An Analytical, Intertemporal, Three-Country Model of Global Imbalances in a Deleveraging World*

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Abstract

We present a simple, microfounded, and intertemporal, model of global imbalances, and extend this to the case of three countries. We follow Blanchard and Milesi Ferretti (2011) in using this model to reveal the effects of a zero bound to world interest rates, in both the two-country and three-country versions of the model. We then discuss the global effects of deleveraging in one region of the world.

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1 Introduction

In this paper we present a microfounded, and intertemporal, model of global imbalances. The model is simple enough to have an analytical solution, even when it is extended to the case of three countries. We are aware of two other papers, Obstfeld and Rogoff (2005) and Cavallo and Tille (2006), that build three-country models, but neither of these models is derived from intertemporal optimisation, and so neither can show the implications for the present of what happens in the future.

We follow Blanchard and Milesi Ferretti (2011) in using this model to reveal the effects on exchange rates and external imbalances of a zero bound to world interest rates, in both the two-country and the three-country versions of the model. We then develop the model by supposing, in the manner of Eggertsson and Krugman (2011), that one region of the world is highly leveraged. In such a setup, we discuss the global outcomes which emerge if financial markets force that region to deleverage by a significant amount.

We begin our analysis with only two countries, and without any analysis of the zero bound or of deleveraging. This basic, preliminary, setup can be explained very simply as follows.

There are two countries, home and foreign, which will sometimes be called US and China. When we add a third country this will sometimes be called Europe. Each of these countries is an endowment economy, which produces only one kind of good. Consumers wish to consume both kinds of goods. Demand for goods comes only from consumers; because this is an endowment economy there are no firms, and there is no investment by firms in any capital stock. This means that the only way in which a country can save for the future is by running a current account surplus.

The model is intertemporal but is, for simplicity, a two-period model: there are two periods of time denoted by $t = 0$ (period zero) and $t = 1$ (period one). Initially we assume that there are no nominal rigidities – this means that we present a real model – and we assume that all markets clear. In each period there is a market for the goods produced in each of the countries; there are thus four goods markets. There is also a market for debt since the first country can run a current account deficit in the first period and increase its debt relative to the second country (and vice versa). There are thus five markets in total. As a result we seek to determine four (relative) prices; the terms of trade between the goods of the countries in each of the two periods and the real interest rate in each of the two countries. We will denote the terms of trade in the two periods by S_0 and S_1 , and we will let a rise in S denote a worsening of the terms of trade of the foreign country. We denote the real interest rate of the home and foreign countries by R and R^*, respectively.

In order to use this model to think about current global circumstances we will examine what happens if one of the countries, the foreign one – which we think of as China – temporarily increases the amount which it produces relative to the amount which its consumers wish to consume. This is equivalent to examining what happens if this country saves more. Our intuition is that in these circumstances, the terms of trade for the foreign country must worsen – *i.e.* its real exchange rate must depreciate in order to adequately increase the world's demand for its goods, and the real interest rate in both countries must fall, in order to ensure that the demand for the home country's goods remains adequate, even although the change in the terms of trade means that this country's good have become relatively more expensive. This is the outcome is depicted by Blanchard and Milesi Ferretti (2011), and we confirm their findings.

Our intertemporal model enables us to keep track of what happens in these circumstances to the current account surplus of the foreign country (*i.e.* China): we will show that it improves. We will also show what happens to the terms of trade between the countries in the future: we will show that the terms of trade of the foreign country improve relative to what happens in the first period; there are circumstances in which the terms of trade improve relative to the initial position, but there are also circumstances in which the terms of trade merely return to their initial position. In addition we will also be able to make clear the way in which the real interest rates facing the two countries come to differ as a result of this shock: the terms of trade of the foreign country worsen in period 0 but then strengthen between period 0 and period 1, and this causes the real interest rates facing the two countries to diverge.

Having set this model up carefully in this manner, we are then able to explore what happens when the interest rate cannot fall in the way supposed above.[1] Obviously this constraint on the solution of the model means that the market for at least one of the goods in the world cannot clear; we depict the outcome as one in which unemployed resources emerge in the home economy (*i.e.* in the US). We are able to show the way in which demand in the US falls through something rather like a Keynesian multiplier process.

It is straightforward to generalise this zero-bound setup to one with three countries, so that our analysis displays outcomes for all of the US, China and Europe. We show what insights can be obtained from doing so. In particular, we are able to describe what determines the distribution of unemployed resources as between the US and Europe. Rather obviously, this outcome depends on the outcome for the terms of trade between the US and Europe. (*i.e.* the outcome for the real Eurodollar exchange rate.)

The final part of this section introduces a leverage constraint into the model. We do this by taking the analysis of Eggertsson and Krugman of 'balance-sheet macroeconomics' (Eggertsson and Krugman (2010)) and applying it to the global economy. In their paper, there are some leveraged consumers, who borrow as much as markets will allow them to borrow and so are effectively income-constrained, whereas other consumers have normal access to capital markets. In our treatment consumers in the US are subject to a leverage constraint of this kind, whereas consumers in the other country, or countries, have normal

[1] We follow Blanchard and Milesi Ferretti (2011) in assuming that the model is a real model. Because of this simplification, all we mean by 'a zero bound' is that the real interest rate cannot fall.

access to international capital markets. In such a world, we explore the way in which this leverage constraint modifies the effects on the world economy of a positive productivity shock in China. We show that the real interest rate must fall further in such a world when there is a positive shock to production in China. And we show that, if there is a zero bound, the leverage constraint increases the magnitude of the global responses. In particular, it increases the size of the shortfall in demand which emerges. That is, the size of the Keynesian multiplier becomes larger.

We then study what happens if US consumers are leveraged, but capital markets force them to deleverage by a significant amount. We explore the general equilibrium consequences of such deleveraging for terms of trade and for the real interest rate. We can show what happens in these circumstances when there is a zero bound to interest rates. Again, it is the case that unemployed resources emerge in what is effectively a Keynesian multiplier process. It is straightforward to generalise this deleveraging setup to a world with three countries, and we do this. We are able to describe what determines the distribution of unemployed resources as between US and Europe.

The paper is set out as follows. In Section 2 we set out the two-country model, solve for the equilibrium outcome, and present the diagram of Blanchard and Milesi Ferretti (2011) which depicts this outcome. We describe comparative static results in response to the shock to Chinese production described above, and depict the outcome diagrammatically. We also explore what happens to these comparative static results in the presence of a zero bound. In Section 3 we set out the three-country model and analyse analytically the equilibrium outcomes when the zero bound does not bind and when it does. Section 4 introduces a leverage constraint into the model. Section 5 studies the effects of deleveraging in such a world, in both a two-country case and a three-country case. Section 6 discusses what we would need to discuss international policy coordination, and concludes that our model is not yet well enough developed for this to be possible. Section 7 concludes.

2 A Two-country Model Without a Leverage Constraint

In what follows, we set out the model in the body of the paper. Details of the deviations are found in the appendices.

2.1 The allocation of Consumption between Home and Foreign Goods

Consumers consume both home and foreign goods. We use a CES aggregator for simplicity.

$$
\begin{aligned}
C_t &= \left(\alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} && (1)\\
C_t^* &= \left((1-\alpha)^{\frac{1}{\eta}} \left(C_{Ht}^*\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{Ft}^*\right)^{\eta-1}\eta\right)^{\frac{\eta}{\eta-1}} && (2)
\end{aligned}
$$

Here, C_{Ht} and C_{Ft} denote the consumption of home and foreign goods by home consumers, and C_{Ht}^* and C_{Ft}^* denote the consumption of home and foreign goods by foreign consumers. Home and foreign consumers both have home bias in their consumption, which is captured by the parameter α, where $\alpha \geq 0.5$. When $\alpha = 0.5$, there is no home bias in consumption. We assume that the elasticity of substitution between home and foreign goods is larger than unity, that is $\eta > 1$.

We denote P_{Ht} and P_{Ft} the price of home and foreign goods in the home country; and similarly, we denote P_{Ht}^* and P_{Ft}^* the price of home and foreign goods in the foreign country. The home and foreign CPI are

$$
\begin{aligned}
P_t &= \left(\alpha P_{Ht}^{1-\eta} + (1-\alpha) P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} && (3)\\
P_t^* &= \left((1-\alpha)\left(P_{Ht}^*\right)^{1-\eta} + \alpha\left(P_{Ft}^*\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} && (4)
\end{aligned}
$$

We assume that the law of one price holds, *i.e.* after conversion at the ruling nominal exchange rate, e_t, each good sells at the same price in each country:

$$
e_t = \frac{P_{Ht}^*}{P_{Ht}} = \frac{P_{Ft}^*}{P_{Ft}} \tag{5}
$$

A rise in e_t is a depreciation of the foreign currency.

We let S_t denote the terms of trade (from the foreign country's perspective),

$$
S_t \equiv \frac{P_{Ht}^*}{P_{Ft}^*} = \frac{P_{Ht}}{P_{Ft}} \tag{6}
$$

We let Y_t and Y_t^* denote the exogenous endowments in the home and foreign country in period t respectively. As a result, the goods market clearing conditions are

$$
\begin{aligned}
Y_0 &= C_{H0} + C_{H0}^*, \qquad Y_1 = C_{H1} + C_{H1}^* && (7)\\
Y_0^* &= C_{F0} + C_{F0}^*, \qquad Y_1^* = C_{F1} + C_{F1}^* && (8)
\end{aligned}
$$

2.2 Intertemporal Choice

There is a real debt market between the two countries. Consumers can borrow from the other country in period 0 and repay, with interest, in period 1. The budget constraint for the home country in period 0 is

$$P_0 D = P_0 C_0 - P_{H0} Y_0 \tag{9}$$

where D denotes real debt in the home country.

In period 1, the budget constraint for the home country is

$$0 = P_1 C_1 - P_{H1} Y_1 + P_1 D R \tag{10}$$

where R is the gross real interest rate in the home country. We combine these two equations to obtain the intertemporal budget constraint. It can be expressed in terms of the terms of trade as follows:

$$C_1 = \left(\alpha + (1-\alpha) S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} R Y_0 + \left(\alpha + (1-\alpha) S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1 - R C_0 \tag{11}$$

For simplicity we assume that home consumers have perfect foresight and a logarithmic utility function of the following form:[2]

$$U_0 = \ln C_0 + \beta \ln C_1 \tag{12}$$

Taking their endowment as given, home consumers maximise utility subject to their budget constraint. This gives the consumption Euler equation for home consumers:

$$C_1 = \beta R C_0 \tag{13}$$

We can substitute the Euler equation into the budget constraint to solve explicitly consumption demands in the two periods.

The intertemporal choice problem for foreign consumers is similar to the problem for home consumers. The budget constraint for the foreign country in period 0 is

$$P_0^* D^* = P_0^* C_0^* - P_{F0}^* Y_0^* \tag{14}$$

where D^* is the real debt by foreign consumers. The budget constraint for the foreign country in period 1 is

$$0 = P_1^* C_1^* - P_{F1}^* Y_1^* + R^* P_1^* D^* \tag{15}$$

[2]This means that the intertemporal elasticity of substitution is constrained to have a value of unity. This assumption could be relaxed by, for example, using a CES utility function. That generalisation would influence the nature of outcomes. In particular, the larger the elasticity of intertemporal substitution, the less the interest rate would need to move after a shock.

where R^* is the gross real interest rate in the foreign economy.

Foreign consumers have the utility of the following form:

$$U_0^* = \ln C_0^* + \beta \ln C_1^* \tag{16}$$

Taking the endowments as given, foreign consumers maximise utility subject to the budget constraint. This gives the consumption Euler equation for foreign consumers:

$$C_1^* = \beta R^* C_0^* \tag{17}$$

Again, by substituting the consumption Euler equation into the budget constraint, we can solve explicitly for consumption demands in the two periods.

