulu, Hawaii: Association for Computing Machinery.
Challet, Damien and Yi-Cheng Zhang. 1998. "On the minority game: Analytical and numerical studies." *Physica A*, 3-4(1), pp. 514-32.
Chapa, Joaquin; Ryan J. Bourgo; Geoffrey L. Greene; Swati Kulkarni and Gary An. 2013. "Examining the Pathogenesis of Breast Cancer Using a Novel Agent-Based Model of Mammary Ductal Epithelium Dynmaics." *PLoS ONE*, 8(5).
Chatterjee, Arnab; Sudhakar Yarlagadda and Bikas K. Chakrabarti eds. **Econophysics of Wealth Distributions**. Milan, Italy: Springer-Verlag Italia, 2005.
Chen, Jing and Silvio Micali. 2013. "The order independence of iterated dominance in extensive games." *Theoretical Economics*, 8, pp. 125-63.
Chen, Shu-Heng. 2012. "Varieties of agents in agent-based computational economics: A historical and and interdisciplinary perspective." *Journal of Economic Dynamics and Control*, 36, pp. 1-25.
Chen, Shu-Heng and Umberto Gostoli. 2014. "Behavioral Macroeconomics and Agent-Based Macroeconomics," In Distributed Computing and Artificial Intelligence, 11th International Conference, ed. S. Omatu, H. Bersini, J. M. Corchado, S. Rodríguez, P. Pawlewski and E. Bucciaralli, 47-54. Springer.
Chen, Shu-Heng; Takeo Terano; Ryuichi Yamamoto and Chung-Ching Tai eds. **Advances in Computational Social Science: The Fourth World Congress**. Tokyo, Japan: Springer Japan, 2014.
Cheng, J. and Michael Wellman. 1998. "The WALRAS Algorithm: A Convergent Distributed Implementation of General Equilibrium Outcomes." *Computational Economics*, 12(1), pp. 1-24.
Cincotti, Silvano; Marco Raberto and Andrea Teglio. 2010. "Credit Money and Macroeconomic Instability in the Agent-Based Model and Simulator Eurace." *Economics: The Open-Access, Open-Assessment E-journal*, 4(2010-26).
Clarkson, Geoffrey P.E. and Herbert A. Simon. 1960. "Simulation of Individual and Group Behavior." *American Economic Review*, 50(5), pp. 920-32.
Clearwater, Scott H. ed. **Market-Based Control**. World Scientific, 1996.
Cliff, Dave and Janet Bruten. 1997a. "Less Than Human: Simple Adaptive Trading Agents for CDA Markets," In. Bristol, UK: Hewlett-Packard Laboratories.
____. 1997b. "Minimal-Intelligence Agents for Bargaining Behaivors in Market-Based Environments," In. Bristol, UK: Hewlett-Packard Labs.
Cohen, Kalman J.; Robert M. Conroy and Steven F. Maier. 1985. "Order flow and the quality of the market," In Market Making and the Changing Structure of the Securities Industry, ed. Y. Amihud, T. S. Ho and R. A. Schwartz. Lexington, Mass.: D.C. Heath and Company.
Cohen, Kalman J. *et al.* 1960. "The Carnegie Tech Management Game." *The Journal of Business*, XXXIII(4), pp. 303-21.
Cohen, Kalman J. and Richard Michael Cyert. 1961. "Computer Models in Dynamic Economics." *The Quarterly Journal of Economics*, LXXV(1), pp. 112-27.

Cohen, Kalman J. and Eric Rhenman. 1961. "The Role of Management Games in Education and Research." *Management Science*, 7(2), pp. 131-66.
Cohen, Michael D.; James G. March and Johan P. Olsen. 1972. "A Garbage Can Model of Organizational Choice." *Administrative Science Quarterly*, 17(1), pp. 1-25.
Colander, David C. ed. **The Complexity Vision and the Teaching of Economics**. Northampton, Mass.: Edward Elgar, 2000.
Colander, David C. *et al.* 2009. "The Financial Crisis and the Systemic Failure of the Economics Profession." *Critical Review: A Journal of Politics and Society*, 21(2-3), pp. 249-67.
Colander, David C. and Roland Kupers. 2014. **Complexity and the Art of Public Policy: Solving Society's Problems from the Bottom Up**. Princeton, N.J.: Princeton University Press.
Coleman, James S. 1964. **Introduction to Mathematical Sociology**. Glencoe, Ill.: Free Press.
Comer, Ken. 2014. "Who Goes First? An Examination of the Impact of Activation on Outcome Behavior of Applied Agent-Based Models," In Department of Systems Engineering and Operations Research. Fairfax, Virginia: George Mason University.
Conitzer, Vincent and Tuomas Sandholm. 2002. "Complexity of Mechanism Design," In Proceedings of the Uncertainty in Artificial Intelligence Conference. Edmonton, Canada.
Cont, Rama. 2006. "Volatility clustering in financial markets: Empirical facts and agent-based models," In Long Memory in Economics, ed. A. P. Kirman and G. Teyssiere. New York, N.Y.: Springer.
Conway, R.W. 1963. "Some tactical problems in digital simulation." *Management Science*, 10(1), pp. 47-61.
Conway, R.W.; B.M. Johnson and W.L. Maxwell. 1959. "Some problems of digital systems simulation." *Management Science*, 6(1), pp. 92-110.
Cotla, Chenna Reddy. 2016. "Social Preferences, Learning, and the Dynamics of Cooperation in Networked Societies: A Dialogue Between Experimental and Computational Approaches," In Department of Computational and Data Sciences. Fairfax, Virginia: George Mason University.
Couclelis, H. 1985. "Cellular worlds: a framework for modeling micro-macro dynamics." *Environment and Planning A*, 17, pp. 585-96.
____. 1989. "Macrostructure and microbehavior in a metropolitan area." *Environment and Planning B: Planning and Design*, 16, pp. 141-54.
Couzin, Iain D. *et al.* 2011. "Uninformed Individuals Promote Democratic Consensus in Animal Groups." *Science*, 334(6062), pp. 1578-80.
Covi, G.; M. Montagna and G. Torri. 2019. "On the Origins of Systemic Risk," In European Central Bank Working Paper.
Crockett, Sean; Stephen Spear and Shyam Sunder. 2008. "Learning competitive equilibrium." *Journal of Mathematical Economics*, 44, pp. 651-71.
Crooks, Andrew T. 2010. "Constructing and implementing an agent-based model of residential segregation through vector GIS." *Int. J. Geogr. Inf. Sci*, 24(5), pp. 661-75.
Crooks, Andrew T.; Nick Malleson; Ed. Manley and Alison J. Heppenstall. 2019. **Agent-Based Modelling & Geographic Information Systems: A Practical Primer**. New York, N.Y.: SAGE Publications.
Cusack, T.R. and R.J. Stoll. 1990. **Exploring Realpolitik: Probing International Relations Theory with Computer Simulation**. Boulder, Colo.: Lynne Rienner.
Cutler, D.M.; J.M. Poterba and Lawrence Summers. 1989. "What Moves Stock Prices?" *Journal of Portfolio Management*, pp. 4-12.
Cyert, Richard Michael and James G. March. 1963. **A Behavioral Theory of the Firm**. Englewood Cliffs, N.J.: Prentice-Hall.
d'Inverno, Mark and Michael Luck. 2001. **Understanding Agent Systems**. New York, N.Y.: Springer.
D'Souza, R.M.; M. Lysenko and K. Rahmani. 2007. "Sugarscape on Steroids: Simulation over a Million Agents at Interactive Rates," In. Michigan Technological University.
Dal Forno, Arianna and Ugo Merlone. 2004. "Personnel Turnover in Organizations: An Agent-Based Simulation Model." *Nonlinear Dynamics, Psychology, and Life Sciences*, 8(2), pp. 205-30.
Dall'Asta, L.; C. Castellano and M. Marsili. 2008. "Statistical physics of the Schelling model of segregation." *Journal of Statistical Mechanics: Theory and Experiment*.
Danielson, Peter. 1992. **Artificial Morality: Virtuous Robots for Virtual Games**. New York, N.Y.: Routledge.
____ ed. **Modelling Rationality, Morality and Evolution**. New York, N.Y.: Oxford University Press, 1996.
Darley, Vince; A. Outkin; T. Plate and F. Gao. 2001. "Learning, Evolution and Tick Size Effects in a Simulation of the NASDAQ Stock Market," In Proceedings of the 5th World Multi-Conference on Systemics, Cybernetics and Informatics (SCI 2001). Orlando, Fl.: International Institute for Informatics and Systematics.
Darley, Vince and Alexander V. Outkin. 2007. **A NASDAQ Market Simulation: Insights on a Major Market from the Science of Complex Adaptive Systems**. World Scientific.
Das, Sanmay. 2016. "Multiagent Systems Modeling." *Tutorials in Operations Research*, pp. 1-18.
Daskalakis, Constantinos; Paul W. Goldberg and Christos H. Papadimitriou. 2006. "The Complexity of Computing a Nash Equilibria," In Proceedings of the 38th Annual ACM Symposium on Theory of Computing, 71-78.
____. 2009. "The Complexity of Computing a Nash Equilibrium." *SIAM Journal on Computing*, 39(1), pp. 195-259.

Dautenhahn, Kerstin ed. **Human Cognition and Social Agent Technology**. Philadelphia, Penn.: John Benjamins Publishing Company, 1999.
Davis, Steven J.; John C. Haltiwanger and Scott Schuh. 1996. **Job Creation and Job Destruction**. Cambridge, Mass.: MIT Press.
Dawid, Herbert. 1999. **Adaptive Learning by Genetic Algorithms: Analytical Results and Applications to Economic Models**. New York, N.Y.: Springer-Verlag.
Dawid, Herbert and Mauro Gallegati. 2018. "Agent-based Macroeconomics," In Handbook of Computational Economics, Volume 4: Heterogeneous Agent Modeling, ed. C. Hommes and B. LeBaron. Amsterdam, Netherlands: Elsevier, North-Holland.
Dawid, Herbert and Simon Gemkow. 2014. "How do social networks contribute to wage inequality? Insights from an agent-based analysis." *Ind. and Corp. Change*, 23(5), pp. 1171-200.
Dawid, Herbert; Simon Gemkow; Philipp Harting; Sander van der Hoog and Michael Neugart. 2012. "The Eurace@Unibi Model: An Agent-Based Macroeconomic Model for Economic Policy Analysis," In Bielefeld Working Papers in Economics and Management, 62. Bielefeld, Germany.
Dawid, Herbert; Philipp Harting and Michael Neugart. 2018. "Cohesion policy and inequality dynamics: Insights from a heterogeneous agents macroeconomic model." *Journal of Economic Behavior and Organization*, 150, pp. 220-55.
de Marchi, Scott. 2005. **Computational and Mathematical Modeling in the Social Sciences**. New York, N.Y.: Cambridge University Press.
De Vany, Arthur. 1993a. "Edgeworth and the Travelling Salesman: Bounded Rationality and the Complexity of Economic Organization," In Institute for Mathematical Behavioral Sciences working paper. Irvine, Calif.: University of California, Irvine.
____. 1993b. "Hard Cores and Soft Cores: Evolving Coalitions, Edgeworth, and the Boltzmann Machine," In Department of Economics working paper. Irvine, Calif.: University of California, Irvine.
____. 1996a. "The Emergence and Evolution of Self-Organized Coalitions," In Computational Economic Systems: Models, Methods & Econometrics, ed. M. Gilli. Boston, Mass.: Kluwer Academic Publishers.
____. 1996b. "Information, Bounded Rationality, and the Complexity of Economic Organization." *Taiwan Journal of Political Economy*, 1(3).
____. 1996c. "Information, Chance, and Evolution: Alchian and the Economics of Self-Organization." *Economic Inquiry*, XXXIV(3), pp. 427-43.
de Wit, Gerrit. 2005. "Firm Size Distributions: An Overview of Steady-State Distributions Resulting from Firm Dynamics Models." *International Journal of Industrial Organization*, 23, pp. 423-50.
DeAngelis, D.L.; D.K. Cox and C.C. Coutant. 1980. "Cannibalism and size dispersal in young-of-the-year largemouth bass: Experiment and model." *Ecological Modelling*, 8, pp. 133-48.
Deffaunt, G.; F. Amblard; Gerard Weisbuch and T Faure. 2002. "How Can Extremism Prevail? A Study Based on the Relative Agreement Interaction Model." *Journal of Artificial Societies and Social Simulation*, 5(4).
Deissenberg, Christophe; Sander van der Hoog and Herbert Dawid. 2008. "EURACE: A massively parallel agent-based model of the European economy." *Applied Mathematics and Computation*, 204(2), pp. 541-52.
Delli Gatti, Domenico; Saul Desiderio; Eduardo Gaffeo; Pasquale Cirillo and Mauro Gallegati. 2011. **Macroeconomics from the Bottom Up**. Milan: Springer.
Delli Gatti, Domenico; Eduardo Gaffeo; Mauro Gallegati; Gianfranco Giulioni and Antonio Palestrini. 2008. **Emergent Macroeconomics: An Agent-Based Approach to Business Fluctuations**. Milano, Italia: Springer-Verlag Italia.
Dibble, Catherine. 2006. "Computational Laboratories for Spatial Agent-Based Models," In Handbook of Computational Economics, ed. L. Tesfatsion and K. L. Judd, 1511-48. New York, N.Y.: Elsevier.
Domowitz, Ian and Jianxin Wang. 1994. "Auctions as algorithms: Computerized trade execution and price discovery." *Journal of Economic Dynamics and Control*, 18(1), pp. 29-60.
Donier, J.; J. Bonart; I. Mastromatteo and J.P. Bouchaud. 2015. "A fully consistent, minimal mondel for nonlinear market impact." *Quantitative Finance*, 15(7), pp. 1109-21.
Dorfman, Robert. 1951. **Application of Linear Programming to the Theory of the Firm, Including an Analysis of Monopolistic Firms by Non-Linear Programming**. Berkeley, California: University of California Press.
Dorfman, Robert; Paul A. Samuelson and Robert M. Solow. 1958. **Linear Programming and Economic Analysis**. New York: McGraw-Hill.
Dosi, G.; Marcelo C. Pereira; Andrea Roventini and Maria Enrica Virgillito. 2018. "The effects of labour market reforms upon unemployment and income inequalities: an agent-based model." *Socio-Economic Review*, 16(4), pp. 687-720.
Dosi, Giovanni; Andrea Roventini and Emanuele Russo. 2019. "Endogenous growth and global divergence in a multi-country agent-based model." *Journal of Economic Dynamics and Control*, 101, pp. 101-29.
Dowlatabadi, Hadi and M. Granger Morgan. 1993a. "Integrated assessment of climate change." *Science*, 259, pp. 1813, 932.
____. 1993b. "A model framework for integrated studies of the climate problem." *Energy Policy*, 21(3), pp. 209-21.

