Introduction to Special Issue on
‘Applications of Statistical Physics in Economics and Finance’

Doyne Farmer and Thomas Lux, eds.

The papers collected in this special issue are intended to provide a representative sample of current research at the interface between physics and economics. The relationship between both disciplines has been a recurrent topic in the history of science, and recently the interaction between the two fields has been rekindled by a resurgence of physicists working on problems in social science. It has often been remarked that the mathematical apparatus developed by (neo-) classical theory, and in particular general equilibrium theory, was strongly influenced by Newtonian mechanics. Indeed, economics has been criticized for getting stuck in a formal apparatus that physics has long since outgrown (Mirowski, 1989). A neoclassical economist might rightly respond that the two fields have diverged for a good reason, i.e. because of the fundamental differences between the study of inert matter and sentient beings, and that the analogies that might have inspired the original neoclassicists are no longer relevant. What is relevant is the development of useful scientific theories that correctly describe the world as it is, and allow us to make normative predictions that might help change it for the better.

While it had been repeatedly pointed out that Walras and Pareto were deeply influenced by the success of Newtonian physics (Ingrao and Israel, 2000), later analogies between physical modeling and economic phenomena have had more of an engineering flavor, e.g. Phillips’ hydraulic apparatus for the simulation of macroeconomic dynamics (Phillips, 1950), or the use of electronic circuits for the same purpose in Morehouse et al. (1950) and Enke (1951). With the introduction of dynamic optimization for representative agent models the link between the economics and engineering literature became even stronger.

Twentieth century physics was dominated by the study of the big and the small, relativity and cosmology on one hand, and quantum mechanics and particle physics on the other. In both cases the scales are very far removed from those of human beings. Any relevance of the methods and concepts developed in modern physics and those in economics would be purely coincidental. There is, however, one branch of post-Newtonian physics whose methods and fundamental ideas are potentially very useful for economics, and that is *statistical physics*. The fundamental idea of statistical physics is that microscopic interactions are constrained by physical law, but because of our inability to measure microscopic states, can otherwise be regarded as random. This has proved to be a very powerful concept and has created an important field in physics (which also goes under the synonym statistical mechanics). Statistical mechanics reproduces all the results of thermodynamics, but is much more powerful, providing important insights and quantitative theories across an enormous range of applications. It remains a field of active research.

Of course, just because the concepts of statistical physics have been useful for physics does not guarantee that they can be modified to apply to social and economic phenomena. Unlike physics, the laws that constrain social interactions are still unknown, and assuming they exist, may be quite different in character. Nonetheless, the research path is suggestive, and resonates with Adam Smith’s classic idea that economic phenomena are emergent properties of the low-level interactions of selfishly motivated agents who may be entirely unaware of the consequences of these actions at a higher level. As articulated by the brilliant physicist Ettore Majorana in the 1930’s, social and economic phenomena can be described as a statistical ensemble of possible microscopic configurations whose exact realization one cannot determine given our incomplete knowledge of all pertinent factors, and whose exact realization one does not need to know. Instead, in a statistical mechanics approach, one makes predictions based on statistical laws that characterize the ensemble average.

Majorana did not have the chance to develop his ideas (translated into English in Majorana, 2005) due to his mysterious and still unsolved disappearance during a ferry trip in 1938, at the age of 32. It was left to others to develop this research program independently and decades later. One of the leaders was undoubtedly Herbert Simon. He proposed that economics should focus more attention on problems such as the distribution of firm sizes, in which "social phenomena … exhibit some of the same simplicity and regularity of pattern as is seen so commonly in physics" (Simon 1955, Ijiri and Simon 1977). Simon and his collaborators developed theories that are very much in the style of statistical mechanics. Minor modifications of these remain our best models for problems such as firm size, city size, word frequencies, or the distribution of links between web sites.

Another important current in this vein started in the early eighties within the synergetics research group at the University of Stuttgart. Weidlich and Haag (1983) adopted a variety of concepts from statistical physics (phase transitions, Master equations, Fokker-Planck equations) to various problems from sociology, political science and economics. While their work and that of their followers has not received much attention from the 'mainstream' in economics at that time, and has perhaps been more successful in its applications for geography and demography, the research of the Stuttgart school conducted over about a quarter of a century constitutes a rich source of inspiration and a wealth of interesting applications of the synergetic approach (cf. Weidlich, 2002, for a summary of this research).