The debt market must clear in both periods. This implies the uncovered interest parity (UIP) holds. The uncovered interest parity can be expressed in terms of the terms of trade as

$$R^* = R \frac{\left(\alpha + (1-\alpha) S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left((1-\alpha) + \alpha S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}} \times \frac{\left((1-\alpha) + \alpha S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + (1-\alpha) S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}} \tag{18}$$

When $\alpha = 0.5$, home and foreign consumers have identical preferences, the real exchange rate is always equal to unity, so home and foreign interest rates are always equal.

2.3 The Full System

In equilibrium the home and foreign goods markets in both periods must clear. The debt market also clears, which implies that uncovered interest parity holds. The full system is derived in detail in Appendix A. By Walras' Law, one of the five market clearing conditions is redundant. We have four equations to solve for the four relative prices (R, R^*, S_0, S_1).

In general, this system is complex and the analytical solution for the relative prices in levels cannot be computed. However, when there is no home bias, *i.e.* $\alpha = 0.5$, the solution is simple since changes in the relative income level of the two countries are of the kind which will not redistribute demand for the two goods. The solution is

$$R = R^* = \frac{1}{\beta} \times \left(\frac{Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}}}{Y_0^{\frac{\eta-1}{\eta}} + (Y_0^*)^{\frac{\eta-1}{\eta}}} \right)^{\frac{\eta}{\eta-1}}, \qquad S_0 = \left(\frac{Y_0^*}{Y_0}\right)^{\frac{1}{\eta}}, \qquad S_1 = \left(\frac{Y_1^*}{Y_1}\right)^{\frac{1}{\eta}}. \tag{19}$$

Obviously, when $Y_0 = Y_0^* = Y_1 = Y_1^* = 1, R = R^* = 1/\beta$, and the terms of trade are unity. We call this the symmetric steady state.

For $\alpha > 0.5$, we log-linearise the system around this symmetric steady state and compute the first-order approximated solution for this system. (A 'hat' denotes the log-deviation around the symmetric steady state.)

$$\hat{R} = -\frac{(1-\alpha)(1+2\alpha(\eta-1))}{1+4\alpha(1-\alpha)(\eta-1)}\hat{Y}_0^* \tag{20}$$

$$\hat{R}^* = -\frac{\alpha(1+2(1-\alpha)(\eta-1))}{1+4\alpha(1-\alpha)(\eta-1)}\hat{Y}_0^* \tag{21}$$

$$\hat{S}_0 = \frac{1+\beta(2\alpha(\eta-1)+1)+4\alpha(1-\alpha)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+4\alpha(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{22}$$

$$\hat{S}_1 = -\frac{2\alpha(2\alpha-1)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+4\alpha(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{23}$$

When foreign endowment in period 0 rises, its terms of trade in the same period worsen so that foreign goods become more attractive to consumers. Moreover, foreign interest rate falls, which makes foreign consumers spend more in period 0. In period 1, the terms of trade in the foreign country strengthen. This substitution effect partly offsets the positive income effect due to the increase in endowment. Home interest rate falls to satisfy the UIP.

The elasticity of substitution between home and foreign goods, η, will affect the movements of the relative prices. For a higher η, the terms of trade move by less. The foreign interest rate falls by less because a bigger fraction of the rise in the demand for foreign goods in period 0 comes from intratemporal substitution. The home interest rate falls by more to keep resources fully utilised in the home country.

We can compute how debt varies when foreign endowment, Y_0^*, is shocked.

$$D = \frac{2\beta\alpha(1-\alpha)(\eta-1)}{(1+\beta)(1+4\alpha(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{24}$$

When foreign endowment rises, if the elasticity of substitution between home and foreign goods, η, is greater than unity, the home country runs a current account deficit and accumulate debt.[3]

2.4 A Diagrammatic Representation

Blanchard and Milesi Ferretti (2011) depict the outcome of their model in a revealing two-dimensional diagram. We can see from the above equations that the demand for the home good depends positively on the terms of trade and negatively on the interest rate. Similarly, the demand for the foreign good in period 0 depends negatively on the terms of trade and negatively on the interest rate. They use these two relationships to describe

[3]If $\eta = 1$, that is the consumption aggregator is Cobb-Douglas, debt is zero and invariant to shocks. This is because when foreign endowment rises, the relative price for foreign goods falls by the same proportion, leaving the income of the consumers unchanged. This is the famous Cole and Obstfeld (1991) result.

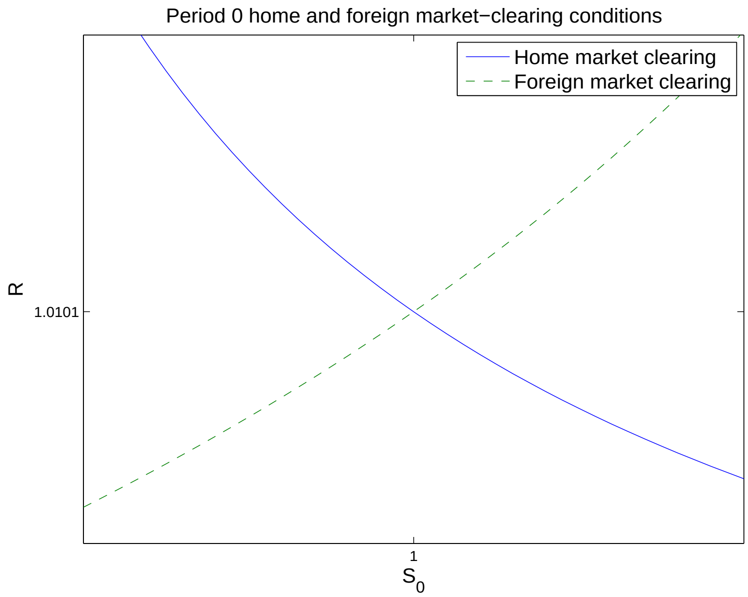

Figure 1: Period 0 goods market equilibrium in two-country model

equilibrium in what is in effect period 0 in our model using a two-dimensional diagram, with the interest rate on the vertical axis and the terms of trade on the horizontal axis. We can do something similar for the model without home bias, *i.e.* when $\alpha = 0.5$, but only after substituting out for the terms of trade in period 1. After doing the substitution we can describe the market clearing conditions for the home market and the foreign market in the following two equations.

$$\text{Home market} \quad : \quad R = \frac{\left(Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \left(1 + S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}}{\left((1+\beta) - \left(1 + S_0^{-(1-\eta)}\right)^{-1}\right) Y_0 - \left(1 + S_0^{-(1-\eta)}\right)^{-1} S_0^{-1} Y_0^*} \tag{25}$$

$$\text{Foreign market} \quad : \quad R = \frac{\left(Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \left(1 + S_0^{1-\eta}\right)^{\frac{\eta}{1-\eta}}}{\left((1+\beta) - \left(1 + S_0^{1-\eta}\right)^{-1}\right) Y_0^* - \left(1 + S_0^{1-\eta}\right)^{-1} S_0 Y_0} \tag{26}$$

We can depict these equations in Figure 1. We assume the discount factor, β, is 0.99 per quarter, and the elasticity of substitution between home and foreign goods, η, is 2. The market clearing condition for the home country is an upward sloping line and that for the foreign country is downward sloping line. Where these two lines cross depicts global equilibrium.

2.5 Comparative Statics

We consider a shock in Y_0^* around the symmetric steady state. The effects of the shock are described in the Equations (20) to (23), and the special case in which $\alpha = 0.5$ is depicted in Equations (19).

These results can be understood as follows. When the endowment in the foreign country rises in period 0, the terms of trade strengthen in period 0 (S_0 rises) so that foreign goods become relatively cheaper, causing home and foreign consumers to consume a larger proportion of foreign goods. The interest rates fall so that consumers in both countries are willing to bring their consumption forward to the present period. As Equations (19) show, the terms of trade will remain unchanged in period 1 in the special case in which $\alpha = 0.5$. Figure 2 fully represents what happens in the system when $\alpha = 0.5$, because in this case S_1 does not change. We can depict the increase in the supply of the foreign good as a rightward shift of the locus showing equilibrium in the foreign market in period 0. However this shift will be offset by an income effect, and the locus showing equilibrium in the home market in period 0 will shift to the right because of an income effect. Because of this increase in the supply of the foreign good, the foreign currency must depreciate. But as a result, to keep resources fully employed in the rest of the world the global interest rate must fall, as shown in the figure.

When $\alpha > 0.5$, the change in S_1 will shift the two curves meaning that the diagram does not fully depict what happens. However, by substituting the solution for S_1 into the log-linearised foreign market clearing condition in period 0, one can show that although a rise in foreign endowment leads to a fall in S_1, which, *ceteris paribus*, reduces country B consumers' income, this effect only partly offsets the expansionary effect of a rise in endowment. Consequently, the results depicted in Figure 2 remains robust for $\alpha > 0.5$.

2.6 A Policy Process

We now move away from the flex-price assumption. Instead we assume, following Blanchard and Milesi Ferretti, that policy is used to maintain the demand in both countries in both periods. To give the setup contemporary relevance we assume that the home country is the US and the foreign country is China. The authorities in China control the terms of trade between the two countries in the two periods, and the authorities in the US control the interest rate linking the periods.[4] The objectives of the policy-makers are that output in the US, Y, is equal to its full employment level, and that the level of output in China, Y^* is equal to its full-employment level. In order for the objectives of policy to be achieved, the home terms of trade strengthen in the period 0 (*i.e.* the renminbi depreciates) and the interest rate falls. It will be obvious from previous discussion

[4] It is a peculiarity of our two period model that there are only three policy instruments. China controls the terms of trade between the two countries in each period, but the US only manipulates the interest rate. This $n-1$ feature would disappear in the limit in an infinite-horizon model.

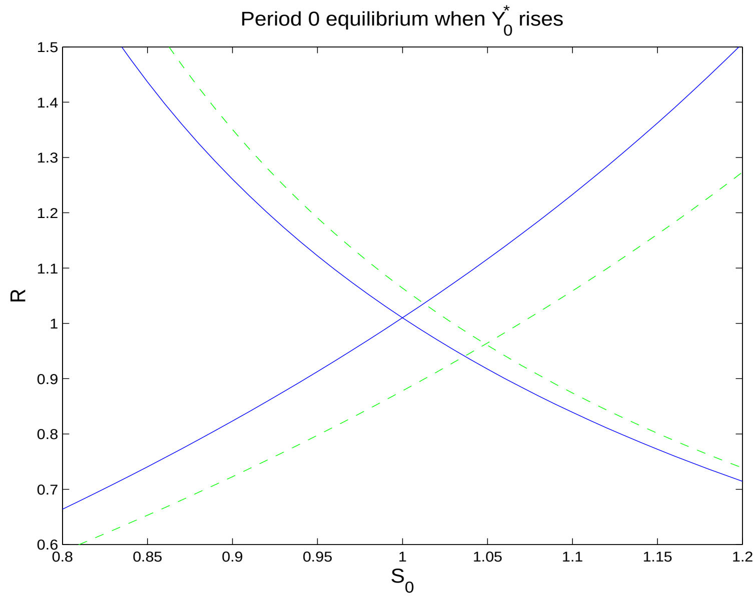

Figure 2: A figure showing the original period 0 equilibrium when supply of foreign goods rises by 10%. (Downward-sloping line = home market clearing condition; upward-sloping line = foreign market clearing condition; solid line = old equilibrium; dashed line = new equilibrium.)

that, when $\alpha = 0.5$, the home terms of trade will remain at their base-run level in period 1, but when $\alpha > 0.5$, the home terms of trade must become weaker than in the initial equilibrium in period 1.

2.7 A Zero Bound in the Two-country Model Without a Leverage Constraint

Now, suppose instead a zero bound is binding in the home economy so that the real interest rate is not allowed to move, *i.e.* $\hat{R} = 0$. This means that we have fixed one of the relative prices in the system exogenously. In order to solve this system, we have to have an additional degree of freedom. We choose this to be the level of demand in period 0, $\hat{Y}_0$.

The foreign country controls the terms of trade to ensure that the level of demand equals the unchanged supply in both periods 0 and 1. We further assume that the demand for home goods in period 1 is equal to the supply, *i.e.* $\hat{Y}_1 = 0$, due to price flexibility. Then, as we will show, some country A goods in period 0 are not demanded. We assume that these goods are disposed costlessly.