Downey, Allen B. 2012. **Think Complexity: Exploring Complexity Science with Python**. Sebastopol, Calif.: O'Reilly Media, Inc.

Downing, Thomas; Scott Moss and Claudia Pahl-Wostl. 2001. "Understanding Climate Policy Using Participatory Agent-Based Social Simulation," In Multi-Agent-Based Simulation, ed. S. Moss and P. Davidsson, 198-213. Heidelberg, Germany: Springer-Verlag.

Duffy, John. 2006. "Agent-Based Models and Human Subject Experiments," In Haodbook of Computational Economics, ed. L. Tesfatsion and K. L. Judd, 949-1011. New York, N.Y.: North-Holland.

Duraisamy, Karthik; Gianluca Iaccarino and Heng Xiao. 2019. "Turbulence Modeling in the Age of Data." *Annual Review of Fluid Mechanics*, 51, pp. 357-77.

Durlauf, Steven N. 2012. "Complexity, economics, and public policy." *Politics, Philosophy & Economics*, 11(1), pp. 45-75.

Edwards, Paul N. 1996. **The Closed World: Computers and the Politics of Discourse in Cold War America**. Cambridge, Mass.: MIT Press.

____. 2010. **A Vast Machine: Computer Models, Climate Data and the Politics of Global Warming**. Cambridge, Mass.: MIT Press.

Einav, Liran and Jonathan Levin. 2014. "Economics in the age of big data." *Science*, 346(6210).

Eisenbroich, Corinna and Nigel Gilbert. 2014. **Modelling Norms**. Dordrecht, Netherlands: Springer Netherlands.

Elliott, C.D. 1992. "The affective reasoner: A process model of emotions in a multi-agent system," In Computer Science. Evanston, Ill.: Northwestern University.

Engel, J.H. 1954. "A Verification of Lanchester's Law." *Journal of the Operations Research Society of America*, 2(2), pp. 163-71.

Engle, Robert. 2001. "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics." *Journal of Economic Perspectives*, 15(4), pp. 157-68.

Epstein, Joshua M. 2001. "Learning to be thoughtless: Social norms and individual computation." *Computational Economics*, 18(1), pp. 9-24.

Epstein, Joshua M. and Robert Axtell. 1996. **Growing Artificial Societies : Social Science from the Bottom Up**. Washington, D.C./Cambridge, Mass.: Brookings Institution Press/MIT Press.

Erev, Ido and Alvin E. Roth. 2007. "Multi-agent learning and the descriptive value of simple models." *Artificial Intelligence*, 171, pp. 423-28.

Erlingsson, Einar Jon; Andrea Teglio; Silvano Cincotti; Hlynur Steffansson; Jon Thor Sturluson and Marco Raberto. 2014. "Housing Market Bubbles and Business Cycles in an Agent-Based Credit Economy." *Economics*, 8(1), pp. 43.

Ermentrout, G.B. and L. Edelstein-Keshet. 1993. "Cellular Automata Approaches to Biological Modeling." *Journal of Theoretical Biology*, 160, pp. 97-113.

Eubank, Stephen *et al.* 2004. "Modelling Disease Outbreaks in Realistic Urban Social Networks." *Nature*, 429, pp. 180-84.

Fagiolo, G.; G. Dosi and R. Gabriele. 2004. "Matching, bargaining, and wage setting in an evolutionary model of labor market and output dynamics." *Advances in Complex Systems*, 7(2).

Fahse, Lorenz; Christian Wissel and Volker Grimm. 1998. "Reconciling Classical and Individual-based Approaches in Theoretical Population Ecology: A Protocol for Extracting Population Parameters from Individual-Based Models." *American Naturalist*, 152, pp. 838-52.

Faith, Joe. 1998. "Why Gliders Don't Exist: Anti-Reductionism and Emergence," In Artificial Life VI, ed. C. Adami, R. K. Belew, H. Kitano and C. E. Taylor, 389-92. Cambridge, Mass.: MIT Press.

Farkas, Illes J.; Dirk Helbing and Tamas Vicsek. 2002. "Mexican waves in an excitable medium." *Nature*, 419, pp. 131-32.

Farmer, J. Doyne. 2002. "Market force, ecology and evolution." *Industrial and Corporate Change*, 11(5), pp. 895-953.

Farmer, J. Doyne and Duncan Foley. 2009. "The economy needs agent-based modelling." *Nature*, 460, pp. 685-86.

Farmer, J. Doyne and John Geanakoplos. 2009. "The virtues and vices of equilibrium and the future of financial economics." *Complexity*, 14(3), pp. 11-38.

Farmer, J. Doyne; Cameron Hepburn; Penny Mealy and Alexander Teytelboym. 2015. "A Third Wave in the Economics of Climate Change." *Environmental and Resource Economics*, 62, pp. 329-57.

Farmer, J. Doyne and Shareen Joshi. 2002. "The price dynamics of common trading strategies." *Journal of Economic Behavior and Organization*, 49(2), pp. 149-71.

Farmer, J. Doyne; Alissa M. Kleinnijenhuis; Paul Nahai-Williamson and Thom Wetzer. 2020. "Foundations of system-wide financial stress testing with heterogeneous institutions," In Bank of England Working Paper 861. London, U.K.

Feldman, Alan. 1973. "Bilateral Trading Processes, Pairwise Optimality, and Pareto Optimality." *Review of Economic Studies*, XL(4), pp. 463-73.

Ferber, Jacques. 1999. **Multi-Agent Systems: An Introduction to Distributed Artificial Intelligence**. Harlow, U.K.: Addison-Wesley.

Ferguson, Neil M. *et al.* 2020. "Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand," In Imperial College COVID-19 Response, 20. London, U.K.

Filatova, T.; P.H. Verburg; D.C. Parker and C.A. Stannard. 2013. "Spatial agent-based models for socio-ecological systems: Challenges and prospects." *Environmental Modeling & Software*, 45, pp. 1-7.
Filatova, Tatiana V. 2009. "Land Markets from the Bottom Up: Micro-macro linkes in economics and implications for coastal risk management," In, 185. University of Twente.
Filatova, Tatiana V.; Dawn C. Parker and A. van der Veen. 2007. "Agent-based land markets: Heterogeneous agents, land prices, and urban land use change," In Proceedings of the 4th Conference of the European Social SImulation Association (ESSA '07). Toulouse, France.
____. 2009. "Agent-based Urban Land Markets: Agent's Pricing Behavior, Land Prices and Urban Land Use Change." *Journal of Artificial Societies and Social Simulation*, 12(13).
Fishwick, Paul A. 1995. **Simulation Model Design and Execution: Building Digital Worlds**. Englewood Cliffs, N.J.: Prentice Hall.
Fix, Julia; Christian von Scheve and Daniel Moldt. 2006. "Emotion-based norm enforcement and maintenance in multi-agent systems: foundations and petri net modeling," In Autonomous Agents and Multi-Agent Systems, ed. H. Nakashima and M. Wellman, 105-07. Hakodate, Japan: Association for Computing Machinery.
Foley, Duncan K. 1994. "A Statistical Equilibrium Theory of Markets." *Journal of Economic Theory*, 62, pp. 321-45.
Follmer, Hans. 1974. "Random Economies with Many Interacting Agents." *Journal of Mathematical Economics*, 1(1), pp. 51-62.
Forrester, Jay W. 1958. "Industrial Dynamics—A Major Breakthrough for Decision-Makers." *Harvard Business Review*, 36, pp. 37-66.
____. 1969. **Urban Dynamics**. Cambridge, Mass.: MIT Press.
Fostel, Ana and John Geanakoplos. 2008. "Leverage Cycles and the Anxious Economy." *American Economic Review*, 98(4), pp. 1211-44.
Foster, Dean P. and H. Peyton Young. 2001. "On the Impossibility of Predicting the Behavior of Rational Agents." *Proc Natl Acad Sci USA*, 98(22), pp. 12848-53.
Frank, Robert. 1988. **Passions within Reason: The Stretegic Role of the Emotions**. New York, N.Y.: W.W. Norton.
Franke, R. and F. Westerhoff. 2012. "Structural stochastic volatility in asset pricing dynamics: Estimation and model contest." *Journal of Economic Dynamics and Control*, 36(8), pp. 1193-211.
Fricke, Daniel and Thomas Lux. 2015. "The effects of a financial transaction tax in an artificial financial market." *Journal of Economic Interaction and Coordination*, 10(1), pp. 119-50.
Friedman, Daniel and John Rust eds. **The Double Auction Market: Institutions, Theories, and Evidence**. Westview Press, 1993.
____ eds. **The Double Auction Market: Institutions, Theories, and Evidence**. Reading, Mass.: Addison-Wesley Publishing, 1994.
Friedman, Milton. 1953. **Essays in Positive Economics**. Chicago, Ill.: University of Chicago Press.
Fudenberg, Drew and David K. Levine. 2007. "An economist's perspective on multi-agent learning." *Artificial Intelligence*, 171, pp. 378-81.
Fuku, Tomoko; Akira Namatame and Taisei Kaizoji. 2006. "Collective Efficiency in Two-Sided Matching," In Artificial Economics: Agent-Based Methods in Finance, Game Theory and Their Applications, ed. P. Mathieu, B. Beaufils and O. Brandouy. Berlin, Germany: Springer-Verlag.
Fürnkranz-Prskawetz, Alexia. 2010. "Agent Based Models on Social Interaction and Demographic Behaviour," In. Vienna, Austria: Technical University of Vienna.
Gai, Prasanna and Sujit Kapadia. 2010. "Contagion in Financial Networks." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 466(2120).
Gale, David and Lloyd S. Shapley. 1962. "College admissions and the stability of marriage." *The American Mathematical Monthly*, 69(1), pp. 9-15.
Gallegati, Mauro and Alan Kirman. 2012. "Reconstructing economics: Agent based models and complexity." *Complexity Economics*, 1, pp. 5-31.
Garcia, Rosanna. 2005. "Uses of Agent-Based Modeling in Innovation/New Product Development Research." *Journal of Product Innivation Management*, 22(5), pp. 380-98.
Gardner, Martin. 1970. "The Fantastical Combinations of John Conways New Solitaire Game "Life"." *Scientific American*, 223(4-6), pp. 120, 14, 18.
Gasser, Les and Michael N. Huhns. 1989. **Distributed Artificial Intelligence**. San Mateo, Calif.: Morgan Kaufmann.
Gaylord, Richard J. and Louis J. D'Andria. 1998. **Simulating Society: A Mathematica Toolkit for Modeling Socioeconomic Behavior**. New York, N.Y.: Telos (Springer-Verlag).
Ge, Jiaqi. 2017. "Endogenous rise and collapse of housing price: An agent-based model of the housing market." *Computers, Environment and Urban Systems*, 62, pp. 182-98.
Geanakoplos, John. 2010. "The Leverage Cycle," In NBER Macroeconomics Annual 2009, ed. D. Acemoglu, K. S. Rogoff and M. Woodford, 1-66. Chicago, Ill.: The University of Chicago Press.
Geanakoplos, John *et al.* 2012. "Getting at Systemic Risk via an Agent-Based Model of the Housing Market." *American Economic Review: Papers and Proceedings*, 102(3), pp. 53-58.
Gelernter, David. 1992. **Mirror Worlds or: The Day Software Puts the Universe in a Shoebox...How it Will Happen and What it Will Mean**. New York, N.Y.: Oxford University Press.