A broader interest in methods from statistical physics emerged in the nineties when some economists became suspicious of the neglect of heterogeneity and interaction in representative agent models and started to look for alternatives to this standard approach (e.g. Kirman, 1993). This genuine interest from certain branches of economics (Brock, 1991, Aoki, 1994, 1995, Foley, 1994) appeared almost simultaneously with the first papers by physicists on economic issues (Mantegna, 1991, Takayasu et al., 1992) that might be counted as the first "econophysics" publications. This movement has gained momentum over the last decade and has established itself as a visible branch of scientific activity in physics with regular scientific meetings and a sizable body of published research in outlets like *Nature*, *Physical Review Letters*, *Physical Review E* , *Physica A*, and *Quantitative Finance*, to name a few. (For a brief review see Farmer, Shubik and Smith, 2005). The contact between econophysics and economics has, however, been hampered by several factors. The very different culture of scientific publishing in physics and economics has generally prevented publications from econophysics in economics journals. This is partly a matter of style of presentation, but it also reflects fundamental differences in the epistemology of the two fields, in particular different views about the objectives of science. Physicists have a very different view about how work should be presented, and in particular about mathematical rigor (which they generally disdain). In addition, physics has a laissez-faire attitude about publication, believing that it is better to err on the side of letting as many new ideas in as possible, and to let the market eventually decide what is good and what is bad through a Darwinian process that selects what is useful and forgets what is not. As a result there are many econophysics papers of poor quality, which shocks economists. When combined with the fact that the best econophysics papers are published in journals that most economists never read, this body of work remains almost unknown outside the sphere of econophysics.

Communication between physicists and economists has been poor. Physicists are perhaps the only group of scientific professionals who are even more arrogant than economists, and in many cases the arrogance and emotions of both sides have been strongly on display. Many physicists have given the impression that they think that economists know little or nothing about their business, at the same time that they are asking for admission into their club. Many economists have interacted with apprehension to what they view as an attempted invasion by aliens, and have scornfully rejected any work by physicists out of hand, without bothering to have even a passing familiarity with it.

This special issue tries to overcome the lack of communication between the two camps. We believe that the statistical approach to modeling economic activity has enormous potential, and that there is much to be gained from the interactions of physicists with economists. For example, the importance of uncertainty in individual decision-making is already well appreciated by game theorists. Statistical mechanics suggests that uncertainty is not just important at the level of individual decision making, but also at the level of collective behavior, and in circumstances where agents may not be fully rational. For example, the microscopic laws of physics may be viewed as analogous to social institutions, such as the double auction, which constrain the behavior of individuals in much the same way that electric fields constrain the behavior of atoms. Individual decision-making is thus shaped by institutions. It may well be the case that many important economic phenomena depend more strongly on institutional constraints than on individual choice. One approach to understanding this is to modify and extend the ideas of statistical physics so that they can apply to microscopic entities that display preferences, make informed decisions, and anticipate the decisions of others.

Economies depend on the distributed activities of a large number of heterogeneous actors, and the notion that an economy can be described in terms of a single representative agent is at best a very rough approximation. In the absence of detailed knowledge of the endowments, preferences, and level of rationality and deliberation of all the agents, it should be advantageous to approach the overall system behavior via what is already known from other fields about the statistical laws for highly complex systems of interacting subunits. Statistical physics offers a wealth of methodological approaches for doing this.

A particularly valuable set of insights from physics concerns the notions of scaling and universality. As expressed by Stanley et al., ‘Statistical physicists have determined that physical systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details.’ (Stanley et al., 1996). Economies have a large number of interacting particles (i.e., agents) and in economics and finance there are many ‘stylized facts’, such as volatility clustering and the heavy tails of the distributions of financial returns, wealth, income, and firm size, that exhibit robust properties in time and space that cry out for fundamental explanation. Macroeconomic ‘laws’ like Okun’s or the Phillips curve can be considered as emergent properties of the dispersed microeconomic activity of an economy. They appear to be quite robust across countries with different microscopic details in their industrial relations and labor market organization.

The papers collected in this issue make serious efforts to apply statistical physics methods to important issues in economics and finance. They span a range of issues, ranging from economic theory to applications in finance. In ‘Classical Thermodynamics and General Equilibrium Theory’, in mathematically precise terms *Eric Smith* and *Duncan Foley* point out that the three axioms of thermodynamics are in close analogy to the central axioms of general equilibrium theory, while showing that the ways in which the two theories have been used are entirely different. *Masanao Aoki*, in his ‘Thermodynamic Limits of Macroeconomic or Financial Models’ promotes the use of concepts from probabilistic combinatorics and population dynamics in order to characterize stationary statistical distributions of clusters of agents. With his contribution ‘Inter-Pattern Speculation: Beyond Minority, Majority and $-Games’, *Damien Challet* bridges the gap between the huge literatures on the so-called minority game and asset pricing models with heterogeneous agents. Taking into account the time needed to open and close positions, he demonstrates minority-type effects in speculative trading and reinterprets the minority game in the financial setting as a competition for predictability. In ‘Time-variation of Higher Moments in a Financial Market with Heterogeneous Agents: An Analytical Approach’, *Simone Alfarano*, *Thomas Lux*, and *Friedrich Wagner* embed a simplified herding model into a simple equilibrium asset pricing model, and show that it can be used to produce realistic predictions of the time variation of higher moments of prices. The paper by *Moshe Levy*, ‘Stock Market Crashes as Social Phase Transitions’, develops a simple model of financial markets in which investor heterogeneity plays the role of temperature, and argues that this variable is an important factor contributing to crashes. ‘Continuous Cascade Models for Asset Returns’, by *Emmanuel Bacry*, *Alexey Kozhemyak* and *Jean-Francois Muzy*, reviews continuous time multi-fractal models for volatility clustering (which are close to models originally developed to understand vorticity in fluid turbulence). *Szabolcs Mike* and *J. Doyne Farmer* develop a simple agent-based model for order placement in a continuous double auction based on high frequency financial data, and show that there are precise quantitative relationships between order flow and price fluctuations. In ‘Cluster Analysis for Portfolio Optimization’, *Vincenzo Tola*, *Fabrizio Lillo*, *Mauro Gallegati* and *Rosario Mantegna* demonstrate how cluster analysis can be used to improve portfolio optimization. In ‘A Network Analysis of the Italian Overnight Money Market’, *Giulia Iori*, *Giulia de Masi*, *Ovidiu Precup*, *Giampaolo Gabbi* and *Guido Caldarelli* study the topology of the network structure of Italian banks as revealed by a large data set consisting of all overnight transactions within the electronic market for interbank deposits over the period 1999 to 2002. *Bernd Rosenow* in his ‘Determining the Optimal Dimensionality of Multivariate Volatility Models with Tools from Random Matrix Theory’ combines the framework of a multivariate GARCH model with concepts from random matrix theory, allowing him to reduce the vast variance-covariance matrix of large portfolios to a small number of significant components that facilitates parameter estimation and forecasting of portfolio volatility. Finally, *Xavier Gabaix*, *Parameswaran Gopikrishnan*, *Vasiliki Plerou* and *H. Eugene Stanley* address the issue of identifying and interpreting statistical regularities of financial data, which have been at the core of much of the econophysics literature. Expanding on their previous work, they discuss the robustness or universality of power laws and of their potential explanations on the base of a behavioral model of transactions by large investors.