From the two foreign goods market clearing conditions, we can solve for the percentage changes in the terms of trade necessary to clear the foreign goods markets.[5] We can then substitute the terms of trade into the home market clearing condition in period 0 to compute the demand for home goods in that period:

$$\hat{Y}_0 \quad = \quad \frac{1}{1+\beta}\hat{Y}_0 - \frac{\beta}{1+\beta} \times \frac{(1-\alpha)}{\alpha}\frac{(1+2\alpha(\eta-1))}{(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{27}$$

where the first term on the right hand side reflects the fact that the income from the sales of goods forces its demand to be less than the supply. The 'marginal propensity to consume', where by this phrase we mean the marginal addition to the demand for home goods created by a marginal increase in home income, is $1/(1+\beta)$. This is smaller than unity because consumers spend in period 1 a fraction, $\beta/(1+\beta)$ of their marginal increase in income in period 0 due to consumption smoothing. Therefore, there is a solution for the home demand in period 0, $\hat{Y}_0$. The solution is:

$$\hat{Y}_0 = -\frac{(1-\alpha)}{\alpha}\frac{(1+2\alpha(\eta-1))}{(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{28}$$

The home demand is negative, but it falls by less than the rise in foreign demand.

We can also compute the terms of trade and the foreign interest rate:

$$\hat{S}_0 \quad = \quad \frac{1+\beta(1+2\alpha(\eta-1))+4\alpha(1-\alpha)(\eta-1)}{\alpha(1+\beta)(1+2\alpha(\eta-1))(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{29}$$

$$\hat{S}_1 \quad = \quad -\frac{2(2\alpha-1)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{30}$$

$$\hat{R}^* \quad = \quad -\frac{2\alpha-1}{\alpha(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{31}$$

In a zero bound environment, when the foreign endowment rises, the foreign terms of trade unambiguously worsen in period 0 and strengthen in period 1. The foreign interest rate falls, consistent with the UIP, in order to shift foreign consumption to the present period.

In response to a rise in foreign endowment in period 0, the terms of trade in the foreign country in that period worsen to induce demand towards foreign goods. However, this leads to insufficient demand in the home country. Since the home interest rate cannot fall to redistribute consumption towards period 0, the income of home consumers will fall. But this means that home country's demand for foreign goods will fall, and the foreign country will worsen its terms of trade by more, further reducing the demand for home goods and the income of home consumers. This downward spiral only stops when the demand for home goods line (Equation (27)) intersects the 45-degree line.

Because of the downward spiral in period 0 describe above, the foreign terms of trade in period 0 have to worsen by more than the previous case in which the home interest

[5] See Appendix A for a detailed derivation of the results in this section.

rate is not constrained. We show this by comparing the coefficients in Equation (22) and Equation (29):

$$\begin{aligned}
&\left.\frac{d\hat{S}_0}{d\hat{Y}_0^*}\right|_{\text{R constrained}} - \left.\frac{d\hat{S}_0}{d\hat{Y}_0^*}\right|_{\text{R unconstrained}} \\
= \;& \frac{(1-\alpha)[1+\beta(1+2\alpha(\eta-1))+4\alpha(1-\alpha)(\eta-1)]}{\alpha(1+\beta)(1+2(1-\alpha)(\eta-1))(1+4\alpha(1-\alpha)(\eta-1))} > 0
\end{aligned} \tag{32}$$

In addition, debt rises more than in the case in which the home interest rate is not at the zero bound because the foreign terms of trade in period 0 worsen by more and the demand for home goods falls.

$$D = \frac{\beta}{1+\beta}\left(\frac{1+2\alpha(1-\alpha)(\eta-1)}{\alpha(1+2(1-\alpha)(\eta-1))}\right)\hat{Y}_0^* \tag{33}$$

Finally, we can verify that the home goods market in period 1 clears. In fact, we are only verifying the Walras' Law, which requires that the home goods market in period 1 clears when the other three goods markets clear.

We can depict the response of the demand for home goods in period 0 and the terms of trade when the home interest rate is at the zero bound in Figure 3. This figure is similar to the one in Blanchard and Milesi Ferretti (2011).

This figure shows an outcome without home bias, *i.e.* when $\alpha = 0.5$, because we know the solution in levels for this parameterisation. The solution is

$$Y_0 = \left(Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}} - (Y_0^*)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \quad R = R^* = 0, \quad S_0 = \left(\frac{Y_0^*}{Y_0}\right)^{\frac{1}{\eta}}, \quad S_1 = \left(\frac{Y_1^*}{Y_1}\right)^{\frac{1}{\eta}}. \tag{34}$$

That is, S_1 is unchanged after a shock to Y_0^*.

We increase the productivity of the foreign country by 10%, the same as before. The foreign terms of trade worsen in order to sell the additional production. This means that home resources are not fully utilised.

When there is home bias, *i.e.* $\alpha > 0.5$, Equation (30) shows that S_1 is negative. We can substitute the solution for $\hat{S}_1$ into the log-linearised foreign market clearing condition. It will be the case that the worsening of the terms of trade in the foreign country in period 1 only partly offsets the rightward shift of the foreign market clearing condition. This means that Figure 3 remains a valid representation of what happens when there is home bias.

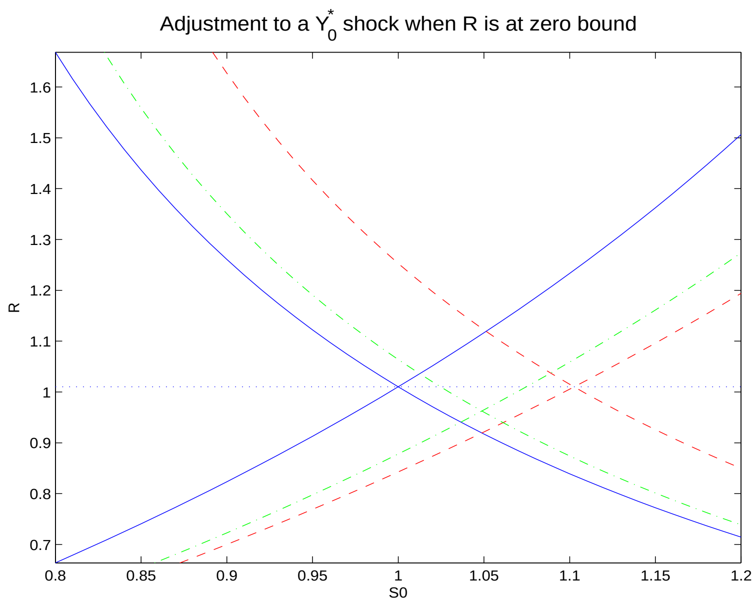

Figure 3: A figure showing the original period 0 equilibrium when the zero bound binds at home and when supply of foreign goods rises by 10%. (Downward-sloping line = home market clearing condition; upward-sloping line = foreign market clearing condition; solid line = old equilibrium; dashed line = new equilibrium when zero bound binds; dashed-dotted line = new equilibrium, no zero bound.)

3 A Three-country Model Without a Leverage Constraint

In what follows, we set up the three-country model and examine the response of the model to a Chinese productivity shock. Details of the derivations are provided in Appendix B.

3.1 Intratemporal allocation of Goods

Suppose there are three countries labeled $J = \{A, B, C\}$, which we will sometimes denote as the US, China and Europe. Denote C_t^J the consumption aggregate in country J in period t. For simplicity, we use a CES aggregator with home bias in consumption.

$$C_t^A = \left(\alpha^{\frac{1}{\eta}}(C_{At}^A)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Bt}^A)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Ct}^A)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{35}$$

$$C_t^B = \left((\alpha^*)^{\frac{1}{\eta}}(C_{At}^B)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{Bt}^B)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Ct}^B)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{36}$$

$$C_t^C = \left((\alpha^*)^{\frac{1}{\eta}}(C_{At}^C)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Bt}^C)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{Ct}^C)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{37}$$

Throughout the model, the superscripts of the consumption indexes denote the country of the consumer and the subscripts denote the country of the endowment. For instance, C_B^A denotes consumption of good B by the consumers of country A. $t = \{0, 1\}$ is the time subscript. α is the consumption home bias, and $\alpha \geq 1/3$. Consumption of foreign goods are spread equally between the two foreign countries for simplicity and

$$\alpha^* \equiv \frac{1-\alpha}{2}. \tag{38}$$

We denote P_{It}^J as country J's price of country I's good in period t and P_t^J is the aggregate price level in country J, defined by

$$P_t^A = \left(\alpha(P_{At}^A)^{1-\eta} + \alpha^*(P_{Bt}^A)^{1-\eta} + \alpha^*(P_{Ct}^A)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{39}$$

$$P_t^B = \left(\alpha^*(P_{At}^B)^{1-\eta} + \alpha(P_{Bt}^B)^{1-\eta} + \alpha^*(P_{Ct}^B)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{40}$$

$$P_t^C = \left(\alpha^*(P_{At}^C)^{1-\eta} + \alpha^*(P_{Bt}^C)^{1-\eta} + \alpha(P_{Ct}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{41}$$

We assume that the law of one price holds. We define three nominal exchange rates e_{BA}, e_{CA} and e_{CB} as follows:

$$e_{BAt} = \frac{P_{Jt}^B}{P_{Jt}^A}, \qquad \text{where } J = \{A, B, C\} \tag{42}$$

$$e_{CAt} = \frac{P_{Jt}^C}{P_{Jt}^A}, \qquad \text{where } J = \{A, B, C\} \tag{43}$$

$$e_{CBt} = \frac{P_{Jt}^C}{P_{Jt}^B}, \qquad \text{where } J = \{A, B, C\} \tag{44}$$

Clearly,

$$e_{CAt} = e_{BAt} \times e_{CBt} \tag{45}$$

We also define the terms of trade as follows:

$$S^J_{It} = \frac{P^J_{It}}{P^J_{Jt}} \tag{46}$$

Then,

$$S^J_{It} = \frac{P^J_{It}}{P^J_{Jt}} = \frac{P^I_{It}}{e_{IJt}} \times \frac{e_{IJt}}{P^I_{Jt}} = \frac{P^I_{It}}{P^I_{Jt}} = \frac{1}{S^I_{Jt}}, \qquad \text{for } I, J = \{A, B, C\} \tag{47}$$

Also,

$$S^C_{Bt} = \frac{P^C_{Bt}}{P^C_{Ct}} = \frac{P^A_{Bt} e_{CAt}}{P^A_{Ct} e_{CAt}} = \frac{P^A_{Bt}}{P^A_{Ct}} = \frac{P^A_{Bt}}{P^A_{At}} \times \frac{P^A_{At}}{P^A_{Ct}} = S^A_{Bt} \times \frac{1}{S^A_{Ct}} = S^C_{At} \times \frac{1}{S^B_{At}} \tag{48}$$

We let Y^A_t, Y^B_t and Y^C_t denote the exogenous endowments in the three countries in period t. As a result, the goods market clearing conditions are:

$$Y^A_0 = C^A_{A0} + C^B_{A0} + C^C_{A0}, \qquad Y^A_1 = C^A_{A1} + C^B_{A1} + C^C_{A1} \tag{49}$$
$$Y^B_0 = C^A_{B0} + C^B_{B0} + C^C_{B0}, \qquad Y^B_1 = C^A_{B1} + C^B_{B1} + C^C_{B1} \tag{50}$$
$$Y^C_0 = C^A_{C0} + C^B_{C0} + C^C_{C0}, \qquad Y^C_1 = C^A_{C1} + C^B_{C1} + C^C_{C1} \tag{51}$$

3.2 Intertemporal Choice

Similar to the case of the two-country model, we assume that there is a real debt market between the three countries. Consumers can borrow from the rest of the world in period 0 and repay, with interest, in period 1. We consider the intertemporal choice in country J, where $J = \{A, B, C\}$.