Gemann, Timothy C.; Kai Kadau; Ira M. Longini Jr. and Catherine A. Macken. 2006. "Mitigation strategies for pandemic influenza in the United States." *Proc Natl Acad Sci USA*, 103(15), pp. 5935-40.
Gemkow, Simon and Michael Neugart. 2011. "Referral hiring, endogenous social networks, and inequality: an agent-based analysis." *Journal of Evolutionary Economics*, 21, pp. 703-19.
Gerhold, S.; L. Glebsky; C. Schneider and H. Weiss. 2008. "Limit states for one-dimensional Schelling segregation models." *Communications in Nonlinear Science and Numerical Simulation*, 13(10), pp. 2236-45.
Gershenson, Carlos and Dirk Helbing. 2015. "When Slower is Faster." *Complexity*, 21(2), pp. 9-15.
Gerst, Michael D. *et al.* 2013. "Agent-based modeling of climate policy: An introduction to the ENGAGE multi-level model framework." *Environmental Modelling & Software*, 44, pp. 62-75.
Ghoulmie, Francois; Rama Cont and Jean-Pierre Nadal. 2005. "Heterogeneity and feedback in an agent-based market model." *Journal of Physics: Condensed Matter*, 17(14), pp. S1259-S68.
Gigerenzer, Gerg. 2000. **Adaptive Thinking: Rationality in the Real World**. New York, N.Y.: Oxford University Press.
Gigerenzer, Gerg and Reinhard Selten eds. **Bounded Rationality: The Adaptive Toolbox**. Cambridge, Mass.: MIT Press, 2001.
Gigerenzer, Gerg; P.M. Todd and ABCResearchGroup. 1999. **Simple Heuristics That Make Us Smart**. New York: Oxford University Press.
Gilbert, Nigel. 2008. **Agent-Based Models**. Thousand Oaks, CA: Sage Publications, Inc.
Gilbert, Nigel and Steven Bankes. 2002. "Platforms and Methods for Agent-Based Modeling." *Proc Natl Acad Sci USA*, 99(supplement 3), pp. 7197-98.
Gilbert, Nigel; John C. Hawksworth and Paul A. Swinney. 2009. "An Agent-Based Model of the English Housing Market," In AAAI.
Gintis, Herbert. 2007. "The Dynamics of General Equilibrium." *Economic Journal*, 117(523), pp. 1280-309.
Gintis, Herbert and Dirk Helbing. 2013. "*Homo Socialis*: An Analytical Core for Sociological Theory," In.
Glavatskiy, Kirill S.; Mikhail Prokopenko; Adrian Carro; Paul Ormerod and Michael Harre. 2021. "Explaining herding and volatility in the cyclical price dynamics of urban housing markets using a large-scale agent-based model." *SN Business & Econoimcs*, 1.
Glushko, Robert J.; Jay M. Tenenbaum and Bart Meltzer. 1999. "An XML framework for agnet-based E-commerce." *Communications of the ACM*, 42(3), pp. 106.
Gneitling, Tilmann and Adrian E. Raftery. 2005. "Weather Forecasting with Ensemble Methods." *Science*, 310(5746), pp. 248-49.
Gode, D.K. and Shyam Sunder. 1993. "Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality." *Journal of Political Economy*, CI, pp. 119-37.
____. 1997. "What Makes Markets Allocationally Efficient?" *Quarterly Journal of Economics*, 112(2), pp. 603-30.
Goertzel, Ben; Ruiting Lian; Itamar Arel; Hugo de Garis and Shuo Chen. 2010. "A world survey of artificial brain projects, Part II: Biologically inspired cognitive architectures." *Neurocomputing*, 74, pp. 30-49.
Goldberg, Arthur P.; Balázs Szigeti; Yin Hoon Chew; John A.P. Sekar; Yosef D. Roth and Jonathan R. Karr. 2018. "Emerging whole-cell modeling principles and methods." *Current Opinion in Biotechnology*, pp. 97-102.
Goldenberg, Jacob; Barak Libai and Eitan Muller. 2001. "Talk of the Network: A Complex Systems Look at the Underlying Proces of Word-of-Mouth." *Marketing Letters*, 12(3), pp. 211-23.
Gomes, Marcelo F.C. *et al.* 2014. "Assessing the International Spreading Risk Associated with the 2014 West African Ebola Outbreak." *PLOS Current Outbreaks*.
Granovetter, Mark. 1973. "The Strength of Weak Ties." *American Journal of Sociology*, 78, pp. 1360-80.
____. 1995. **Getting a Job: A Study of Contacts and Careers**. Chicago, Ill.: The University of Chicago Press.
Green, Jerry R. 1972. "On the Inequitable Nature of Core Allocations." *Journal of Economic Theory*, 4(2), pp. 132-43.
Grieves, Michael and John Vickers. 2017. "Digital Twin: Mitigating Unpredictable, Undesirable Emergent Behavior in Complex Systems," In Transdisciplinary Perspectives on Complex Systems: New Findings and Approaches, ed. F.-J. Kahlen, S. Flumerfelt and A. Alves. Switzerland: Springer Nature.
Grimm, Volker and Steven F. Railsback. 2005. **Individual-based Modeling and Ecology**. Princeton, N.J.: Princeton University Press.
Grimm, Volker *et al.* 2005. "Pattern-Oriented Modeling of Agent-Based Complex Systems: Lessons from Ecology." *Science*, 310, pp. 987-91.
Grodzins, Morton. 1958. **The Metropolitan Area as a Racial Problem**. Pittsburgh, Penn.: University of Pittsburgh Press.
Grow, André and Jan Van Bavel. 2015. "Assortative Mating and the Reversal of Gender Inequality in Education in Europe: An Agent-Based Model." *PLoS ONE*, 10(6).
Guerrero, Omar A. and Robert L. Axtell. 2013. "Employment Growth through Labor Flow Networks." *PLoS ONE*, 8(5), pp. e60808.
Guerrero, Omar A. and Eduardo López. 2015. "Firm-to-firm labor flows and the aggregate matching function: A network-based test using employer-employee matched records." *Economic s Letters*, 136(9-12).
Gulden, Tim. 2004. "Adaptive Agent Modelling in a Policy Context," In School of Public Affairs. College Park, Maryland: University of Maryland.

Gutowitz, H. 1990. **Cellular Automata: From Theory to Practice**. Cambridge, Mass.: MIT Press.
____ ed. **Cellular Automata: Theory and Experiment**. Cambridge, Mass.: MIT Press, 1991.
Guvenen, Fatih. 2011. "Macroeconomics with Heterogeneity: A Practical Guide," In NBER Working Paper Series. Cambridge, Mass.
Haas, Christian; Steven O. Kimbrough and Clemens van Dinther. 2013. "Strategic Learning by (e-Service) Suppliers in Service Value Networks." *Journal of Service Research*, 16(3), pp. 259-76.
Hahn, Frank H. 1992. "The Next One Hundred Years," In The Future of Economics, ed. J. D. Hey. Cambridge, Mass.: Blackwell Publishers.
Hailu, Atakelty and Steven Schilizzi. 2004. "Are Auctions More Efficient Than Fixed Price Schemes When Bidders Learn?" *Australian Journal of Management*, 29(2), pp. 147-68.
____. 2005. "Learning in a "Basket of Crabs": An Agent-Based Computational Model of Repeated Conservation Auctions," In Nonlinear Dynamics and Heterogeneous Interacting Agents, ed. T. Lux, S. Reitz and E. Samanidou, 27-39. New York, N.Y.: Springer.
Hailu, Atakelty and Sophie Thoyer. 2006. "Multi-unit auction formal design." *Journal of Economic Interaction and Coordination*, 1, pp. 129-46.
____. 2007. "Designing Multi-unit Multiple Bid Auctions: An Agent-based Computational Model of Uniform, Discriminatory and Generalised Vickrey Auctions." *Economic Record*, 83(Supplement s1), pp. S57-S72.
Haldane, Andrew G. and Arthur E. Turrell. 2018. "Drawing on different disciplines: macroeconomic agent-based models." *Journal of Evolutionary Economics*.
Hales, David. 2002. "Group Reputation Supports Beneficent Norms." *Journal of Artificial Societies and Social Simulation*, 5(4).
Halloran, M. Elizabeth; Ira M. Longini Jr.; Azhar Nizam and Yang Yang. 2002. "Containing Bioterrorist Smallpox." *Science*, 298(5597), pp. 1428-32.
Hammond, Ross A. 2009. "Complex Systems Modeling for Obesity Research." *Preventing Chronic Disease*, 6(3), pp. 1-10.
Harrison, J. Richard and Glenn R. Carroll. 2006. **Culture and Demography in Organizations**. Princeton, N.J.: Princeton University Press.
Hayek, F. A. von. 1945. "The Use of Knowledge in Society." *American Economic Review*, 35(4), pp. 519-30.
____. 1964. "Kinds of Order in Society." *New Individualist Review*, 3(2), pp. 3-12.
Heard, Daniel. 2014. "Statistical Inference Utilizing Agent-Based Models," In Department of Statistical Science. Durham, N.C.: Duke University.
Heard, Daniel; Georgiy Bobashev and Robert J. Morris. 2014. "Reducing the Complexity of an Agent-Based Local Heroin Market Model." *PLoS ONE*, 9(7).
Heard, Daniel; Gelonia Dent; Tracy Schifeling and David Banks. 2015. "Agent-Based Models and Microsimulation." *Annual Review of Statistics and Its Applications*, (2), pp. 259-72.
Hedstrom, Peter. 2005. **Dissecting the Social: On the Principles of Analytical Sociology**. New York, N.Y.: Cambridge University Press.
Hedstrom, Peter and Richard Swedberg eds. **Social Mechanisms: An Analytical Approach to Social Theory**. New York, N.Y.: Cambridge University Press, 1998.
Hegselman, R. and U. Krause. 2002. "Opinion Dynamics and Bounded Confidence Models: Analysis and Simulation." *Journal of Artificial Societies and Social Simulation*, 5(3).
Hegselmann, Rainer. 2012. "Thomas C. Schelling and the Computer: Some Notes on Schelling's Essay "On Letting a Computer Help with the Work"." *Journal of Artificial Societies and Social Simulation*, 15(4), pp. 9.
____. 2017. "Thomas C. Schelling and James M. Sakoda: The Intellectual, Technical, and Social History of a Model." *Journal of Artificial Societies and Social Simulation*, 20(3).
Helbing, Dirk ed. **Social Self-Organization: Agent-Based Simulations and Experiments to Study Emergent Social Behavior**. Berlin: Springer-Verlag, 2012.
Helbing, Dirk; Illes J. Farkas and Tamas Vicsek. 2000. "Simulating Dynamical Features of Escape Panic." *Nature*, 407, pp. 487-90.
Hemelrijk, Charlotte K. and Hanno Hildenbrandt. 2011. "Some Causes of the Variable Shape of Flocks of Birds." *PLoS ONE*.
Henrickson, Leslie. 2002. "Old Wine in a New Wineskin: College Choice, College Access Using Agent-Based Modeling." *Social Science Computer Review*, 20(4), pp. 400-19.
Heppenstall, Alison J.; Andrew T. Crooks; Linda M. See and Michael Batty eds. **Agent-Based Models of Geographical Systems**. Netherlands: Springer, 2012.
Higdon, David M. *et al.* 2016. **From Maps to Models: Augmenting the Nation's Geospatial Intelligence Capabilities**. Washington, D.C.: The National Academies Press.
Hillebrand, E. and J. Stender eds. **Many-Agent Simulation and Artificial Life**. Washington, D.C.: IOS Press, 1994.
Hirsch, M.D.; Christos H. Papadimitriou and S.A. Vavasis. 1989. "Exponential Lower Bounds for Finding Brouwer Fixed Points." *Journal of Complexity*, 5, pp. 379-416.
Hoegeweg, P. and B. Hesper. 1990. "Individual-oriented modeling in ecology." *Mathematical and Computer Modelling*, 13, pp. 83-90.