We hope that this selection of papers offers an impression of the scope and breadth of the growing literature in the interface between economics/finance and physics, that it will help readers to get acquainted with these new approaches and that it will stimulate further collaborations between scientists of both disciplines.

We would like to express our sincere thanks to JEDC editors *Carl Chiarella* and *Cars Hommes* for their kind and welcome offer to devote a special issue of the *Journal of Economic Dynamics & Control* to this burgeoning new field of interdisciplinary research and we are extremely grateful for their continuous encouragement and for sympathetically accompanying the gestation process of this issue. We also wish to express our gratitude to the referees for their very detailed and substantive reports that often were extremely instrumental in shaping papers in a way to make them better comprehensible for scientists from another discipline. We also would like to thank the authors for participating in this project.

Doyne Farmer
Thomas Lux

References:

Aoki, M., 1994, New Macroeconomic Modeling Approaches: Hierarchical Dynamics and Mean Field Approximations, *Journal of Economic Dynamics and Control* 18, 805-877.

Aoki, M., 1995, New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibrium and Externalities as Field Effects. Cambridge, University Press.

Brock, W., 1991, Understanding Macroeconomic Time Series Using Complex Systems Theory, *Structural Change and Economic Dynamics* 2, 119-141.

Enke, S., 1951, Equilibrium between Spatially Separated Markets: Solution by Electronic Analogue, *Econometrica* 19, 40-47.

Farmer, J.D., M. Shubik and E. Smith, 2005, Is Economics the Next Physical Science?, *Physics Today* 58, 9, 37-42.

Foley, D., 1994, A Statistical Equilibrium Theory of Markets, *Journal of Economic Theory* 62, 321-345.

Ijiri, Y. and H. A. Simon, 1977, Skew Distributions and the Sizes of Business Firms, Studies in Mathematical Economics. North- Holland, Amsterdam.

Ingrao, B. and G. Israel, 2000, The Invisible Hand: Economic Equilibrium in the History and Science. Boston, MIT-Press.

Kirman, A., 1993, Ants, Rationality and Recruitment, *Quarterly Journal of Economics* 108, 137-156.

Majorana, E., 2005, The Value of Statistical Laws in Physics and Social Sciences, Quantitative Finance 2, 135-140.

Mantegna, K., 1991, Levy Walks and Enhanced Diffusion in Milan Stock Exchange, *Physica A* 179, 232-242.

Mirowski, P., 1989, More Heat Than Light. Economics as Social Physics: Physics as Nature’s Economics. Cambridge, University Press.

Morehouse, N., R. Strotz and S. Horwitz, 1950, An Electro-Analogue Method for Investigating Problems in Economic Dynamics: Inventory Oscillations, *Econometrica* 18, 313-328.

Phillips, A., 1950, Mechanical Models in Economic Dynamics, Economica 17, 283-305.

Simon, H.A., 1955, On a class of skew distribution functions, Biometrika, 42(3/4):425–440.

Stanley, H. et al., 1996, Can Statistical Physics Contribute to the Science of Economics? *Fractals* 4, 415-425.

Takayasu, H., H. Miara, T. Hirabuyashi and K. Hamada, 1992, Statistical Properties of Deterministic threshold Elements – The Case of Market Prices, *Physica A* 184, 127-134.

Weidlich, W. and G. Haag, 1983, Concepts and Models of Quantitative Sociology, Springer. Berlin.

Weidlich, W., 2002, Socio-Dynamics: A Systematic Approach to Mathematical Modeling in the Social Sciences. Taylor & Francis. London.