Country J's consumers sell their endowment, borrow from or lend to the rest of the world and consume in period 0, so the budget constraint in period 0 is the following:

$$P^J_0 D^J = P^J_0 C^J_0 - P^J_{J0} Y^J_0 \tag{52}$$

where D^J denotes real debt in country J.

In period 1, the budget constraint for country J is

$$0 = P^J_1 C^J_1 - P^J_{J1} Y^J_1 - P^J_1 D^J R^J \tag{53}$$

where R^J is the gross real interest rate in country J. We can combine the two budget constraints the obtain the intertemporal budget constraint for country J as follows:

$$C^J_1 \;\; = \;\; \frac{P^J_{J0}}{P^J_0} R^J Y^J_0 + \frac{P^J_{J1}}{P^J_1} Y^J_0 - R^J C^J_0 \tag{54}$$

We assume that country J consumers have perfect foresight and a logarithmic utility function of the following form:

$$U_0^J = \ln C_0^J + \beta \ln C_1^J \tag{55}$$

Taking their endowment as given, country J consumers maximise utility subject to their budget constraint. This gives the consumption Euler equation for country J consumers:

$$C_1^J = \beta R^J C_0^J \tag{56}$$

We can substitute the Euler equation into the budget constraint to solve explicitly consumption demands in the two periods.

3.3 Uncovered Interest Parity

The debt market must clear in both periods. This implies the uncovered interest parity holds between any pairs of the three countries. The uncovered interest parities can also be expressed in terms of the terms of trade as follows:

$$\begin{aligned}
\frac{R^B}{R^A} &= \frac{S_{A1}^B}{S_{A0}^B} \frac{\left(\alpha + \alpha^*(S_{A0}^B)^{1-\eta} + \alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)})^{\frac{1}{1-\eta}}} \\
&\quad \times \frac{\left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{(\alpha + \alpha^*(S_{A1}^B)^{1-\eta} + \alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)})^{\frac{1}{1-\eta}}} && (57) \\
\frac{R^C}{R^A} &= \frac{S_{A1}^C}{S_{A0}^C} \frac{\left(\alpha + \alpha^*(S_{A0}^C)^{1-\eta} + \alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)})^{\frac{1}{1-\eta}}} \\
&\quad \times \frac{\left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{(\alpha + \alpha^*(S_{A1}^C)^{1-\eta} + \alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta})^{\frac{1}{1-\eta}}} && (58)
\end{aligned}$$

3.4 The Full System

The full system contains the six goods market clearing conditions and two debt market clearing conditions. The debt market clearing conditions are the uncovered interest parities, Equations (57) and (58). The goods market clearing conditions are the resource constraints, Equations (51), with demands equal to optimal consumptions. Appendix B shows the full system with detailed derivations.

By Walras' Law, one of the eight market clearing conditions is redundant. We have seven equations to solve for the seven relative prices $\{R^A, R^B, R^C, S_{A0}^B, S_{A0}^C, S_{A1}^B, S_{A1}^C\}$.

It is easy to see that when there is no home bias, *i.e.* $\alpha = \alpha^* = 1/3$, the solution is

$$\begin{aligned} R^A &= R^B = R^C = \frac{1}{\beta} \times \left(\frac{(Y_1^A)^{\frac{\eta-1}{\eta}} + (Y_1^B)^{\frac{\eta-1}{\eta}} + (Y_1^C)^{\frac{\eta-1}{\eta}}}{(Y_0^A)^{\frac{\eta-1}{\eta}} + (Y_0^B)^{\frac{\eta-1}{\eta}} + (Y_0^C)^{\frac{\eta-1}{\eta}}} \right)^{\frac{\eta}{\eta-1}} \\ S_{A0}^B &= \left(\frac{Y_0^B}{Y_0^A}\right)^{\frac{1}{\eta}},\ S_{A0}^C = \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{1}{\eta}},\ S_{A1}^B = \left(\frac{Y_1^B}{Y_1^A}\right)^{\frac{1}{\eta}},\ S_{A1}^C = \left(\frac{Y_1^C}{Y_1^A}\right)^{\frac{1}{\eta}}. \end{aligned} \tag{59}$$

The effects of an increase in endowment in country B is obvious in this case without home bias. World interest rates fall and the terms of trade in country B deteriorate. Nothing else changes. This is just like it was in the two-country case.

For $\alpha > 1/3$, we log-linearise the system around the symmetric steady state in which all endowments are unity, the terms of trade are unity and the real interest rates are $1/\beta$. We consider a rise in endowment in country B in period 0, Y_0^B.

The solution is

$$\hat{R}^A = R^C = -\frac{\alpha^*(1 + (\eta - 1)(1 + \alpha - \alpha^*))}{1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*)}\hat{Y}_0^B \tag{60}$$

$$\hat{R}^B = -\frac{(1 - 3\alpha^*) + \alpha^*(1 + (\eta - 1)(1 + \alpha - \alpha^*))}{1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*)}\hat{Y}_0^B \tag{61}$$

$$\hat{S}_{A0}^B = \frac{(1 + \beta) + (\beta + 3\alpha^*)(\eta - 1)(1 + \alpha - \alpha^*)}{(1 + \beta)(1 + (\eta - 1)(1 + \alpha - \alpha^*))(1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*))}\hat{Y}_0^B \tag{62}$$

$$\hat{S}_{A1}^B = -\frac{(1 - 3\alpha^*)(\eta - 1)(1 + \alpha - \alpha^*)}{(1 + \beta)(1 + (\eta - 1)(1 + \alpha - \alpha^*))(1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*))}\hat{Y}_0^B \tag{63}$$

$$\hat{S}_{A0}^C = \hat{S}_{A1}^C = 0 \tag{64}$$

And the debts are

$$D^A = D^C = \frac{\beta}{(1 + \beta)}\frac{\alpha^*(\eta - 1)(1 + \alpha - \alpha^*)}{(1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*))}\hat{Y}_0^B \tag{65}$$

$$D^B = -\frac{2\beta}{(1 + \beta)}\frac{\alpha^*(\eta - 1)(1 + \alpha - \alpha^*)}{(1 + 3\alpha^*(\eta - 1)(1 + \alpha - \alpha^*))}\hat{Y}_0^B \tag{66}$$

In this system, country A and C are symmetric, so there is no reason for the terms of trade between these two countries to change in either of the two periods. The interest rates in both countries will fall by the same amount. The movements in other relative prices resemble those in the two-country model.

The solution shows how the elasticity of substitution between home and foreign goods, η, affects these variables. As η rises, smaller moves in the terms of trade are sufficient to shift demands towards country B goods. This means that the interest rate in country B, $\hat{R}^B$, which shifts consumption intertemporally, moves by less. To keep the demand for their goods unchanged, interest rates in country A and C have to fall by more.

The solutions of the three-country model are summarised in Table 1.

As in the two-country model, it remains true that the demand for each country's goods depends positively on the country's terms of trade and negatively on the interest rate. Similarly, the demand for other country's goods depends negatively on the country's terms of trade and negatively on the interest rate. We can thus depict the equilibrium by plotting the market clearing conditions for country A and B in period 0, but only after imposing the conditions that markets in period 1 and markets in country C clear. We do this for the model without home bias, *i.e.* when $\alpha = 1/3$. After doing the substitution, we can describe the market clearing conditions for country A and B's markets in the following two equations:

$$R^A = \frac{\left(\left(Y_1^A\right)^{\frac{\eta-1}{\eta}} + \left(Y_1^B\right)^{\frac{\eta-1}{\eta}} + \left(Y_1^C\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \left(1 + \left(S_{A0}^B\right)^{-(1-\eta)} + \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{1-\eta}}}{\left[1 + \beta - \frac{1+\left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}}{1+\left(S_{A0}^B\right)^{-(1-\eta)}+\left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}}\right] Y_0^A - \left(1 + \left(S_{A0}^B\right)^{-(1-\eta)} + \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}\right)^{-1} \left(S_{A0}^B\right)^{-1} Y_0^B} \tag{67}$$

$$R^A = \frac{\left(\left(Y_1^A\right)^{\frac{\eta-1}{\eta}} + \left(Y_1^B\right)^{\frac{\eta-1}{\eta}} + \left(Y_1^C\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \left(1 + \left(S_{A0}^B\right)^{1-\eta} + \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}} \left(S_{A0}^B\right)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}}{\left[1 + \beta - \left(1 + \left(S_{A0}^B\right)^{1-\eta} + \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}} \left(S_{A0}^B\right)^{1-\eta}\right)^{-1}\right] Y_0^B - \left[\frac{1+\left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}}{1+\left(S_{A0}^B\right)^{1-\eta}+\left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{\eta-1}{\eta}}\left(S_{A0}^B\right)^{1-\eta}}\right] S_{A0}^B Y_0^A} \tag{68}$$

Figure 4 depicts the outcome in this $\alpha = 1/3$ case. The market clearing condition for country A is the downward-sloping solid line. The market clearing condition for country B is the upward-sloping solid line. We can draw these lines in this case because S_{A0}^C, S_{A1}^C and S_{A1}^B are unity. Where these lines intersect depicts the global equilibrium. After a positive endowment shock in country B, the terms of trade worsen in country B in period 1 (S_{A0}^B rises) so that its goods become relative cheaper, causing consumers to consume a larger proportion of its goods. The interest rates fall so that consumers are willing to bring their consumption forward to the present period. This figure shows the interest rate and the terms of trade between the US and China. The symmetry of the setup means that we can draw an identical diagram between Europe and China.

We can consider the case when $\alpha > 1/3$. In this case, it remains true that the symmetry ensures that S_{A0}^C and S_{A1}^C are unity. It is no longer the case, however, that S_{A1}^B is unity in period 1. In fact, China's terms of trade worsen in period 0 relative to the baseline and strengthen in period 1 relative to the baseline. Nevertheless, we can substitute out the log-linearised solution for S_{A1}^B in the log-linearised home and foreign market clearing conditions and show that this term only partly offsets the rightward shifts of the market clearing conditions. The figure remains a valid representation of what happens when there

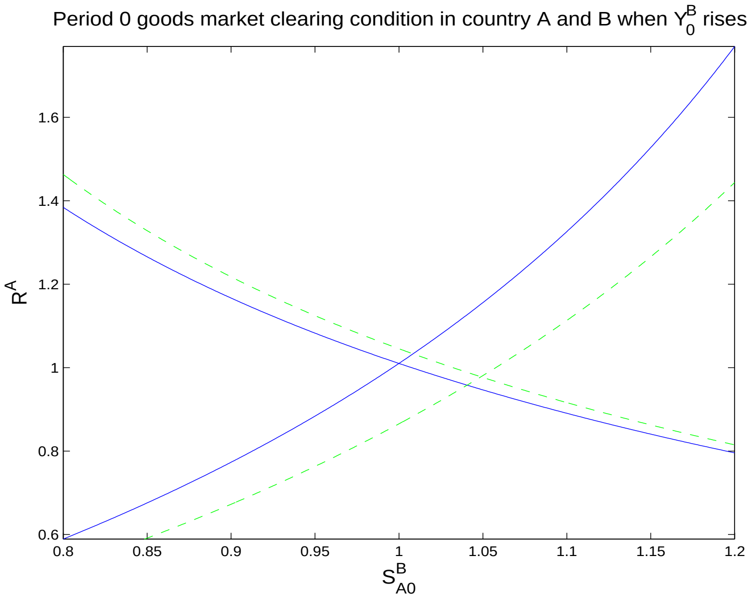

Figure 4: Period 0 goods market equilibrium in country A and B in the three-country model when $\alpha = 1/3$. Dashed lines show the equilibrium when Y_0^B rises by 10%.

is home bias: interest rates fall and the terms of trade worsen in country B.

There is an identical picture for China and Europe. This arises directly out of the symmetry present everywhere in the model.

3.5 A Zero Bound in three-country model

We continue to consider a Chinese productivity shock in this section. But now we suppose that the zero bound is binding in the US.