Hoffer, Lee D.; Georgiy Bobashev and Robert J. Morris. 2009. "Research a Local Heroin Market as a Complex Adaptive System." *Am J. Community Pshchol*, 44, pp. 273-86.
Hoffman, Johan and Claes Johnson. 2007. **Computational Turbulent Incompressible Flow**. New York, N.Y.: Springer.
Hofman, Jake M.; Amit Sharma and Duncan J. Watts. 2017. "Prediction and explanation in social systems." *Science*, 355(6324), pp. 486-88.
Hokamp, Sascha and Michael Pickhardt. 2010. "Income Tax Evasion in a Society of Heterogeneous Agents—Evicence from an Agent-Based Model." *International Economic Journal*, 24(4), pp. 541-53.
Holland, John Henry. 1998. **Emergence: From Chaos to Order**. Reading, Mass.: Perseus.
____. 2012. **Signals and Boundaries: Building Blocks for Complex Adaptive Systems**. Cambridge, Mass.: MIT Press.
____. 2014. **Complexity: A Very Short Introduction**. Oxford, UK: Oxford University Press.
Holland, John Henry and John Miller. 1991. "Artificial Adaptive Agents in Economic Theory." *American Economic Review*, 81(2), pp. 363-70.
Holt, Richard P.F.; Jr. Rosser, J. Barkley and David C. Colander. 2011. "The Complexity Era in Economics." *Review of Political Economy*, 23(3), pp. 357-69.
Hommes, Cars H. 2002. "Modeling the stylized facts in finance through simple nonlinear adaptive systems." *Proc Natl Acad Sci USA*, 99(3), pp. 7221-28.
____. 2011. "The Heterogeneous Expectations Hypothesis: Some Evidence From the Lab." *Journal of Economic Dynamics and Control*, 35(1), pp. 1-24.
Hommes, Cars H. and Blake LeBaron eds. **Handbook of Computational Economics**. New York, N.Y.: Elsevier, 2018.
Hommes, Cars and Thomas Lux. 2013. "Individual Expectations and Aggregate Behavior in Learning-to-Forecast Experiments." *Macroeconomic Dynamics*, 17, pp. 373-401.
Hommes, Cars; J. Sonnemans; J. Tuinstra and H. van de Velden. 2007. "Learning in cobweb experiments." *Macroeconomic Dynamics*, 11(S1), pp. 8-33.
Hooten, M.B. and C.K. Wikle. 2010. "Statistical agent-based models for discrete spatio-temporal systems." *Journal of the American Statistical Association*, 105(236-248).
Horty, John F. 2001. **Agency and Deontic Logic**. New York, N.Y.: Oxford University Press.
Howitt, Peter and Robet Clower. 2000. "The Emergence of Economic Organization." *Journal of Economic Behavior and Organization*, 41(1), pp. 55-84.
Huberman, Bernardo A. ed. **The Ecology of Computation**. New York, N.Y.: North-Holland, 1987.
Huberman, Bernardo A. and Natalie S. Glance. 1993. "Evolutionary Games and Computer Simulations." *Proc Natl Acad Sci USA*, 90, pp. 7716-18.
Huberman, Bernardo A. and Tadd Hogg. 1994. "Distributed Computation as an Economic System." *Journal of Economic Perspectives*, 9(1), pp. 141-52.
Hume, David. 1896 [1739]. **A Treatise of Human Neature**. Oxford, U.K.: Oxford at the Clarendon Press.
Huston, M.; D.L. DeAngelis and W. Post. 1988. "New computer models unify ecological theory." *BioScience*, 38(10), pp. 682-91.
Huttegger, Simon M.; Brian Skyrms and Kevin J.S. Zollman. 2012. "Probe and Adjust in Information Transfer Games." *Erkenntnis*, 79(4), pp. 835-53.
Ilachinski, Andrew. 2004. **Artificial War: Multiagent-Based Simulation of Combat**. Singapore: World Scientific Publishing.
Ingram, Gregory K.; John F. Kain and J. Royce Ginn. 1972. **The Detroit Protype of the NBER Urban Simulation Model**. Cambridge, Mass.: NBER.
Izumi, K. 2001. "Complexity of Agents and Complexity of Markets," In Meeting the Challenge of Social Problems via Agent-Based Simulation, ed. T. Terano, Deguchi, H., and Takadama, K. Tokyo: Springer-Verlag.
____. 2002. "Does Learning by Market Participants Make Financial Markets Complicated?," In Multi-Agent Modeling and Simulation of Economic Systems, ed. K. Kurumatani, S.-H. Chen and A. Ohuchi. AAAI Press.
Jackson, J.R. 1959. "Learning from Experience in Business Decision Games." *California Management Review*, 1(2), pp. 92-107.
Jackson, Matthew O. 2008. **Social and Economic Networks**. Princeton, N.J.: Princeton University Press.
Janes, Kevin A. and Douglas A. Lauffenburger. 2013. "Models of signalling networks—what cell biologists can gain from them and give to them." *Journal of Cell Science*, 126, pp. 1913-21.
Janssen, Marco A. ed. **Complexity and Ecosystem Management: The Theory and Practice of Multi-Agent Systems**. Northampton, Mass.: Edward Elgar Publishing, Inc., 2002.
Jefferies, P.; M.L. Hart and Neil F. Johnson. 2001. "Deterministic dynamics in the minority game." *Physical Review E*, 65, pp. 016105.
Jennings, Nicholas R.; Katia Sycara and Michael Wooldridge. 1998. "A Roadmap of Agent Research and Development." *Autonomous Agents and Multi-Agent Systems*, 1(1), pp. 7-38.
Johnson, Neil F.; Paul Jefferies and Pak Ming Hui. 2003. **Financial Market Complexity: What Physics Can Tell Us About Market Behavior**. New York, N.Y.: Oxford University Press.

Joulin, Armand; Augustin Lefevre; Daniel Grunberg and Jean-Philippe Bouchaud. 2008. "Stock price jumps: news and volume play a minor role," In arXiv. Ithaca, N.Y.: Cornell University.
Judd, Ken. 1997. "Computational Economics and Economic Theory: Substitutes or Complements?" *Journal of Economic Dynamics and Control*, 21(6), pp. 907-42.
____. 1998. **Numerical Methods in Economics**. Cambridge, Mass.: MIT Press.
Kahneman, Daniel. 2011. **Thinking, Fast and Slow**. New York, N.Y.: Farrar, Straus and Giroux.
Kahneman, Daniel and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decisions Under Risk." *Econometrica*, 47, pp. 313-27.
Kandori, Michio; George Mailath and Raphael Rob. 1993. "Learning, Mutation, and Long-Run Equilibria in Games." *Econometrica*, 61(1), pp. 29-56.
Kao, Albert B.; Noam Miller; Colin Torney; Andrew Hartnett and Iain D. Couzin. 2014. "Collective learning and optimal consensus decisions in social animal groups." *PLoS Computational Biology*, 10(8).
Karr, Jonathan R. *et al.* 2012. "A Whole-Cell Computational Model Predicts Phenotype from Genotype." *Cell*, 150, pp. 389-401.
Kearns, Michael and Y. Mansour. 2002. "Efficient Nash Computation in Large Population Games with Bounded Influence," In Conference on Uncertainty in Artificial Intelligence. Morgan Kaufmann.
Keila, P.S. and D.B. Skillicorn. 2005. "Structure in the Enron Email Dataset." *Computational & Mathematical Organization Theory*, 11, pp. 183-99.
Kemeny, J.G.; Oskar Morgenstern and Gerald L. Thompson. 1956. "A Generalization of the von Neumann Model of an Expanding Economy." *Econometrica*, 24(2), pp. 115-35.
Kermack, W.O. and A.G. McKendrick. 1927. "A Contribution to the Mathematical Theory of Epidemics." *Proc. Roy. Soc. Lond. A*, 115, pp. 700-21.
Khandani, Amir E. and Andrew W. Lo. 2011. "What happened to the quants in August 2007? Evidence from factors and transactions data." *Journal of Financial Markets*, 14, pp. 1-46.
Kiesling, Elmar; Markus Günther; Christian Stummer and Lea M. Wakolbinger. 2012. "Agent-based simulation of innovation diffusion: a review." *Central European Journal of Operations Research*, 20(2), pp. 183-230.
Kimbrough, Steven O. and Frederic H. Murphy. 2009. "Learning to Collude Tacitly on Produciton Levels by Oligopolistic Agents." *Computational Economics*, 33(1), pp. 47-78.
____. 2013. "Strategic bidding of offer curves: An agent-based approach to exploring supply curve equilibria." *European Journal of Operational Research*, 229(1), pp. 165-78.
Kiran, Miriam; Paul Richmond; Mike Holcombe; Lee Shawn Chin; David Worth and Chris Greenough. 2010. "FLAME: Simulating Large Populations of Agents on Parallel Hardware Architectures," In Proceedings of the 9th International Conference on Autonomous Agent and Multiagent Systems (AAMAS 2010), ed. W. van der Hoek, G. A. Kaminka, Y. Lespérance, M. Luck and S. Sen. Toronto, Canada: Internatoinal Foundation for Autonomous Agents and Multiagent Systems.
Kirman, Alan. 2004. "Economics and complexity." *Advances in Complex Systems*, 7(2).
____. 2011. **Complex economics: Individual and collective rationality**. New York, N.Y.: Routledge.
Kirman, Alan P. 1989. "The Intrinsic Limits of Modern Economic Theory: The Emperor Has No Clothes." *Economic Journal*, 99(395), pp. 126-39.
____. 2010. "The Economic Crisis is a Crisis for Economic Theory." *CESifo Economic Studies*, 56(4), pp. 498-535.
Kirman, Alan P and Nicholaas J. Vriend. 2000. "Learning to be Loyal: A Study of the Marseille Fish Market," In Interaction and Market Structure. Essays on Heterogeneity in Economics, ed. D. Delli Gatti and A. P. Kirman. Berlin, Germany: Springer-Verlag.
____. 2001. "Evolving Market Structure: An ACE Model of Price Dispersion and Loyalty." *Journal of Economic Dynamics and Control*, 25(3-4), pp. 459-502.
Kirman, Alan P. 1992. "Whom or What Does the Representative Individual Represent?" *Journal of Economic Perspectives*, 6(2), pp. 117-36.
____. 1997. "The Economy as an Interactive System," In The Economy as an Evolving Complex System II, ed. W. B. Arthur, S. N. Durlauf and D. A. Lane. Reading, Mass.: Addison-Wesley.
Klein, Lawrence R. and Arthur S. Goldberger. 1955. **An Econometric Model of the United States**. New York: North-Holland Publishing Company.
Klimt, Bryan and Yiming Yang. 2004. "The Enron Corpus: A NEw Dataset for Email Classification Research," In European Conference on Machine Learning (ECML 2004), ed. J.-F. Boulicaut, F. Esposito, F. Giannotti and D. Pedreschi, 217-26. Pisa, Italy: Springer.
Klusch, Matthias and Onn Shehory. 1996a. "Coalition Formation Among Rational Information Agents," In Agents Breaking Away - Proceedings of the Seventh European Workshop on Modeling Autonomous Agents in a Multi-Agent World, ed. W. Van de Velde and J. Perram, 204-17. New York, N.Y.: Springer-Verlag.
____. 1996b. "A Polynomial Kernel-Oriented Coalition Algorithm for Rational Information Agents," In Proceedings of the Second International Conference on Multi-Agent Systems, ed. M. Tokoro. Menlo Park, Calif.: AAAI Press.
Knuth, Donald E. 1976. **Marriages Stables**. Montreal, Canada: Les Presses de l'Universite de Montreal.
____. 2005. **The Art of Computer Programming, Volume 4, Fascicle 3**. Upper Saddle River, N.J.: Pearson Education, Inc.

Kochugovindan, S. and Nicholaas J. Vriend. 1998. "Is the Study of Complex Adaptive Systems Going to Solve the Mystery of Adam Smith's 'Invisible Hand'?" *Independent Review*, 3(1), pp. 53-66.

Koesrindartoto, Deddy. 2004. "Treasury Auctions, Uniform or Discriminatory?: An Agent-Based Approach," In Iowa State University, Department of Economics Working Paper Series. Ames, Iowa.

Kohler, Hans-Peter. 2001. **Fertility and Social Interactions**. New York, N.Y.: Oxford University Press.

Kohler, Timothy A. and George J. Gumerman eds. **Dynamics in Human and Primate Societies: Agent-Based Modeling of Social and Spatial Processes**. New York, N.Y.: Oxford University Press, 2000.

Kollman, Ken; John H. Miller and Scott E. Page. 1992. "Adaptive Parties in Spatial Elections." *American Political Science Review*, 86, pp. 929-37.

____. 1997a. "Computational Political Economy," In The Economy as an Evolving Complex System II, ed. W. B. Arthur, S. N. Durlauf and D. A. Lane. Reading, Mass.: Addison-Wesley.

____. 1997b. "Political Institutions and Sorting in a Tiebout Model." *American Economic Review*, 87(5), pp. 977-92.

Kollman, Ken; John Miller and Scott E. Page eds. **Computational Models in Political Economy**. Cambridge, Mass.: MIT Press, 2003.

Korobow, Adam; Chris Johnson and Robert Axtell. 2007. "An Agent-Based Model of Tax Compliance with Social Networks." *National Tax Journal*, 60(3), pp. 589-610.

Kouwenberg, Roy and Remco C.J. Zwinkels. 2015. "Endogenous price bubbles in a multi-agent system of the housing market." *PLoS ONE*, 10(6), pp. e129070.

Kravari, Kalliopi and Nick Bassiliades. 2015. "A Survey of Agent Platforms." *Journal of Artificial Societies and Social Simulation*, 18(1), pp. 11.

Krugman, Paul. 1996. **The Self-Organizing Economy**. New York, N.Y.: Blackwell.

Kuznar, Lawrence A. 2006. "High-Fidelity Computational Social Science in Anthropology." *Social Science Computer Review*, 24(1), pp. 15-29.

Lagunoff, Roger and Akihiko Matsui. 1997. "Asynchronous Choice in Repeated Coordination Games." *Econometrica*, 65(6), pp. 1467-77.

Laird, John E.; Paul S. Rosenbloom and Allen Newell. 1986. "Chunking in Soar: The Anatomy of a General Learning Mechanism." *Machine Learning*, 1(1).

Lamperti, Francesco *et al.* 2018. "Towards agent-based integrated assessment models: examples, challenges, and future devleopments." *Regional Environmental Chhange*, 19, pp. 747-62.

Lanchester, F.W. 1916. **Aircraft in Warfare: The Dawn of the Fourth Arm**. London, U.K.: Constable and Company, Ltd.

Lane, David A. 1993a. "Artificial Worlds and Economics, Part 1." *Journal of Evolutionary Economics*, 3, pp. 89-107.

____. 1993b. "Artificial Worlds and Economics, Part 2." *Journal of Evolutionary Economics*, 3, pp. 177-97.

Langton, Christopher G. ed. **Artificial Life: Proceedings of an Interdisciplinary Workshop on the Synthesis and Simulation of Living Systems**. Reading, Mass.: Addison-Wesley Publishing, 1989.

____ ed. **Artificial Life III**. Reading, Mass.: Addision-Wesley Publishing, 1994.

Langton, Christopher G.; Charles Taylor; J. Doyne Farmer and Steen Rasmussen eds. **Artirficial Life II**. Redwood City, Calif.: Addison-Wesley Publishing, 1992.

Lansing, J. Stephen. 1991. **Priests and Programmers: Technologies of Power in the Engineered Landscape of Bali**. Princeton, N.J.: Princeton University Press.

Latek, M.; B. Kaminski and Robert L. Axtell. 2009. "Bounded rationality via recursion," In Proceedings of the Eighth International Conference on Autonomous Agents and Multiagent Systems. Budapest, Hungary.

Latek, Maciej M.; Seyed M. Mussavi Rizi and Armando Geller. 2013. "Persistence in the Political Economy of Conflict: The Case of the Afghan Drug Industry," In AAAI Fall Symposium. Arlington, VA: AAAI.

Laver, Michael and Ernest Sergenti. 2011. **Party Competition: An Agent-Based Model**. Princeton, NJ: Princeton University Press.

Lawson, Barry G. and Steve Park. 2000. "Asynchronous Time Evolution in an Artificial Society Model." *Journal of Artificial Societies and Social Simulation*, 3(1).

Lazer, David *et al.* 2009. "Computational Social Science." *Science*, 323(5915), pp. 721-23.

Leal, Sandrine Jacob; Mauro Napoletano; Andrea Roventini and Giorgio Fagiolo. 2016. "Rock around the clock: An agent-based model of low- and high-frequency trading." *Journal of Evolutionary Economics*, 26(1), pp. 49-76.

LeBaron, Blake. 2001a. "Empirical Regularities from Interacting Long and Short Memory Investors in an Agent-Based Stock Market." *IEEE Transactions on Evolutionary Computation*, 5, pp. 442-55.

____. 2001b. "Evolution and Time Horizons in an Agent-Based Stock Market." *Macroeconomic Dynamics*, 5, pp. 225-54.

____. 2001c. "Financial Market Efficiency in a Coevolutionary Environment," In Agents 2000: The Simulation of Social Agents: Architectures and Institutions, ed. D. Sallach and T. Wolsko. Chicago, Ill.: Argonne National Laboratory.

____. 2001d. "Volatility Magnification and Persistence in an Agent Based Financial Market," In Working paper. Waltham, Mass.: Brandeis University.

____. 2002. "Short-Memory Traders and Their Impact on Group Learning in Financial Markets." *Proc Natl Acad Sci USA*, 99(suppl 3), pp. 7201-06.

____. 2006. "Agent-based Computational Finance," In Handbook of Computational Economics, ed. L. Tesfatsion and K. L. Judd, 1187-233. New York, N.Y.: North-Holland.
LeCun, Yann; Yoshua Bengio and Geoffrey Hinton. 2015. "Deep learning." *Nature*, 521, pp. 436.
Lee, Hau L.; V. Padmanaabhan and Seungjin Whang. 1997. "Information Distortion in a Supply Chain: The Bullwhip Effect." *Management Science*, 43(3), pp. 546-58.
Leijonhufvud, Axel. 1967. "Keynes and the Keynesians: A Suggested Interpretation." *American Economic Review, Papers and Proceedings*, 57(2), pp. 401-10.
____. 1993. "Towards a Not-To-Rational Macroeconomics." *Southern Economic Journal*, 61(1), pp. 1-13.
Leitao, Paulo. 2009. "Agent-based distributed mnufacturing control: A state-of-the-art survey." *Engineering Applications of Artificial Intelligence*, 22(7), pp. 979-91.
Lempert, Robert. 2002. "Agent-based modeling as organizational and public policy simulators." *Proc Natl Acad Sci USA*, 99(3), pp. 7195-96.
Leontief, Wassily W. 1951. "Input-Output Economics." *Scientific American*, 185(4), pp. 15-21.
LePage, Christophe; Francois Bousquet; Innocent Bakam; Alassane Bah and Christian Baron. 2000. "CORMAS: A Multiagent Simulation toolkit to Model Natural and Social Dynamics at Multiple Scales."
Levy, H.; Moshe Levy and Sorin Solomon. 2000. **Microscopic Simulation of Financial Markets: From Investor Behavior to Market Pheonomena**. New York, N.Y.: Academic Press.
Lewars, Errol G. 2011. **Computational Chemistry: Introduction to the Theory and Applications of Molecular and Quantum Mechanics**. New York, N.Y.: Springer.
Lewis, David. 1969. **Convention: A Philosophical Study**. Cambridge, Mass.: Harvard University Press.
Lindgren, K. 1992. "Evolutionary Phenomena in Simple Dynamics," In Artificial Life II, ed. C. G. Langton, C. Taylor, J. D. Farmer and S. Rasmussen. Redwood City, Calif.: Addison-Wesley.
Liu, Jiming. 2001. **Autonomous Agents and Multi-Agent Systems: Explorations in Learning, Self-Organization and Adaptive Computation**. Singapore: World Scientific.
Lo, Andrew W. 2004. "The Adaptive Markets Hypothesis." *Journal of Portfolio Management*, 30(5), pp. 15-29.
Lomi, Alessandro and Erik R. Larsen eds. **Dynamics of Organizations: Computational Modeling and Organization Theories**. Cambridge, Mass.: MIT Press, 2001.
Lomuscio, Alessio and Marek Sergot. 2002. "On Multi-agent Systems Specification vis Deontic Logic," In Intelligent Agents VIII, ed. J.-J. C. Meyer and M. Tambe, 86-99. Berlin: Springer-Verlag.
Longini Jr., Ira M. *et al.* 2005. "Containing Pandemic Influenza at the Source." *Science*, 309, pp. 1083-87.
Lucas, Robert E., Jr. 1978. "On the Size Distribution of Business Firms." *Bell Journal of Economics*, 9, pp. 508-23.
____. 1986. "Adaptive Behavior and Economic Theory," In Rational Choice: The Contrast between Economics and Psychology, ed. R. M. Hogarth and M. W. Reder. Chicago, Ill.: University of Chicago Press.
Luke, Sean. 2013. **Essentials of Metaheuristics**. lulu.com.
Luke, Sean; Claudio Cioffi-Revilla; Liviu Panait; Keith Sullivan and Gabriel Balan. 2005. "MASON: A Multiagent Simulation Environment." *SIMULATION*, 81(7), pp. 517-27.
Luna, Francesco. 2000. "Induction and Firm Creation," In Economic Simulations in Swarm: Agent-Based Modelling and Object Oriented Programming, ed. F. Luna and B. Stefansson. Dordrecht: Kluwer Academic Publishers.
Luna, Francesco and Benedikt Stefansson eds. **Economic Simulations in Swarm: Agent-Based Modelling and Object Oriented Programming**. Dordrecht: Kluwer Academic Publishers, 2000.
Lustick, Ian S. and Dan Miodownik. 2009. "Abstractions, Ensembles, and Virtualizations: Simplicity and Complexity in Agent-Based Modeling." *Comparative Politics*, 41(2), pp. 223-44.
Luttmer, Erzo G.J. 2011. "On the Mechanics of Firm Growth." *Review of Economic Studies*, 78(3), pp. 1042-68.
____. 2012. "Technology Diffusion and Growth." *Journal of Economic Theory*, 147, pp. 602-22.
Lux, Thomas. 1998. "The Socioeconomic Dynamics of Speculative Markets: Interacting Agents, Chaos and the Fat Tails of Return Distributions." *Journal of Economic Behavior and Organization*, 33, pp. 143-65.
____. 2015. "Emergence of a core-periphery structure in a simple dynamic model of the interbank market." *Journal of Economic Dynamics and Control*, 52, pp. A11-A23.
Lux, Thomas and Michele Marchesi. 1999. "Scaling and Criticality in a Stochastic Multi-Agent Model of a Financial Market." *Nature*, 397, pp. 498-500.
____. 2000. "Volatility Clustering in Financial Markets: A Microsimulation of Interacting Agents." *International Journal of Theoretical and Applied Finance*, 3(4), pp. 675-702.
Lux, Thomas and Remco C.J. Zwinkels. 2018. "Empirical Validation of Agent-Based Models," In Handbook of Computational Economics, ed. C. Hommes and B. LeBaron, 437-88. Amsterdam: North-Holland.
Macal, C.M. 2016. "Everything you need to konw about agent-based modelling and simulation." *Journal of Simulation*, 10, pp. 144-56.
Mackenzie, Donald. 2006. **An Engine, Not a Camera: How Financial Models Shape Markets**. Cambridge, Mass.: MIT Press.
Macy, Michael W. and Cristiano Castelfranchi. 1998. "Social Order in Artificial Worlds." *Journal of Artificial Societies and Social Simulation*, 1(1).
Macy, Michael W. and Andreas Flache. 2002. "Learning Dynamics in Social Dilemmas." *Proc Natl Acad Sci USA*, 99(Suppl. 3), pp. 7229-36.

Macy, Michael W. and Robb Willer. 2002. "From Factors to Actors: Computational Sociology and Agent-Based Modeling." *Annual Review of Sociology*, 28, pp. 143-66.
Maes, Patti. 1990. **Designing Autonomous Agents: Theory and Practice from Biology to Engineering and Back**. Cambridge, Mass.: MIT Press.
Magliocca, Nicholas; Virginia McConnell and Margaret Walls. 2015. "Exploring sprawl: Results from an economic agent-based model of land and housing markets." *Ecological Economics*, 113, pp. 114-25.
Magliocca, Nicholas; Elena Safirova; Virginia McConnell and Margaret Walls. 2011. "An economic agent-based model of coupled housing and land markets (CHALMS)." *Computers, Environment and Urban Systems*, 35, pp. 183-91.
Mandelbrot, Benoit. 1963. "The Variation on Certain Speculative Prices." *Journal of Business*, XXXVI, pp. 394-419.
Manski, Charles. 1993. "Identification of Endogenous Social Effects: The Reflection Problem." *Review of Economic Studies*, 60, pp. 531-42.
____. 1995. **Identification Problems in the Social Sciences**.
____. 1997. "Identification of Anonymous Endogenous Social Interactions," In The Economy as an Evolving Complex System, ed. W. B. Arthur, S. Durlauf and D. A. Lane. Menlo Park, Calif.: Addison-Wesley.
Manzo, Gianluca ed. **Analytical Sociology: Actions and Networks**. Chichester, West Sussex, U.K.: John Wiley & Sons, Ltd., 2014.
Marimon, R.; Ellen McGrattan and Thomas J. Sargent. 1990. "Money as a Medium of Exchange in an Economy with Artificially Intelligent Agents." *Journal of Economic Dynamics and Control*, 14, pp. 329-73.
Markram, Henry. 2006. "The Blue Brain Project." *Nature Reviews Neuroscience*, 7, pp. 153-60.
____. 2012. "A Countdown to a Digital Simulation of Every Last Neuron in the Human Brain." *Scientific American*.
Marks, Robert E. 1992. "Breeding Hybrid Strategies: Optimal Behavior for Oligopolists." *Journal of Evolutionary Economics*, 2, pp. 17-38.
Marshall, Alfred. 1920. **Principles of Economics**. London: Macmillan.
Martin, Cedric; Jean-Christophe Castella; Hoang Lan Anh; Yann Eguienta and Tran Trong Hieu. 2004. "A Participatory Simulation to Facilitate Farmers' Adoption of LIvestock Feeding Systems Based on Conservation Agriculture in Uplands of Northern Vietnam." *International Journal of Agricultural Sustainability*, 2(2), pp. 118-32.
Mas-Colell, Andreu; Michael D. Whinston and Jerry R. Green. 1995. **Microeconomic Theory**. New York, N.Y.: Oxford University Press.
Matthews, Robin B.; Nigel G. Gilbert; Alan Roach; gary Polhill and Nick M. Gotts. 2007. "Agent-based land-use models: a review of applications." *Landscape Ecology*, 22(10), pp. 1447-59.
McFadden, Daniel and Paul A. Ruud. 1994. "Estimation by Simulation." *The Review of Economics and Statistics*, LXXVI(4), pp. 591.
Meadows, Donella H.; Dennis L. Meadows; Jorgen Randers and William W. Behrens, III. 1972. **The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind**. New York, N.Y.: Universe Books.
Mellacher, Patrick and Timon Scheuer. 2021. "Wage Inequality, Labor Market Polarization and Skill-Biased Technological Change: An Evolutionary (Agent-Based) Approach." *Computational Economics*, 58, pp. 233-78.
Mian, Atif; Kamalesh Rao and Amir Sufi. 2013. "Household Balance Sheets, Consumption, and the Economic Slump." *Quarterly Journal of Economics*, 128(4), pp. 1687-726.
Michalewicz, Zbigniew and David B. Fogel. 2000. **How to Solve It: Modern Heuristics**. New York, N.Y.: Springer-Verlag.
Miller, John H. 2015. **A Crude Look at the Whole: The Science of Complex Systems in Business, Life, and Society**. New York, N.Y.: Basic Books.
Miller, John H. and Scott E. Page. 2007. **Complex Adaptive Systems: An Introduction to Computational Models of Social Life**. Princeton, N.J.: Princeton University Press.
Miller, Noam; Simon Garnier; Andrew T. Hartnett and Iain D. Couzin. 2013. "Both information and social cohesion determine collective decisions in animal groups." *Proc Natl Acad Sci USA*, 110(13), pp. 5263-68.
Minar, Nelson; Roger Burkhart; Christopher G. Langton and Manor Askenazi. 1996. "The Swarm Simulation System: A Toolkit for Building Multi-agent Simulations," In. Swarm.org.
Minsky, Hyman P. 1982. "The Financial Instability Hypothesis: Capitalistic Processes and the Behavior of the Economy," In Financial Crises: Theory, History and Policy, ed. C. P. Kindleberger and J.-P. Laffargue, 13-29. Cambridge, U.K.: Cambridge University Press.
____. 1986. **Stabilizing an Unstable Economy**. New Haven, Conn.: Yale University Press.
____. 1992. "The Financial Instability Hypothesis," In Levy Economics Institute of Bard College, 9. Annandale-on-Hudson, N.Y.: Working Paper, No. 74.
Mirowski, Philip. 2001. **Machine Dreams: How Economics Became a Cyborg Science**. New York, N.Y.: Cambridge University Press.
Mirowski, Philip E. 1996. "Do You Know the Way to Santa Fe? Or, Political Economy Gets More Complex," In Interactions in Political Economy: Malvern After Ten Years, ed. S. Pressman, 13-40. New York, N.Y.: Routledge.
Mitchell, Melanie. 2009. **Complexity: A Guided Tour**. New York, N.Y.: Oxford University Press.