Now, again suppose that the zero bound is binding in country A (US) so that the real interest rate is not allowed to move, *i.e.* $\hat{R}^A = 0$, and that the shock is not large enough to make the other two interest rates bind. We study what happens when the endowment in country B (China) in period 0 rises. We further assume that China controls the terms of trade against the US, that is $\hat{S}^B_{A0}$ and $\hat{S}^B_{A1}$, to ensure that the goods market in China clears in each of the two periods.

From the two market clearing conditions for country B, we can compute the change in terms of trade which ensures the goods markets in China clear. The expressions for the terms of trades are reported in Appendix B. We discuss the intuitions as follows: when the incomes in country A and C fall, *ceteris paribus*, the demand for country B's goods falls, so the terms of trade in country B have to worsen by more to make its goods more attractive. A worsening of terms of trade in country C against country A reduces the demand for country B goods by country C, for a given bilateral terms of trade between country A and B. Therefore, in response to this the terms of trade in country B have to worsen.

We can substitute the terms of trade into the period 0 market clearing conditions in country A and country C in order to study the extent of underutilisation of resources that is caused by the zero bound. As we will show, the distribution of the shortfall in demand in country A and country C depends on the ability of these countries to control the bilateral terms of trade. We do not have a good theory of who controls these terms of trade. There has been much popular discussions on whether quantitative easing may be used for this purpose. To model that would require a more elaborate model than we would have here.

In the following, we consider three cases. In the first case, we assume that country A and country C share the shortfall in demand equally. In the second case, we assume that country C controls the terms of trade so that the shortfall in demand only occurs in country A. In the third case, we assume that country A controls the terms of trade so that underutilisation of resources only happens in country C.

3.5.1 Case 1: Country A and Country C have the same shortfall in demand

In this subsection, we assume that the demands for country A and country C goods fall by equal proportion such that $\hat{Y}_0^A = \hat{Y}_0^C$. We will also impose the assumption that the demands in period 1 equal the exogenous supplies of goods , *i.e.* $\hat{Y}_1^A = \hat{Y}_1^C = 0$.

We solve for the bilateral terms of trade between country A and C that is necessary to clear the goods markets in period 1 and substitute these results into the market clearing conditions of country A and C. This enables us to compute the common level of demand in the two countries:

$$\hat{Y}_0^A = \hat{Y}_0^C = -\frac{\alpha^*(1+(\eta-1)(1+\alpha-\alpha^*))}{(1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{69}$$

The terms of trade and interest rates are

$$\hat{S}_{A0}^C = \hat{S}_{A1}^C = 0 \tag{70}$$

$$\hat{S}_{A0}^B = \frac{(1+\beta)+(\beta+3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{71}$$

$$\hat{S}_{A1}^B = -\frac{(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{72}$$

$$\hat{R}^A = \hat{R}^C = 0 \tag{73}$$

$$\hat{R}^B = -\frac{(1-3\alpha^*)}{1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{74}$$

Given our assumption that country A and C share the shortfall in demand equally, the two countries are symmetric. The interest rate in country C will not change, and the

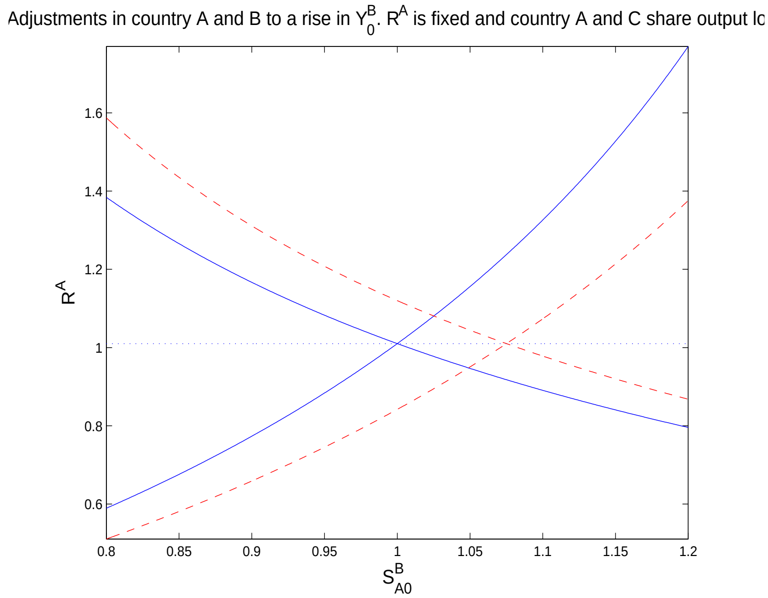

Figure 5: Period 0 goods market equilibrium in country A and B in the three-country model when $\alpha = 1/3$. Dashed lines show the equilibrium when Y_0^B rises by 10%. (Downward-sloping line = country A market clearing condition; upward-sloping line = country B market clearing condition; solid line = old equilibrium; dashed line = new equilibrium when zero bound binds and country A and C share output loss; dashed-dotted line = new equilibrium, no zero bound.)

bilateral terms of trade between these two countries will also remain unchanged.

We can depict the response of the level of demand for country A goods and the terms of trade between country A and B to a 10% positive productivity shock in country B for this case in Figure 5, similar to what we did for the two-country model. The figure shows an outcome when there is no home bias, *i.e.* $\alpha = 1/3$. The solution is

$$R^A = R^B = R^C = 0, \quad Y_0^A = Y_0^C = \left(\frac{(Y_1^A)^{\frac{\eta-1}{\eta}} + (Y_1^B)^{\frac{\eta-1}{\eta}} + (Y_1^C)^{\frac{\eta-1}{\eta}} - (Y_0^B)^{\frac{\eta-1}{\eta}}}{2} \right)^{\frac{\eta}{\eta-1}},$$

$$S_{A0}^B = \left(\frac{Y_0^B}{Y_0^A}\right)^{\frac{1}{\eta}}, \; S_{A0}^C = \left(\frac{Y_0^C}{Y_0^A}\right)^{\frac{1}{\eta}}, \; S_{A1}^B = \left(\frac{Y_1^B}{Y_1^A}\right)^{\frac{1}{\eta}}, \; S_{A1}^C = \left(\frac{Y_1^C}{Y_1^A}\right)^{\frac{1}{\eta}}. \tag{75}$$

This means that S_{A0}^C, S_{A1}^B and S_{A1}^C will not change after the shock. The demands for country A and country C's goods fall (by the same amount, by assumption) because of a deterioration of the terms of trade in country B. The symmetry of this setup means that we can draw an identical diagram between country B and C.

For $\alpha > 1/3$, the symmetry of this setup ensures that S_{A0}^C and S_{A1}^C are unity. Nevertheless, S_{A1}^B will strengthen. By substituting the log-linearised solution for S_{A1}^B into log-linearised market clearing conditions, we can show that the overall effect of a positive productivity shock in country B is a fall in demand for the other countries' goods. The figure remains a valid representation of the system. The symmetry of the model means that there is an identical picture for country B and C.

3.5.2 Case 2: Country C controls bilateral terms of trade

In this subsection, we assume country C controls $\hat{S}^C_{A0}$ and $\hat{S}^C_{A1}$ to keep its resources fully utilised, that is $\hat{Y}^C_0 = \hat{Y}^C_1 = 0$. We further assume that the level of demand for country A goods in period 1 equals to the unchanged supply, *i.e.* $\hat{Y}^A_1 = 0$. From the market clearing conditions of country C, we can compute the terms of trade between these two countries in terms of the income in country A and substitute these terms of trade into the country A market clearing condition in period 0 to obtain:

$$\hat{Y}^A_0 = \frac{1}{1+\beta}\hat{Y}^A_0 - \frac{\beta}{(1+\beta)}\frac{\alpha^*(1+(\eta-1)(1+\alpha-\alpha^*))}{(\alpha+\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{76}$$

where the first term on the right hand side reflects the fact that the income from the sales of goods forces the demand to be less than the exogenous supply. The 'marginal propensity to consume' is $1/(1+\beta)$. The level of demand for country A goods in period 0 in this case is

$$\hat{Y}^A_0 = -\frac{\alpha^*(1+(\eta-1)(1+\alpha-\alpha^*))}{(\alpha+\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{77}$$

The terms of trade and interest rates are

$$\hat{S}^B_{A0} = \frac{((1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))((1+\beta)+(\beta+3\alpha^*)(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))(\alpha+\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{78}$$

$$\hat{S}^B_{A1} = -\frac{(1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))(\alpha+\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{79}$$

$$\hat{S}^C_{A0} = \frac{\alpha^*((1+\beta)+(\beta+3\alpha^*)(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{80}$$

$$\hat{S}^C_{A1} = -\frac{\alpha^*(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{81}$$

$$\hat{R}^B = -\frac{(1-3\alpha^*)(1-\alpha^*+2\alpha^*(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{82}$$

$$\hat{R}^C = -\frac{\alpha^*(1-3\alpha^*)(1+(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}^B_0 \tag{83}$$

As in the previous case, when productivity in country B rises, the terms of trade worsen. *Ceteris paribus*, the demand for country C's (and country A's) goods fall. In this case, however, country C controls the terms of trade vis-a-vis country A, S^C_{A0} and will worsen the terms of trade. This means that the demand for country A's goods will fall further, triggering a downward spiral with a significant fall in demand .

3.5.3 Case 3: Country A controls bilateral terms of trade

In this section, we assume instead country A controls $\hat{S}^C_{A0}$ and $\hat{S}^C_{A1}$ to keep its resources fully utilised, that is $\hat{Y}^A_0 = \hat{Y}^C_0 = 0$. This case is asymmetrical to the previous case because only country A is at the zero bound. As before, we impose the assumption that because of price flexibility, the demand for country C goods in period 1 is equal to the fixed supply.

We can compute from the market clearing conditions of country A the terms of trade between these two countries necessary to keep country A's demand unchanged and then substitute for the terms of trade into country C's market clearing condition in period 0. We obtain:

$$\hat{Y}_0^C = \frac{1}{1+\beta}\hat{Y}_0^C - \frac{\beta}{1+\beta}\hat{Y}_0^B \tag{84}$$

Since this market must clear in equilibrium, this implies

$$\hat{Y}_0^C = -\hat{Y}_0^B \tag{85}$$

That is, a rise of country B endowment in period 0 by one unit leads to an equal fall in demand in country C. The reason is as follows: as the endowment in country B rises, country B's terms of trade against country A worsen. *Ceteris paribus*, the demand in country A falls. As country A is up against the zero bound, there is no intertemporal substitution. To keep the demand at the pre-shock level, country A worsens its terms of trade against country C, by an amount exactly equal to the amount which country B's terms of trade deteriorate against it, in order to transfer the fall in demand to country C. The movements in the terms of trade confirm this:

$$\hat{S}_{A0}^B = \frac{(1+\beta+(3\alpha^*+\beta)(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{86}$$

$$\hat{S}_{A1}^B = -\frac{(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{87}$$

$$\hat{S}_{A0}^C = -\frac{(1+\beta+(3\alpha^*+\beta)(\eta-1)(1+\alpha-\alpha^*))}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{88}$$

$$\hat{S}_{A1}^C = \frac{(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{89}$$

The interest rates are

$$\hat{R}^B = -\frac{(1-3\alpha^*)}{(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{90}$$

$$\hat{R}^C = \frac{(1-3\alpha^*)}{(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{91}$$

The interest rate in country C goes to the wrong way. This interest rate is consistent with the uncovered interest parity – a higher country C interest rate is necessary to compensate for the worsening terms of trade so that the return of holding country C debt equals the fixed return of country A's.

4 Including a Leverage Constraint in the Model

In the following, we move away from models without leverage constraints by assuming that US consumers are impatient and are subject to an exogenous borrowing limit in the initial period. We will study the response to a Chinese productivity shock under this environment for the two-country and three-country models, and for the cases in which a zero bound is binding and is not binding. These responses will be compared with the responses in the unleveraged models.

4.1 Two-country Leveraged Model

Now suppose home consumers are more impatient than foreign consumers. Home consumers are constrained by a debt limit, D, and they borrow up to the limit. Consumptions are computed by the budget constraint:

$$C_0 = \frac{P_{H0}}{P_0}Y_0 + D = \left(\alpha + (1+\alpha)S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0 + D \tag{92}$$

$$C_1 = \frac{P_{H1}}{P_1}Y_1 - RD = \left(\alpha + (1+\alpha)S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1 - RD \tag{93}$$

Foreign consumers do not face any leverage constraint. They choose their demands according to the consumption Euler equation, Equation (17).

This two-country system with a leveraged constraint thus contains five equations, corresponding to the market clearing conditions to the four goods markets and a debt market. We can substitute the home and foreign demands for goods into the resource constraints to derive the goods market clearing conditions. The debt market clearing condition remains unchanged and is given by Equation (18). The full system is presented in Appendix C.

By Walrus law, one of the five market clearing conditions is redundant. Given the exogenous variables including the exogenous debt limit in the home country, D, this system solves for the four relative prices $\{R, R^*, S_0, S_1\}$.

When $\alpha = 0.5$. The solution for this system is

$$R = R^* = \frac{\left(Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \frac{Y_1^{\frac{\eta-1}{\eta}}}{Y_1^{\frac{\eta-1}{\eta}} + (Y_1^*)^{\frac{\eta-1}{\eta}}}}{\beta\left(Y_0^{\frac{\eta-1}{\eta}} + (Y_0^*)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \frac{Y_0^{\frac{\eta-1}{\eta}}}{Y_0^{\frac{\eta-1}{\eta}} + (Y_0^*)^{\frac{\eta-1}{\eta}}} - \left(\frac{1}{2}\right)^{\frac{1}{1-\eta}}(1+\beta)D}, \tag{94}$$

$$S_0 = \left(\frac{Y_0^*}{Y_0}\right)^{\frac{1}{\eta}}, \qquad S_1 = \left(\frac{Y_1^*}{Y_1}\right)^{\frac{1}{\eta}}. \tag{95}$$

There are a few things to note.

First, in the case where $\alpha = 0.5$, the solutions for the terms of trade are identical to the ones in which consumers are not debt-constrained (given in Equation (19)). Without home bias, changes in the terms of trade do not cause income effects to redistribute the relative demands of goods.

Second, when the debt limit D falls exogenously, the real interest rate falls. The intuition is similar to Eggertsson and Krugman (2010). Since the debt-constrained home consumers consume all their borrowings in period 0, a fall in D reduces their consumption. The interest rate R has to fall to induce the foreign consumers, who smooth consumption, to consume more in period 0 in order to clear the goods markets in that period.

Third, the existence of a debt constraint changes the way in which the world respond to the endowment shock studied earlier. Suppose, for simplicity, D is zero, an exogenous rise in foreign endowment in period 0, Y_0^*, leads to a smaller fall in interest rate relative to the efficient solution.[6] This is because the rise in foreign endowment strengthens the home terms of trade in period 0, which allows the debt-constrained home consumers to consume more in that period, relative to the efficient economy in which home consumers smooth consumption. This effect reduces the fall in the interest rates necessary to achieve a sufficient overall increase in consumption in period 0.

When there is home bias, *i.e.* ($\alpha > 0.5$), there is in general no analytical solution to the relative prices.[7] Therefore, we will study this system by calibration and simulation in the next section.

4.1.1 A Zero Bound in the Two-country Leveraged Model

In this section, suppose the home interest rate is at the zero bound, so it cannot fall further. There is no analytical solution for this system in general. Therefore, in the following, we study the special case in which the debt limit in the home country tends to zero and an analytical solution exists.

Suppose the endowments are $Y_0 = Y_0^* = Y_1 = Y_1^* = 1$, then the terms of trade are unity and the foreign interest rate R^* is $1/\beta$ in equilibrium . The home interest rate, implied by the UIP, is equal to the foreign interest rate in equilibrium. Further suppose that the foreign economy controls the terms of trade, S_0 and S_1, to ensure that the foreign goods markets clear and that the demand for home goods in period 1 equals the fixed endowment, *i.e.* $\hat{Y}_1 = 0$, so that any shortfall in demand will occur in the home country

[6] This lower interest rate sensitivity is not general. In fact, when we simulate the system with home bias in the following section, interest rates in the model with a leverage constraint will in general be more volatile. The reason is that there are two effects affecting the interest rate. First, the income-constrained home consumers in the leveraged system spend a bigger fraction of period 0 income on foreign goods than the unconstrained consumers in the unleveraged system. Second, the demand for foreign goods in period 0 in the unleveraged system is more sensitive to interest rate movements because home consumers smooth consumption. When α is large, home consumers spend less on foreign goods, so the first effect is weak. The second effect means that interest rates need to move more in the leveraged system.

[7] The terms of trade S_0, is the solution of $\frac{(\alpha S_0^\eta Y_0 - (1-\alpha)Y_0^*)}{(2\alpha-1)S_0^\eta} = \left(\alpha + (1-\alpha)S_0^{-(1-\eta)}\right)^{-1} Y_0 + \left(\alpha + (1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} D$, and there is no analytical solution in general.

in period 0. We study a positive shock to the foreign endowment in period 0.

From the two foreign goods markets clearing conditions, we can solve for the percentage changes in the terms of trade necessary to clear the foreign goods markets.[8] We then substitute the terms of trade into the home market clearing condition in period 0 to compute the demand for home goods in that period:

$$\begin{aligned}\hat{Y}_0 &= \left(1 - \frac{\alpha(1-\alpha)\beta}{(1+\beta)((1-\alpha)(1+2\alpha(\eta-1))+\alpha^2)}\right)\hat{Y}_0 \\ &\quad - \frac{\beta(1-\alpha)(1-\alpha+2\alpha(\eta-1))}{(1+\beta)((1-\alpha)(1+2\alpha(\eta-1))+\alpha^2)}\hat{Y}_0^* \end{aligned} \tag{96}$$

where the first term on the right hand side reflects the fact that the income from the sales of home goods forces the demand to be less than the exogenous supply. Since $\alpha > 0.5$, the 'marginal propensity to consume', which is the coefficient of $\hat{Y}_0$ on the right hand side of Equation (96), is less than unity. This is because there is leakage to foreign consumers who smooth consumption intertemporally. We can show that the 'marginal propensity to consume' in this case with income-constrained consumers is less than that with consumers who are not income-constrained.

$$\begin{aligned} & \mathrm{MPC}^{\text{income-constrained}} - \mathrm{MPC}^{\text{income unconstrained}} \\ = \; & \frac{\beta(1-\alpha)(1+2\alpha(\eta-1))+\alpha(2\alpha-1)\beta}{(1+\beta)((1-\alpha)(1+2\alpha(\eta-1))+\alpha^2)} > 0 \end{aligned} \tag{97}$$

There are two reasons for this result. First, income-constrained consumers spend a larger proportion of their income on home goods in period 0, so a fall in income reduces the demand for home goods by a larger proportion. Second, a worsening of the foreign terms of trade in each of the two periods reduces the demand for home goods in period 0 per unit fall in the sales of home goods. In contrast, a worsening in the terms of trade in the foreign country in period 1 does not reduce the demand for home goods in period 0 in the model without a leverage constraint. Therefore, as a fall in home income causes the home terms of trade in both periods to strengthen, it leads to a sharper fall in demand for home goods in the leveraged system, causing a more severe downward spiral relative to the unleveraged system.

By equating the supply of and demand for home goods in period 0, we can compute the demand, $\hat{Y}_0$:

$$\hat{Y}_0 = -\left(\frac{1-\alpha}{\alpha} + 2(\eta-1)\right)\hat{Y}_0^* \tag{98}$$

The demand for home goods falls. Furthermore, it falls below the demand in the model

[8] See Appendix C for details of the derivations.

in which home consumers are not debt-constrained.

$$
\begin{aligned}
&\left.\frac{d\hat{Y}_0}{d\hat{Y}_0^*}\right|_{\text{income-constrained}} - \left.\frac{d\hat{Y}_0}{d\hat{Y}_0^*}\right|_{\text{unconstrained}} \\
= \; & -\frac{2(\eta-1)(1-2\alpha(1-\alpha)+2\alpha(1-\alpha)(\eta-1))}{\alpha(1+2(1-\alpha)(\eta-1))} < 0
\end{aligned} \tag{99}
$$

We may compute the Keynesian multiplier. The Keynesian multiplier is the increase in equilibrium demand for home goods when the home demand curve, Equation (96), shifts up, exogenously, by one unit. The Kenyesian multiplier is $1/(1-MPC)$ and is larger than unity because the marginal propensity to consume is positive.

The relative prices in this system with a binding zero bound and debt limit are

$$
\hat{S}_0 = \frac{1}{\alpha}\hat{Y}_0^*, \quad \hat{S}_1 = 0, \quad \hat{R} = 0, \quad \hat{R}^* = -\frac{2\alpha-1}{\alpha}\hat{Y}_0^* \tag{100}
$$

In response to a rise in home endowment in period 0, the terms of trade in the foreign country fall to attract demand for foreign goods in that period. The resulting fall in demand for home goods reduces the income of home consumers. However, home consumers are income-constrained and a fall in income in period 0 does not affect the home demands in period 1. For foreign consumers, the rise in endowment has a positive income effect to the foreign demands in period 1. This income effect is offset by the foreign interest rate alone, so the terms of trade in period 1 does not need to move.

Finally, it can be verified that the demand for home goods in period 1 are unaffected by the shock, which is consistent with our assumption in the beginning of this piece of analysis that the supply of home goods in the same period is unchanged.

4.1.2 Calibration and Simulation

As it is not possible to find an analytical solution in the leveraged model with a positive debt limit in general, we study this system by calibration and simulation.

We take a time period in this model to be ten years. This is because we are interested in the evolution of the debt stock in longer run and are not interested in the business cycle properties of the model. We follow Obstfeld and Rogoff (2005) and Cavallo and Tille (2006) to set the consumption home bias, α, equal to 0.7. We follow these papers to choose the elasticity of substitution between home and foreign goods, η, to be 2. The real GDP of China is about 40% of the US real GDP. The real GDP growth in the US in the past ten years is about 10% and we take the real GDP growth in China to be about 50%. For the real interest rates, we choose the equilibrium real interest rate of 10% in ten years, or 1% per annum, in both the US and China.

We use the equilibrium conditions presented in the previous section to find equilibrium terms of trade S_0, S_1, the debt limit, D, and the discount factor β. The numerical values are given in Table 2.

It turns out that the discount factor, β, is larger than unity. The fact that the size of the economy in China is only a fraction of US's and that China has to lend a lot to the US in the initial period mean that Chinese consumers are very patient. In our view, the preference to future consumption is not unreasonable in China. Precautionary saving motives, liquidity or borrowing constraints, implicit consumption taxes and other institutional distortions contribute to the high household saving rate in China (Ma and Yi, 2010).

We compare this leveraged model with the model without a leverage constraint, *i.e.* the model presented in Section 2. To make a fair comparison, we use the same exogenous parameters as the ones we use for the leveraged model. Furthermore, we allow the US and China to have different discount factors in the model without a leveraged constraint so as to construct the same baseline for the two models.[9] We set the discount factor in China, β, equal to the discount factor in the leveraged model, and calibrate the discount factor in the US, β^{US}, to make the level of debt in the baseline equilibrium in this model equal to the exogenous debt limit in the leveraged model. We get a discount factor in the US of $\beta^{US} = 0.825$, which means an annual discount rate of 2%.

Our strategy is to do comparative statics around the baseline equilibriums. We are interested in a rise in productivity in China. We model this by increasing exogenously the period 0 endowment in the foreign country, or Y_0^*. We study this problem, for both the unleveraged model and the leveraged model, in two different circumstances. In the first set of circumstances, the interest rates and terms of trade can adjust freely to ensure all markets clear. In the second set of circumstances, we assume that the US interest rate is at the zero bound and cannot fall further. In all four cases, we assume that China manipulates the terms of trade such that the goods markets in China clear. We also impose, in all four cases, the requirement that the goods market in the US will clear in future. This means that, in the cases when not all markets clear, we assume that, there is a shortfall in demand for US goods in period 0.