Mittone, L. and Paolo Patelli. 2000. "Imitative Behaviour in Tax Evasion," In Economic Modelling with Swarm: Agent-Based Modelling and Object Oriented Programming, ed. F. Luna and Stefansson. Boston: Kluwer Academic Publishers.
Moore, Cristopher and Stephan Mertens. 2011. **The Nature of Computation**. Oxford, U.K.: Oxford University Press.
Morgan, M. Granger. 2014. "Use (and abuse) of expert elicitation in support of decision making for public policy." *Proc Natl Acad Sci USA*, forthcoming.
Morgan, M. Granger; Max Henrion and Mitchell J. Small. 1990. **Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis**. New York, N.Y.: Cambridge University Press.
Morgenstern, Oskar and Gerald L. Thompson. 1976. **Mathematical Theory of Expanding and Contracting Economies**. Lexington, Mass.: Lexington Books.
Moss, Scott. 2001a. "Game Theory: Limitations and an Alternative." *Journal of Artificial Societies and Social Simulation*, 4(2).
____. 2001b. "Messy Systems - The Target for Multi Agent Based Simulation," In Multi-Agent-Based Simulation, ed. S. Moss and P. Davidsson, 1-14. Heidelberg, Germany: Springer-Verlag.
____. 2002. "Policy Analysis from First Principles." *Proc Natl Acad Sci USA*, 99(suppl. 3), pp. 7267-74.
Moss, Scott; Claudia Pahl-Wostl and Thomas Downing. 2001. "Agent-Based Integrated Assessment Modelling: The Example of Climate Change." *Integrated Assessment*, 2(1), pp. 17-30.
Nagel, Kai and Chris Barrett. 1997. "Using Microsimulation Feedback for Trip Adaptation for Realistic Traffic in Dallas." *International Journal of Modern Physics C*, 8(3), pp. 505-25.
Nagel, Kai; R. Beckman and Chris L. Barrett. 1998. "TRANSIMS for Transportation Planning," In Technical Report. Los Alamos, N.M.: Los Alamos National Laboratory.
Nagel, Kai and Maya Paczuski. 1995. "Emergent Traffic Jams." *Physical Review E*, 51, pp. 2909.
Nagel, Kai and Steen Rasmussen. 1994. "Traffic at the Edge of Chaos," In Artificial Life IV, ed. R. A. Brooks and P. Maes, 224-35. Cambridge, Mass.: MIT Press.
Nagy, Béla; J. Doyne Farmer; Jessika E. Trancik and John Paul Gonzales. 2011. "Superexponential long-term trends in information technology." *Technological Forecasting and Social Change*, 78(8), pp. 1356-64.
Namatame, Akira and Shu-Heng Chen. 2016. **Agent-Based Modeling and Network Dynamics**. Oxford, U.K.: Oxford University Press.
Neugart, Michael. 2004. "Endogenous matching functions: An agent-based computational approach." *Advances in Complex Systems*, 7(2).
____. 2008. "Labor market policy evaluation with ACE." *Journal of Economic Behavior and Organization*, 67(2), pp. 418-30.
Newell, Allen and Herbert A. Simon. 1956. "The Logic Theory Machine: A Complex Information Processing System." *IRE Transactions on Information Theory*, 2(3), pp. 61-79.
____. 1972. **Human Problem Solving**. Englewood Cliffs, N.J.: Prentice-Hall.
Newman, Mark E.J. 2010. **Networks: An Introduction**. New York, N.Y.: Oxford University Press.
Niazi, Muaz and Amir Hussain. 2011. "Agent-based computing from multi-agent systems to agent-based models: A visual survey." *Scientometrics*, 89, pp. 479-99.
Nicolaisen, James; Valentin Petrov and Leigh Tesfatsion. 2000. "Market Power and Efficiency in a Computaitonal Electricity Market with Discriminatory Double-Auction Pricing," In Staff General Reearch Papers. Ames, Iowa: Iowa Stae University.
Nier, Eriend; Jing Yang; Tanju Yorulmazer and Amadeo Alentorn. 2007. "Network Models and Financial Stability." *Journal of Economic Dynamics and Control*, 31, pp. 2033-60.
Nikolett, Mérö Bence-Vágó. 2018. "A Demand-Led Model of the Housing Market for Studying the Macroprudential Means of Regulating Housing Loand." *Közgazdasági Szemle*, 65(11), pp. 1115-53.
Nisan, Noam; Tim Roughgarden; Eva Tardos and Vijay V. Vazirani eds. **Algorithmic Game Theory**. New York, N.Y.: Cambridge University Press, 2007.
Nissen, Mark E. and Raymond E. Levitt. 2004. "Agent-based modeling of knowledge dynamics." *Knowledge Management Research & Practice*, 2(3), pp. 169-83.
Noe, Thomas H.; Michael J. Rebello and Jun Wang. 2003. "Corporate Financing: An Artificial Agent-based Analysis." *The Journal of Finance*, LVIII(3), pp. 943-73.
____. 2006. "The Evolution of Security Designs." *The Journal of Finance*, LXI(5), pp. 2103-35.
Nordhaus, William D. 1993a. "Optimal Greenhouse-Gas Reductions and Tax Policy in the "DICE" Model." *American Economic Review (Papers and Proceedings)*, 83(2), pp. 313-17.
____. 1993b. "Rolling the 'DICE': An optimal transition path for controlling greenhouse gases." *Resource and Energy Economics*, 15, pp. 27-50.
Norman, Michael L. *et al.* 1996. "Galaxies Collide On the I-Way: an Example of Heterogeneous Wide-Area Collaborative Supercomputing." *International Journal of High Performance Computing Applications*, 10(2-3), pp. 132-44.
North, Michael J.; Nicholson T. Collier and Jerry R. Vos. 2006. "Experiences Creating Three Implementations of the Repast Agent Modeling Toolkit." *ACM Transactions on Modeling and Computer Simulation*, 16(1), pp. 1-25.

North, Michael J. and Charles M. Macal. 2007. **Managing Business Complexity: Discovering Strategic Solutions with Agent-Based Modeling and Simulation**. New York, N.Y.: Oxford University Press.
Norton, Kerri-Ann and Aleksander S. Popel. 2014. "An agent-based model of cancer stem cell initiated avascular tumour growth and metastasis: the effect of seeding frequency and location." *Journal of the Royal Society Interface*, 11, pp. 1-13.
Nowak, Martin A. and Robert M. May. 1992. "Evolutionary Games and Spatial Chaos." *Nature*, 359, pp. 827-29.
Nozick, Robert. 1994. "Invisible Hand Explanations." *American Economic Review*, 84(2), pp. 314-18.
O'Hare, G.M.P. and Nicholas R. Jennings. 1996. **Foundations of Distributed Artificial Intelligence**. New York, N.Y.: Wiley.
Oldham, Matthew. 2017. "Introducing a Multi-Asset Stock Market to Test the Power of Investor Networks." *Journal of Artificial Societies and Social Simulation*, 20(4), pp. 22.
Orcutt, Guy H. 1957. "A New Type of Socio-Economic System." *Review of Economics and Statistics*, 39(2), pp. 116-23.
____. 1960. "Simulation of Economic Systems." *American Economic Review*, 50(5), pp. 893-907.
Orcutt, Guy H.; Martin Greenberger; John Korbel and Alice M. Rivlin. 1961. **Microanalysis of Socioeconomic Systems: A Simulation Study**. New York, N.Y.: Harper & Row.
Ossowski, Sascha. 1999. **Co-ordination in Artificial Agent Societies: Social Structure and Its Implications for Autonomous Problem-Solving Agents**. Berlin: Springer-Verlag.
Ostrom, Elinor. 1990. **Governing the Commons: The Evolution of Institutions for Collective Action**. New York, N.Y.: Cambridge University Press.
Ostrom, Elinor; Joanna Burger; Christopher B. Field; Richard B. Norgaard and David Policansky. 1999. "Revisiting the Commons: Local Lessons, Global Challenges." *Science*, 284, pp. 278-82.
Ostrom, Elinor; Roy Gardner and James Walker eds. **Rules, Games, and Common Pool Resources**. Ann Arbor, Michigan: University of Michigan Press, 1994.
Paddrik, Mark; Roy Hayes; Andrew Todd; Steve Yang; Peter Beling and William Scherer. 2012. "An agent based model of the E-Mini S&P 500 applied to flash crash analysis," In IEEE Conference on Computational Intelligence for Financial Engineering & Economics. New York, N.Y.: IEEE.
Page, Scott E. 1997. "On Incentives and Updating in Agent Based Models." *Computational Economics*, 10(1), pp. 67-87.
____. 2007. **The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies**. Princeton University Press.
Palmer, Richard G.; W. Brian Arthur; John Henry Holland; Blake LeBaron and Paul Tayler. 1994. "Artificial economic life: A simple model of a stock market." *Physica D*, 75, pp. 264-74.
Panait, Liviu and Sean Luke. 2005. "Cooperative Multi-Agent Learning; The State of the Art." *Autonomous Agents and Multi-Agent Systems*, 11, pp. 387-434.
Pancs, R. and Nicholaas J. Vriend. 2007. "Schellings's spatial proximity model of segregation revisited." *Journal of Public Economics*, 91(1-2), pp. 1-24.
Pangallo, Marco; Torsten Heinrich and J. Doyne Farmer. 2019. "Best reply structure and equilibrium convergence in generic games." *Science Advances*, 5(2), pp. 13.
Papadimitriou, Christos H. 1994. "On the Complexity of the Parity Argument and Other Inefficient Proofs of Existence." *Journal of Computer and Systems Sciences*, 48, pp. 498-532.
Papadimitriou, Christos and Mihalis Yannakakis. 1994. "On Complexity as Bounded Rationality," In Proceedings of the Twenty-Sixth Annual ACM Symposium on the Theory of Computing, 726-33. New York, N.Y.: ACM Press.
Papadopoulos, Georgios. 2019. "Income inequality, consumption, credit and credit risk in a data-driven agent-based model." *Journal of Economic Dynamics and Control*, 104, pp. 39-73.
Parker, Dawn; Steven Manson; Marco Janssen; Matt Hoffman and Peter Deadman. 2003. "Multi-Agent Systems for the Simulation of Land Use and Land Cover Change: A Review." *Annals of the Association of American Geographers*, 93(2), pp. 314-37.
Parkes, David C. and Michael P. Wellman. 2015. "Economic Reasoning and Artificial Intelligence." *Science*, 349(6245), pp. 267-72.
Parsons, Simon; Piotr J. Gmytrasiewicz and Michael Wooldridge eds. **Game Theory and Decision Theory in Agent-Based Systems**. Boston, Mass.: Kluwer Academic Publishers, 2002.
Parunak, H Van Dyke; Robert Savit and Rick L. Riolo. 1998. "Agent-Based Modeling vs. Equation-Based Modeling: A Case Study and Users' Guide," In Multi-Agent Systems and Agent-Based Simulation, ed. J. S. Sichman, R. Conte and G. N. Gilbert. Berlin: Springer-Verlag.
Pawson, Tony. 1995. "Protein modules and signalling networks." *Nature*, 373, pp. 573-80.
Phelan, Steven E. 2004. "Using Agent-Based Simulation to Examine the Robustness of Up-or-Out Promotion Systems in Universities." *Nonlinear Dynamics, Psychology, and Life Sciences*, 8(2), pp. 177-204.
Polanyi, Michael. 1948. "Planning and Spontaneous Order." *The Manchester School of Economic and Social Studies*, 16, pp. 237-68.
Poledna, Sebastian and Stefan Thurner. 2016. "Elimination of systemic risk in financial networks by means of a systemic risk transaction tax." *Quantitative Finance*, 16(10), pp. 1599-613.

Poledna, Sebastian; Stefan Thurner; J. Doyne Farmer and John Geanakoplos. 2014. "Leverage-induced systemic risk under Basle II and other credit risk policies." *Journal of Banking & Finance*, 42, pp. 199-212.

Potts, Jason D. 2000. **The New Evolutionary Microeconomics: Complexity, Competence and Adaptive Behaviour**. Northampton, Mass.: Edward Elgar.

Prietula, Michael J.; Kathleen M. Carley and Les Gasser eds. **Simulating Organizations: Computational Models of Institutions and Groups**. Cambridge, Mass.: MIT Press, 1998.