The results for a Chinese endowment shock are shown in Figure 6. In this set of figures, we plot on the horizontal axis the deviation of Chinese endowment from the baseline equilibrium. On the vertical axis, we plot the deviations of relative prices and the deviations of the demand for US goods from their respective baseline equilibrium values.

We see two important results by comparing the simulations of the model without leverage and the model with a leverage constraint. First, interest rates are more sensitive in the model with a leveraged constraint. This is because US consumers cannot smooth consumption intertermporally, and all reallocation of resources across the two periods has to be done by Chinese consumers. Second, when the zero bound binds in the US, the fall in demand for US goods is more severe in the leveraged model, because the Keynesian

[9]Although the parameters in the baseline equilibriums of the two models are identical, the vector of relative prices are not exactly identical. See Table 2.

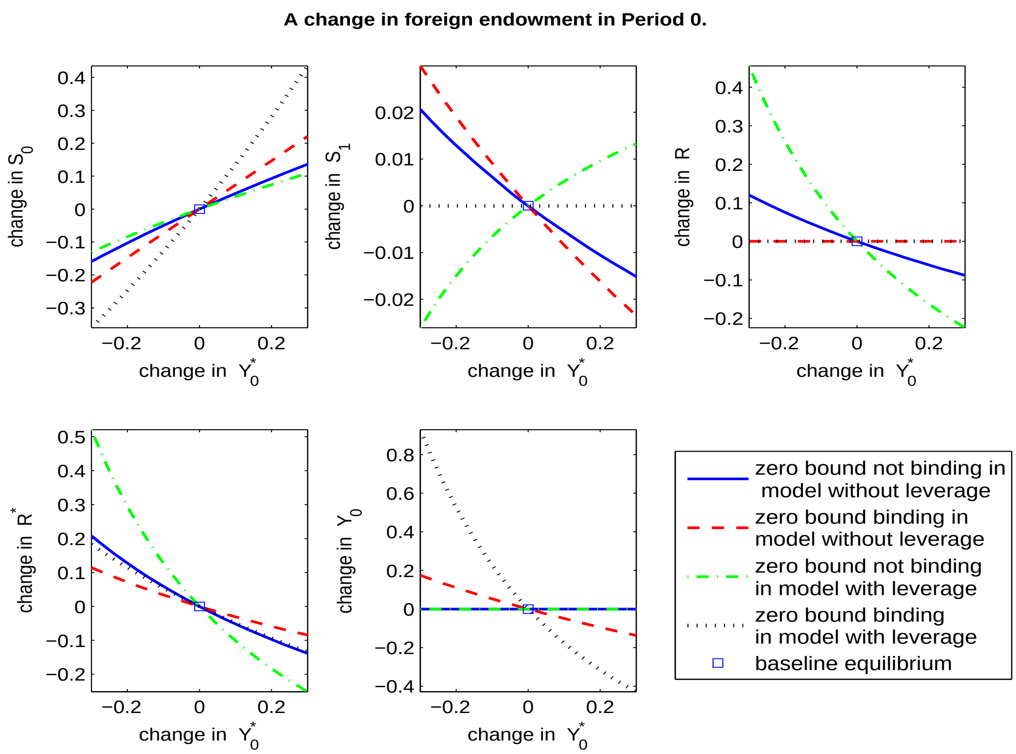

Figure 6: Comparative statics of the calibrated two-period model with different foreign endowment in period 0, Y_0^*

multiplier is larger.

In the following, we explain what happens in the leveraged model. Consider a rise in foreign endowment, Y_0^*, in the model in which the zero bound does not bind. The terms of trade in the foreign country deteriorate and world interest rates fall to induce demands for country B goods in the initial period. The terms of trade deteriorate in period 1 because of the faster growth in country B.

In the model in which the zero bound binds, in the home country, the foreign economy worsens the terms of trade to induce demands for its goods, leading to insufficient demands in the home economy. This triggers a downward spiral and the demand for home goods falls significantly. The terms of trade in period 1 is unchanged. This is because with the debt limit fixed and interest rate at the zero bound, debt interest repayment is unchanged by the endowment shock. The income effect of S_0 and the intertemporal substitution effect R^* exactly offset each other, so markets in period 1 clear.

4.2 A Three-Country Leveraged Model

In this subsection, we consider a model in which country A consumers are more impatient than consumers in the rest of the world. These consumers borrow in the initial period, up to an exogenous debt limit, D^A. This means that country A consumers are no longer following the consumption Euler equation.

Consumptions in country A are

$$C_0^A = \left(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0^A + D^A \tag{101}$$

$$C_1^A = \left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1^A - R^A D^A \tag{102}$$

Consumers in country B and C do not face any leverage constraint. They choose their demands according to the consumption Euler equation.

The full system contains six goods market clearing conditions and two debt market clearing conditions. The debt market clearing conditions are the uncovered interest parities, Equation (57) and (58). The goods market clearing conditions derived by substituting the demands for goods into the resource constraints. We report the full system in Appendix C.

Given the parameters $\{\alpha, \alpha^*, \beta^B, \beta^C, \eta, Y_0^A, Y_1^A, Y_0^B, Y_1^B, Y_0^C, Y_1^C, D^A\}$, these equilibrium conditions solve for the the relative prices $\{S_{A0}^B, S_{A1}^B, S_{A0}^C, S_{A1}^C R^A, R^B, R^C\}$. By Walras' law, one of the equilibrium conditions is redundant.

4.2.1 Three-Country Model with Equal Country Sizes

We can simulate this model with realistic parameters just like what we did for the two-country model. But in order to disentangle the asymmetry coming from different country sizes and the asymmetry coming from the leverage constraint assumption, we choose to simulate the model with equal country sizes.[10] In the baseline simulation, we study a system in which the output in each country is identical and normalised to unity. Each time period is set to equal to 10 years because we are interested in the long-run change in international debt position. We also let the debt limit in country A, D^A, tend to zero as the baseline to maintain the symmetry, so there will be no borrowing at all. We assume that the discount factor is about 0.99 per quarter in country B and C. This implies that β^B and β^C are 0.65 in 10 years. So, in the absence of shocks, the equilibrium terms of trade are unity and interest rates are $1/\beta$. The calibration of the symmetric three-country model is summarised in Table 3.

We simulate this leveraged model and compare this with a model with Euler equation consumers, that is the model presented in Section 6. In that model we assume that consumers in country A are not subject to a debt limit. We set the subjective discount factor in country A, or β^A, equal to 0.65, identical to the discount factor in the other two countries. The baseline equilibrium is identical to the one in Table 3.

We study the a positive endowment shock in country B in period 0, that is a rise in Y_0^B. We have computed analytically the solutions in the unleveraged model. But we are not able to obtain an analytical solution for the leveraged model. Therefore, in this section we compare numerically the comparative statics of the model without a leverage constraint and the model in which the debt limit in country A is fixed.

Figure 7 shows the case in which the zero bound is not binding. In the model without a leverage constraint, country A and C are symmetric, so a rise in country B endowment does not affect the terms of trade between country A and C. This is not true in the model in which country A's debt limit is fixed. As country A cannot smooth consumption intertemporally in that model, consumption demand for each variety of goods is more sensitive to terms of trade movements in that model. When endowment in country B rises and its terms of trade deteriorates, *ceteris paribus*, the demand for country A goods fall more than the demand for country C goods. Therefore, the terms of trade in country A has to worsen against country C to ensure resources are fully utilised in country A. The interest rate in country C has to fall by more relative to the unleveraged model to move consumption to the current period, although it does not fall by as much as country A's in this case. In fact, as implied by the UIP, the interest rates in all three countries fall below the interest rates in the model without the leverage constraint.

In period 1, the interest payment received by country B is reduced compared with the model without a leverage constraint, because the debt limit in country A, or D^A, is now fixed exogenously. Hence, a smaller improvement in terms of trade in country B in period 1 is needed to make demand equal to the supply, compared with the model without a leverage constraint. Since the debt limit in country A is fixed, consumption in country B

[10] We have also simulated the model with more realistic parameters (not presented here). The results do not differ significantly from those presented here.

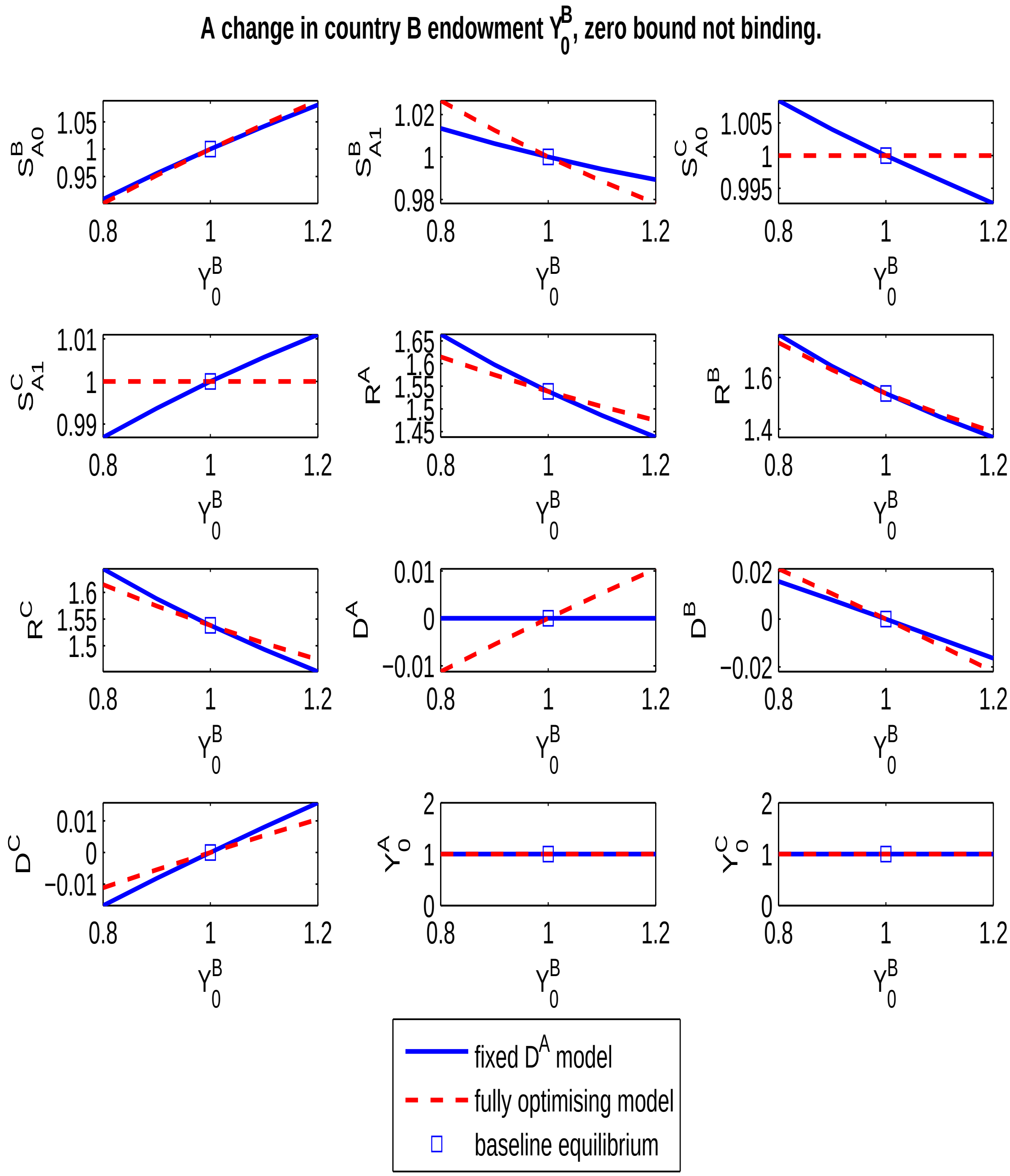

Figure 7: Comparative statics of the symmetric three-country model with leverage-unconstrainted agents and with fixed debt limit for country A consumers when the zero bound does not bind.