Rader, Trout. 1968. "Pairwise Optimality and Non-Competitive Behavior," In Papers in Quantitative Economics, ed. J. P. Quirk and A. M. Zarley. Lawrence, Kansas: University Press of Kansas.

Rahmandad, Hazhir and John Sterman. 2008. "Heterogeneity and Network Structure in the Dynamics of Diffusion: Comparing Agent-Based and Differential Equation Models." *Management Science*, 54(5), pp. 998-1014.

Rai, Varun and Adam Douglas Henry. 2016. "Agent-based modelling of consumer energy choices." *Nature Climate Change*, 6, pp. 556-62.

Railsback, Steven F. and Volker Grimm. 2011. **Agent-Based and Individual-Based Modeling: A Practical Introduction**. Princeton, N.J.: Princeton University Press.

____. 2019. **Agent-Based and Individual-Based Modeling: A Practical Introduction**. Princeton, N.J.: Princeton University Press.

Ramanath, Ana Maria and Nigel Gilbert. 2004. "The Design of Participatory Agent-Based Social Simulations." *Journal of Artificial Societies and Social Simulation*, 7(4).

Rand, William and Roland T. Rust. 2011. "Agent-based modeling in marketing: Guidelines for rigor." *International Journal of Research in Marketing*, 28(3), pp. 181-93.

Rao, Anand S. and Michael P. Georgeff. 1995. "BDI Agents: From Theory to Practice," In Proceedings of the First International Conference on Multiagent Systems (ICMAS-95), ed. V. Lesser, 312-19. San Francisco, Calif.: American Association for Artificial Intelligence.

Ray, Debraj. 2007. **A Game Theoretic Perspective on Coalition Formation**. New York, N.Y.: Oxford University Press.

Ray, Debraj and Raj Vohra. 1999. "A Theory of Endogenous Coalition Structure." *Games and Economic Behavior*, 26, pp. 286-336.

Repenning, Alexander; Andri Ioannidou and John Zola. 2000. "AgentSheets: End-User Programmable Simulations." *Journal of Artificial Societies and Social Simulation*, 3(3).

Reynolds, Craig W. 1987. "Flocks, Herds, and Schools: A Distributed Behavioral Model." *Computer Graphics*, 21(4), pp. 25-34.

Richiardi, Matteo. 2004. "A search model of unemployment and firm dynamics." *Advances in Complex Systems*, 7(2).

Ridker, Ronald G. 1973. "To Grow or Not to Grow: That's Not the Relevant Queston." *Science*, 182(4119), pp. 1315-18.

Roberts, Siobhan. 2015. **Genius at Play: The Curious Mathematical Mind of John Horton Conway**. New York, N.Y.: Bloomsbury Publishing.

Rodríguez, Luis-Felipe and Félix Ramon. 2014. "Development of Computational Models of Emotions for Autonomous Agents: A Review." *Cognitive Computation*, 6, pp. 351-75.

Rogers, Everett. 1995. **Diffusion of Innovations**. New York, N.Y.: Free Press.

Rohaly, Jeffrey; Adam Carasso and Mohammed Adeel Saleem. 2005. "The Urban-Brookings Tax Policy Center Microsimulation Model: Documentation and Methodology for Version 304," In. Washington, D.C.: Urban Institute.

Rosenbloom, Paul S.; John E. Laird; Allen Newell and F. Orciuch. 1985. "R1-Soar: An experiment in knowledge-intensive programming in a problem-solving architecture." *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-7(5).

Rosenschein, Jeffrey S. and Gilad Zlotkin. 1994. **Rules of Encounter: Designing Conventions for Automated Negotiation among Computers**. Cambridge, Mass.: MIT Press.

Rosser, Jr., J. Barkley ed. **Complexity in Economics**. Northampton, Mass.: Edward Elgar Publishing, Inc., 2004.

____. 2010. "Is a transdisciplinary perspectove on economic complexity possible?" *Journal of Economic Behavior and Organization*, 75(1), pp. 3-11.

Roth, Alvin E. and I. Erev. 1995. "Learning in Extensive-Form Games: Experimental Data and Simple Dynamic Models in the Intermediate Term." *Games and Economic Behavior*, 8, pp. 164-212.

Roth, Alvin E. and Marilda A. Oliveira Sotomayor. 1990. **Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis**. Cambridge, U.K.: Cambridge University Press.

Rouchier, Juliette; Francois Bousquet; Olivier Barreteau; Christophe Le Page and Jean-Luc Bonnefoy. 2001. "Multi-Agent Modelling and Renewable Resource Issues: The Relevance of Shared Representations for Interacting Agents," In Multi-Agent-Based Simulation, ed. S. Moss and P. Davidsson, 181-97. Hiedelberg, Germany: Springer-Verlag.

Roughgarden, Tim. 2010. "Computing equilibria: a computational complexity perspective." *Economic Theory*, 42, pp. 193-236.

Russell, Stuart J and Peter Norvig. 2010. **Artificial Intelligence: A Modern Approach**. New York, N.Y.: Pearson.

Russo, Alberto; Luca Riccetti and Mauro Gallegati. 2016. "Increasing inequality, consumer credit and financial fragility in an agent-based macroeconomic model." *Journal of Evolutionary Economics*, 26, pp. 25-47.
Rust, John. 1997. "Dealing with the Complexity of Economic Calculations," In Working paper. New Haven, Conn.: Yale University.
Said, Lamjed Ben; Thierry Bouron and Alexis Drogoul. 2002. "Agent-baed interaction analysis of consumer behavior," In First International Joint Conference on Autonomous Agents and Multiagent Systems, 184-90.
Sakoda, James M. 1949. "Minidoka: An analysis of changing patterns of social interaction," In Sociology. Berkeley, California: University of California, Berkeley.
____. 1971. "The Checkerboard Model of Social Interaction." *Journal of Mathematical Sociology*, 1(1), pp. 119-32.
Salganik, Matthew J. *et al.* 2020. "Measuring the predictability of life outcomes with a scientific mass collaboration." *Proceedings of the National Academy of Sciences*, 117(15), pp. 8398-403.
Sandholm, Tuomas. 1999. "Rational Agent Decision-Making," In Multiagent Systems: A Modern Approach to Distributed Artificial Intelligence, ed. G. Weiss. Cambridge, Mass.: MIT Press.
____. 2007. "Perspectives on multiagent learning." *Artificial Intelligence*, 171(382-391).
Sandholm, Tuomas; Kate Larsen; M. Andersson; Onn Shehory and F. Tohme. 1998. "Anytime Coalition Structure Generation with Worst Case Guarantees." *Proceedings of the American Association for Artificial Intelligence*.
Sargent, Thomas J. 1999. **The Conquest of American Inflation**. Princeton, N.J.: Princeton University Press.
Scarf, Herbert. 1973. **The Computation of Economic Equilibria**. New Haven, Conn.: Yale University Press.
Scarf, Herbert E. 1982. "The Computation of Equilibrium Prices: An Exposition," In Handbook of Mathematical Economics, Volume II, ed. K. J. Arrow and M. D. Intrilligator. New York, N.Y.: North-Holland.
Scarf, Herbert E. and John B. Shoven eds. **Applied General Equilibrium Analysis**. New York, N.Y.: Cambridge University Press, 1984.
Schelling, Thomas C. 1969a. "Models of Segregation," In RAND Memoranda, 1-92. Santa Monica, California.
____. 1969b. "Models of Segregation." *American Economic Review*, 59(2), pp. 488-93.
____. 1971a. "Dynamic Models of Segregation." *Journal of Mathematical Sociology*, 1(2), pp. 143-86.
____. 1971b. "On the Ecology of Micro-Motives." *Public Interest*, 25, pp. 59-98.
____. 1972a. "On Letting a Computer Help with the Work," In Teaching & Research Materials, Public Policy Program, John F. Kennedy School of Govenrment, Harvard University, 54. Cambridge, Mass.
____. 1972b. "A Process of Residential Segregation: Neighborhood Tipping," In Racial Discrimination in Economic Life, ed. A. Pascal, 157-84. Lexington, Mass.: D.C. Heath.
____. 2006. "Some Fun, Thirty-Five Years Ago," In Handbook of Computational Economics, ed. L. Tesfatsion and K. L. Judd, 1639-44. New York, N.Y.: North-Holland.
Schlesinger, Matthew and Domenico Parisi. 2001. "The Agent-Based Approach: A New Direction for Computational Models of Development." *Developmental Review*, 21, pp. 121-46.
Schmidhuber, Jürgen. 2015. "Deep learning in neural networks: An overview." *Neural Networks*, 61, pp. 85-117.
Schreiber, Darren. 2014. "The Emergence of Parties: An Agent-Based Simulation." *Political Research Quarterly*, 67(1), pp. 136-51.
Shehory, Onn and Sarit Kraus. 1993. "Coalition Formation Among Autonomous Agents: Strategies and Complexity," In From Reaction to Cognition: Proceedings of the Fifth European Workshop on Modeling Autonomous Agents in a Multi-Agent World, ed. C. Castelfranchi and J.-P. Muller. New York, N.Y.: Springer-Verlag.
Shoham, Yoav and Kevin Layton-Brown. 2009. **Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations**. New York, N.Y.: Cambridge University Press.
Shoham, Yoav; R. Powers and T. Grenager. 2004. "Multi-agent reinforcement learning: A critical survey," In AAAI Fall Symposium on Artifical Multi-Agent Learning.
Shoham, Yoav; Rob Powers and Trond Grenager. 2007. "If multi-agent learning is the answer, what is the question?" *Artificial Intelligence*, 171, pp. 365-77.
Shoham, Yoav and Moshe Tennenholtz. 1997. "One the Emegence of Social Conventions: Modeling, Analysis and Simulations." *Artificial Intelligence*, 94(1-2), pp. 139-66.
Shoven, John B. and John Whalley. 1992. **Applying General Equilibrium**. New York, N.Y.: Cambridge University Press.
Shubik, Martin. 1960a. "Bibliography on Simulation, Gaming, Artificial Intelligence and Allied Topics." *Journal of the American Statistical Association*, 55(292), pp. 736-51.
____. 1960b. "Simulation of the Industry and the Firm." *American Economic Review*, 50(5), pp. 908-19.
Siebenhüner, Bernd and Volver Barth. 2005. "The role of computer modelling in participatory integrated assessments." *Environmental Impact Assessment Review*, 25, pp. 367-89.
Simon, Herbert A. 1955. "On a Class of Skew Distribution Functions." *Biometrika*, 42, pp. 425-40.
____. 1963. "Problems of Methodology—Discussion." *American Economic Review: Papers and Proceedings*, 53(2), pp. 229-31.
____. 1978a. "On How To Decide What To Do." *Bell Journal of Economics*, 9(2), pp. 494-507.
____. 1978b. "Rationality as Process and as Product of Thought." *American Economic Review, Papers and Proceedings*, 68(2), pp. 1-16.
____. 1996 [1969]. **The Sciences of the Artificial**. Cambridge, Mass.: MIT Press.

____. 1997a. **An Empirically-Based Microeconomics**. Cambridge, U.K.: Cambridge University Press.
____. 1997b. **Models of Bounded Rationality: Behavioral Economics and Business Organizations**. Cambridge, Mass.: MIT Press.
____. 1997c. **Models of Bounded Rationality: Economic Analaysis and Public Policy**. Cambridge, Mass.: MIT Press.
____. 1997d. **Models of Bounded Rationality: Empirically Grounded Economic Reason**. Cambridge, Mass.: MIT Press.
____. 1998. "Comments," In Modeling Bounded Rationality, ed. A. Rubinstein. Cambridge, Mass.: MIT Press.
Slovic, Paul; Baruch Fischoff and Sarah Lichtenstein. 1982. "Facts versus Fears: Understanding Perceived Risk," In Judgement Under Uncertainty: Heuristics and Biases, ed. D. Kahneman, P. Slovic and A. Tversky. New York, N.Y.: Cambridge University Press.
Smaldino, Paul E. and Mark Lubell. 2011. "An Institutional Mechanism for Assortment in an Ecology of Games." *PLoS ONE*, 6(8), pp. 1-7.
____. 2014. "Institutions and cooperation in an ecology of games." *Artificial Life*, 20(2), pp. 207-21.
Smith, Eric; J. Doyne Farmer; Laszlo Gillemot and Supriya Krishnamurthy. 2003. "Statistical theory of the continuous double auction." *Quantitative Finance*, 3(6), pp. 481-514.
Smith, Vernon L. 1962. "An Experimental Study of Competitive Market Behavior." *Journal of Political Economy*, LXX(2), pp. 111-37.
Squazzoni, Flaminio. 2012. **Agent-Based Computational Sociology**. Chichester, U.K.: Wiley.
Stefansson, Benedikt. 2000. "Simulating Economic Agents in Swarm," In Economic Simulations in Swarm: Agent-Based Modelling and Object-Oriented Programming, ed. F. Luna and B. Stefansson, 1-61. Boston: Kluwer Academic Publishers.
Sterman, John. 2000. **Business Dynamics: Systems Thinking and Modeling for a Complex World**. New York, N.Y.: McGraw-Hill.
Stiglitz, Joseph E. 2009. "The Current Economic Crisis and Lessons for Economic Theory." *Eastern Economic Journal*, 35.
Stone, Peter. 2007. "Multiagent learning is not the answer. It is the question." *Artificial Intelligence*, 171, pp. 402-05.
Sun, Ron ed. **Cognition and Multi-Agent Interaction: From Cognitive Modeling to Social Simulation**. New York, N.Y.: Cambridge University Press, 2006.
Tambe, Milind. 2011. **Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned**. New York, N.Y.: Cambridge University Press.
Tassier, Troy and Filippo Menczer. 2001. "Emerging Small-World Referral Networks in Evolutionary Labor Markets." *IEEE Trans. Evol. Comp.*, 5(5), pp. 482-92.
____. 2008. "Social Network Structure, Segregation, and Equality in a Labor Market with Referral Hiring." *Journal of Economic Behavior and Organization*, 66(3-4), pp. 514-28.
Terano, Takao. 2007. "Exploring the Vast Parameter Space of Multi-Agent Based Simulation," In Multi-Agent-Based Simulation VII, ed. L. Antunes and K. Takadama. Heidelberg, Germany: Springer.
Terna, Pietro. 1998. "Simulation Tools for Social Scientists: Building Agent Based Models with SWARM." *Journal of Artificial Societies and Social Simulation*, 1(2), pp. 1-12.
Terranova, Roberta and Enrico M. Turco. 2022. "Concentration, stagnation and inequality: An agent-based approach." *Journal of Economic Behavior and Organization*, 193, pp. 569-95.
Tesfatsion, Leigh. 1998. "Ex Ante Capacity Effects in Evolutionary Labor Markets with Adaptive Search," In Economic Report. Ames, Iowa: Iowa State University.
____. 2001. "Structure, Behavior, and Market Power in an Evolutionary Labor Market with Adaptive Search." *Journal of Economic Dynamics and Control*, 25, pp. 419-57.
____. 2002. "Agent-Based Computational Economics: Growing Economies from the Bottom Up." *Artificial Life*, 8(1), pp. 55-82.
____. 2003. "Agent-Based Computational Economics: Modeling Economies as Complex Adaptive Systems." *Information Sciences*, 149(4), pp. 262-68.
____. 2020. **A New Swing-Contract Design for Wholesale Power Markets**. Wiley/IEEE Press.
Tesfatsion, Leigh and Kenneth L. Judd eds. **Handobok of Computational Economics, Volume 2: Agent-Based Computational Economics**. Elsevier/North-Holland, 2006.
Thagard, Paul. 1988. **Computational Philosophy of Science**. Cambridge, Mass.: MIT Press.
Thompson, Gerald L. and Sten Thore. 1992. **Computational Economics: Economic Modeling with Optimization Software**. Scientific Press, Inc.
Thurner, Stefan; J. Doyne Farmer and John Geanakoplos. 2012. "Leverage causes fat tails and clustered volatility." *Quantitative Finance*, 12(5), pp. 695-707.
Tobler, W.R. 1970. "A Computer Movie Simulating Urban Growth in the Detroit Region." *Economic Geography*, 46(Suppplement), pp. 234-40.
____. 1979. "Cellular geography," In Philosophy in Geography, ed. S. Gale and G. Olsson, 379-86. Dordrecht, The Netherlands: Kluwer.
Todd, Michael J. 1976. **The Computation of Fixed Points and Application**. Berlin, Germany: Springer-Verlag.

Toffoli, T. and N. Margolus. 1987. **Cellular Automata Machines: A New Environment for Modeling**. Cambridge, Mass.: MIT Press.
Tomasiello, Diego Bogado; Mariana Giannotti and Flávia F. Feitosa. 2020. "ACCESS: An agent-based model to explore job accessibility inequalities." *Computers, Environment and Urban Systems*, 81.
Transportation Research Board, x. 1961. "Bibliography on Theory of Traffic Flow and Related Subjects." *Operations Research*, 9(4), pp. 568-74.
Trichet, Jean-Claude. 2010. "Reflections on the nature of monetary policy non-standard measures and finace theory," In ECB 2010 Central Banking Conference. Frankfurt, Germany.
Tullock, Gordon and Colin D. Campbell. 1970. "Computer Simulation of a Small Voting System." *Economic Journal*, LXXX(317), pp. 97-104.
Tversky, Amos and Daniel Kahneman. 1981. "The Framing of Decisions and the Rationality of Choice." *Science*, 211, pp. 453-58.
____. 1986. "Rational Choice and the Framing of Decisions," In Rational Choice: The Contrast between Economics and Psychology, ed. R. M. Hogarth and M. W. Reder. Chicago, Ill.: University of Chicago Press.
____. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5(297-323).
Ulam, Stanislaw. 1952. "Random processes and transformations." *Proceedings of the International Congress on Mathematics, 1950 (American Mathematical Society, Providence, RI)*, 2, pp. 264-75.
Valente, Thomas. 1995. **Network Models of the Diffusion of Innovations**. Cresskill, N.J.: Hampton Press.
____. 1996. "Social Network Thresholds in the Diffusion of Innovations." *Social Networks*, 18, pp. 69-89.
van der Hoek, Wiebe and Michael Wooldridge. 2007. "Multi-Agent Systems," In Handbook of Knowledge Representation, ed. B. Porter, V. Lifschitz and F. van Harmelen, 887-928. Elsevier B.V.
Vandell, Kerry D. and Bennett Harrison. 1978. "Racial Transition among Neighborhoods: A Simulation Model Incorporating Institutional Parameters." *Journal of Urban Economics*, 5, pp. 441-70.
Varian, Hal R. ed. **Economic and Financial Modeling with Mathematica**. New York, N.Y.: Springer, 1992.
Vega-Redondo, Fernando. 2007. **Complex Social Networks**. New York, N.Y.: Cambridge University Press.
Velásquez, Juan D. 1997. "Modeling Emotions and Other Motivations in Synthetic Agents," In AAAI-97 Proceedings.
Vinkovic, D. and Alan P Kirman. 2006. "A physical analogue of the Schelling model." *Proc Natl Acad Sci USA*, 103(51), pp. 19261-5.
Vohra, Rakesh V. and Michael P. Wellman. 2007. "Foundations of multi-agent learning: Introduction to the special issue." *Artificial Intelligence*, 171, pp. 363-64.
von Neumann, John. 1958. **The Computer and the Brain**. New Haven, Conn.: Yale University Press.
von Neumann, John and Arthur W. Burks. 1966. **Theory of Self-Reproducing Automata**. Urbana, Ill.: University of Illinois Press.
von Neumann, John and Oskar Morgenstern. 1944. **Theory of Games and Economic Behavior**. Princeton, N.J.: Princeton University Press.
Vriend, Nicholaas J. 1995. "Self-Organization of Markets: An Example of a Computational Approach." *Computational Economics*, 8(3), pp. 205-31.
____. 2002. "Was Hayek an Ace?" *Southern Economic Journal*, 68(4), pp. 811-40.
____. 2004. "ACE Models of Market Organization." *Revue d'Economie Industrielle*, 107, pp. 63-74.
Waldrop, M. Mitchell. 1992. **Complexity: The Emerging Science at the Edge of Order and Chaos**. New York, N.Y.: Simon & Schuster.
Walker, Adam and Michael Wooldridge. 1995. "Understanding the Emergence of Conventions in Multi-Agent Systems," In International Conference on Multi-Agent Systems, ed. V. Lesser. San Francisco, Calif.: AAAI Press/MIT Press.
Wallace, Robert; Amy Geller and V. Ayano Ogawa eds. **Assessing the Use of Agent-Based Models for Tobacco Regulation**. Washington, D.C.: The National Academies Press, 2015.
Wallick, Richard. 2012. "Agent-based modeling, public choice, and the legacy of Gordon Tullock." *Public Choice*, 152(1), pp. 223-44.
Waltemath, Dagmar *et al.* 2016. "Toward Community Standards and Software for Whole-Cell Modeling." *IEEE Transactions on Biomedical Engineering*, 63(10).
Wang, Zhihui; Joseph D. Butner; Vittorio Cristini and Thomas S. Deisboeck. 2015. "Integrated PK-PD and agent-based modeling in oncology." *Journal of Pharmacokinetics and Pharmacodynamics*, 42(2), pp. 179-89.
Waters, Keith. 2019. "Firm Formation and Regional Labor Allocation," In Schar School of Policy and Government. Arlington, Virginia: George Mason University.
Watts, Duncan. 1999. **Small Worlds: The Dynamics of Networks between Order and Randomness**. Princeton, N.J.: Princeton University Press.
Watts, Duncan J. 2013. "Computational Social Science: Exciting Progress and Future Directions." *The Bridge*, 43(4), pp. 5-10.
Weiss, Gerhard ed. **Multiagent Systems: A Modern Approach to Distributed Artificial Intelligence**. Cambridge, Mass.: MIT Press, 1999.

Wellman, Michael. 1996. "Market-Oriented Programming: Some Early Lessons," In Market Based Control, ed. S. H. Clearwater. World Scientific.
Wellman, Michael P. 2015. "Putting the agent in agent-based modeling." *Autonomous Agents and Multi-Agent Systems*, 30(6), pp. 1175-89.
White, Roger; Guy Engelen and Inge Uljee. 2015. **Modeling Cities and Regions as Complex Systems: From Theory to Planning Applications**. Cambridge, Mass.: MIT Press.
Wilensky, Uri and William Rand. 2015. **An Introduction to Agent-Based Modeling: Modeling Natural, Social, and Engineered Complex Systems with NetLogo**. Cambridge, Mass.: MIT Press.
Wilhite, Alan. 2001. "Bilateral Trade and 'Small-World' Networks." *Computational Economics*, 18(1), pp. 49-64.
Wolfram, Stephen. 1983. "Statistical mechanics of cellular automata." *Reviews of Modern Physics*, 55, pp. 601-44.
____. 1984. "Universality and complexity in cellular automata." *Physica D*, 10, pp. 1-35.
____. 1986. **Theory and Applications of Cellular Automata**. Singapore: World Scientific.
Wooldridge, Michael. 2002. **An Introduction to Multi-Agent Systems**. West Sussex, England: John Wiley & Sons.
Wooldridge, Michael; Michael Fisher; Marc-Philippe Huget and Simon Parsons. 2002. "Model checking multi-agent sytsems with MABLE," In Autonomoust Agents and Multiagent Systems (AAMAS), ed. M. Gini and T. Ishida, 952-59. Bologna, Italy: Association for Computing Machinery.
Wooldridge, Michael J. and Nicholas R. Jennings. 1995a. "Agent theories, architectures, and languages: A survey," In International Workshop on Agent Theories, Architectures, and Languages, ed. M. J. Wooldridge and N. R. Jennings, 1-39. Springer.
____ eds. **Intelligent Agents**. Berlin: Springer-Verlag, 1995b.
Wunder, Michael; Siddharth Suri and Duncan J. Watts. 2013. "Empirical agent based models of cooperation in public goods games," In EC '13 Proceedings of the Fourteenth ACM Conference on Electronic Commerce, 891-908. Philadelphia, Penn.: Association for Computing Machinery.
Wurman, Peter R.; Michael P. Wellman and William E. Walsh. 1998. "The Michigan Internet AuctionBot: A configurable auction server for human and software agents," In Agents '98: Proceedings of the Second International Conference on Autonomous Agents, ed. K. P. Sycara and M. Wooldridge, 301-08. Minneapolis, Minnesota: Association for Computing Machinery.
Yakovenko, Victor M. and Jr. Rosser, J. Barkley. 2009. "Statistical mechanics of money, wealth, and income." *Reviews of Modern Physics*, 81(4), pp. 1703-25.
Yeh, Chia-Hsuan and Chun-Yi Yang. 2010. "Examining the effectiveness of price limits in an artificial stock market." *Journal of Economic Dynamics and Control*, 34, pp. 2089-108.
Young, H. Peyton. 1993a. "The Evolution of Conventions." *Econometrica*, 61(1), pp. 57-84.
____. 1993b. "An Evolutionary Model of Bargaining." *Journal of Economic Theory*, 59(1), pp. 145-68.
____. 1998. **Individual Strategy and Social Structure**. Princeton, N.J.: Princeton University Press.
____. 2007. "The possible and the impossible in multi-agent learning." *Artificial Intelligence*, 171, pp. 429-33.
Young, H. Peyton and Mary A. Burke. 2001. "Competition and Custom in Economic Contracts: A Case Study of Illinois Agriculture." *American Economic Review*, 91, pp. 559-73.
Zhang, Junfu. 2001. "An Evolutionary Approach to Residential Segregation," In Economics. Baltimore, Maryland: Johns Hopkins University.
____. 2003. "Growing Silicon Valley on a landscape: An agent-based approach to high-tech industrial clusters." *Journal of Evolutionary Economics*, 13(5), pp. 529-48.
____. 2004a. "A Dynamic Model of Residential Segregation." *Journal of Mathematical Sociology*, 28, pp. 147-70.
____. 2004b. "Residential Segregation in an All-Integrationist World." *Journal of Economic Behavior and Organization*, 54(4), pp. 533-50.
____. 2011. "Tipping and Residential Segregation." *Journal of Regional Science*, 51, pp. 167-93.
Zhang, Le; Zhihui Wang; Jonathan A. Sagotsky and Thomas S. Deisboeck. 2009. "Multiscale agent-based cancer modeling." *Journal of Mathematical Biology*, 58(4), pp. 545-59.
Zhu, Jun *et al.* 2008. "Integrating large-scale functional genomic data to dissect the complexity of yeast regulatory networks." *Nature Genetics*, 40, pp. 854-61.