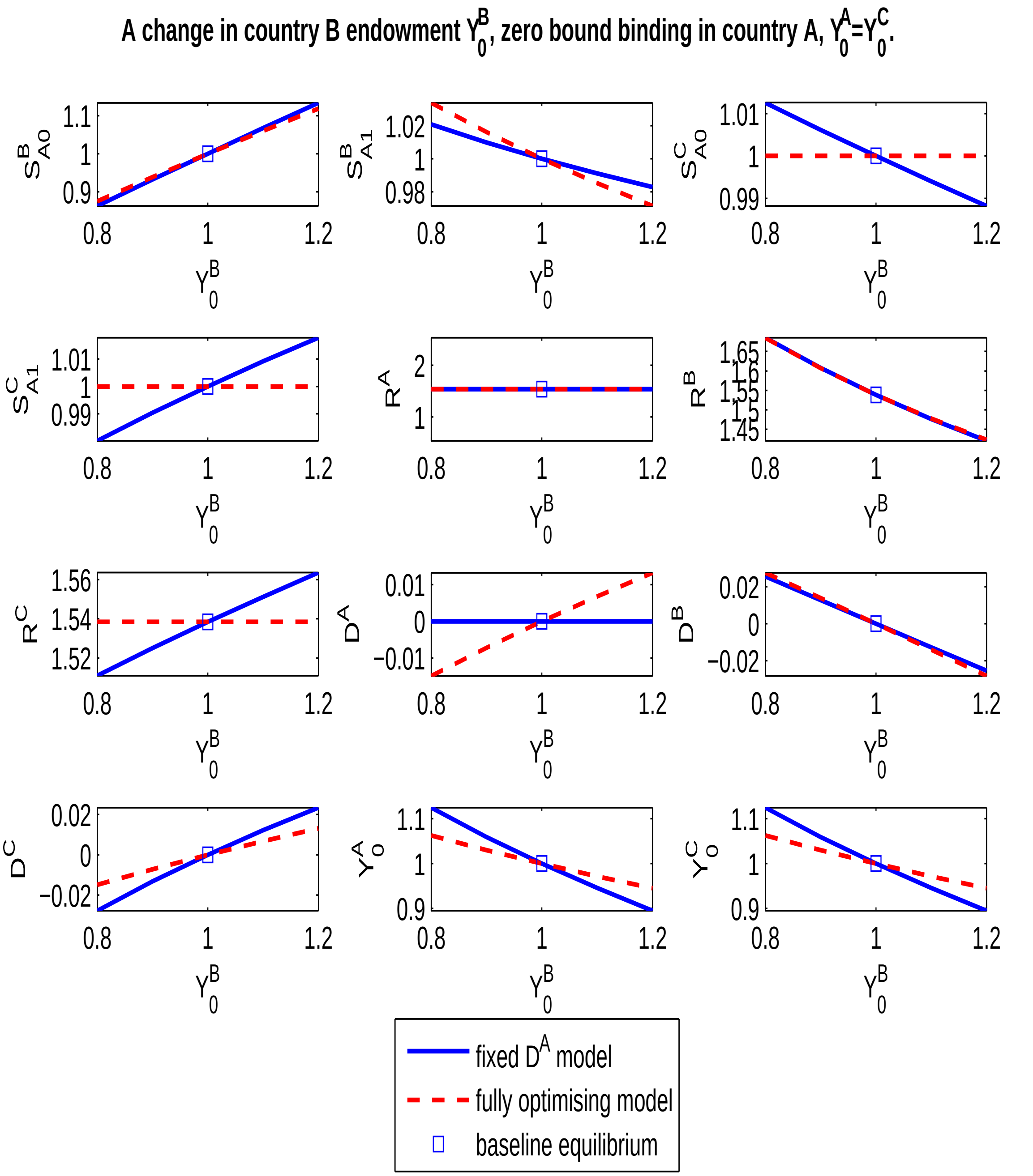

Figure 8: Comparative statics of the symmetric three-country model with leverage-unconstrainted agents and with fixed debt limit for country A consumers when the zero bound binds and country A and C share the shortfall in demand.

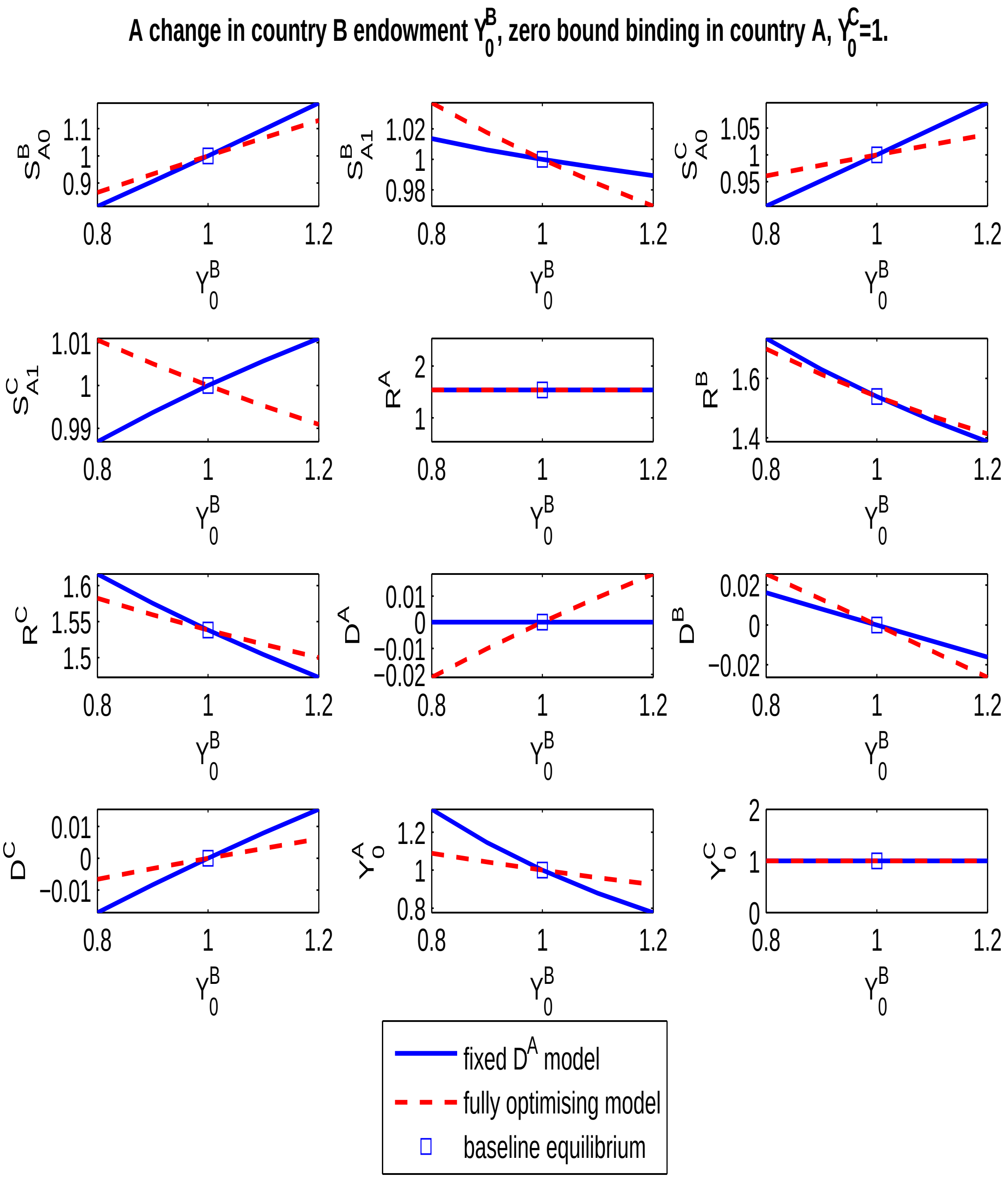

Figure 9: Comparative statics of the symmetric three-country model with leverage-unconstrainted agents and with fixed debt limit for country A consumers when the zero bound binds and country A suffers from a fall in demand.

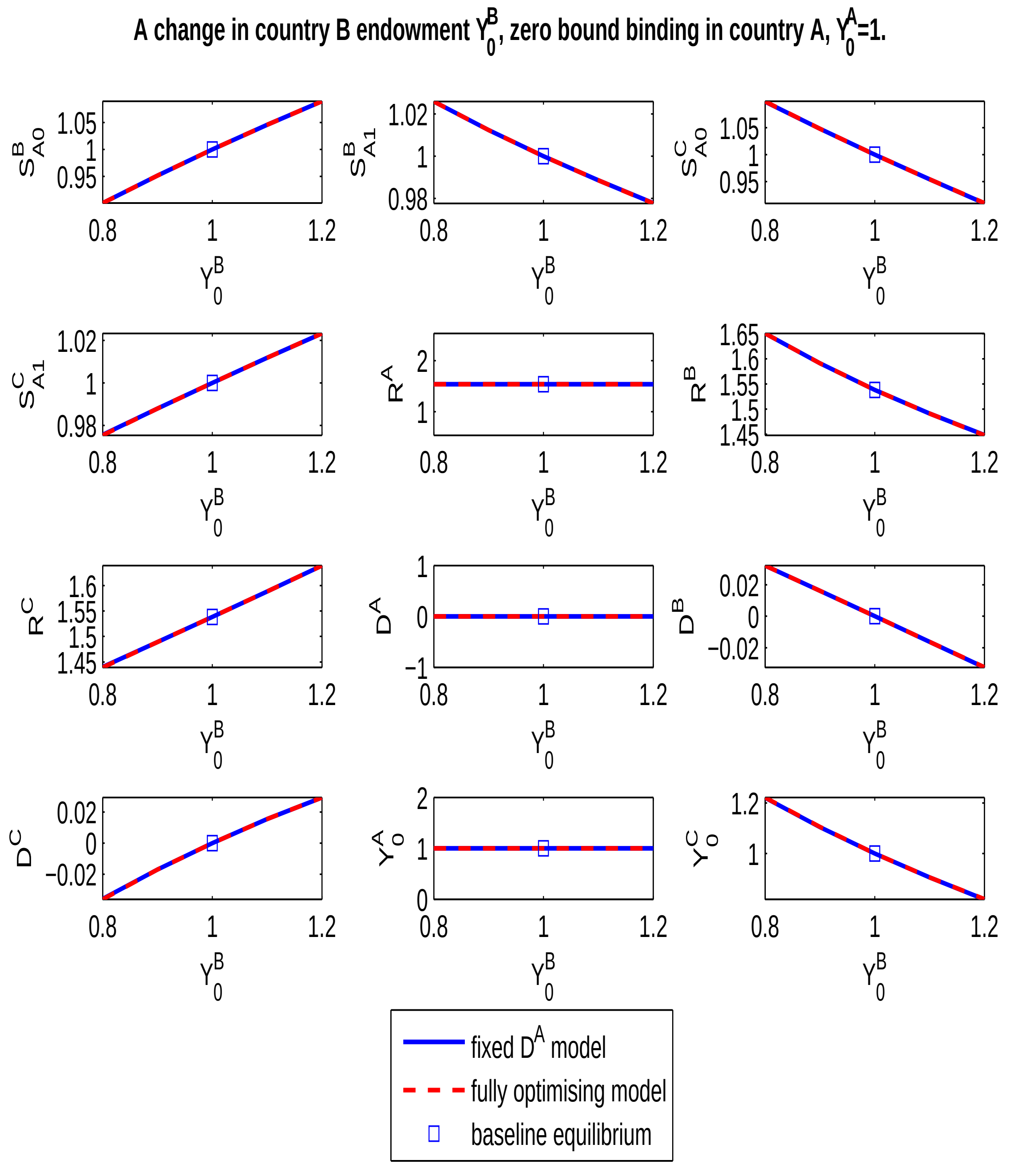

Figure 10: Comparative statics of the symmetric three-country model with leverage-unconstrainted agents and with fixed debt limit for country A consumers when the zero bound binds and country C suffers from a fall in demand.

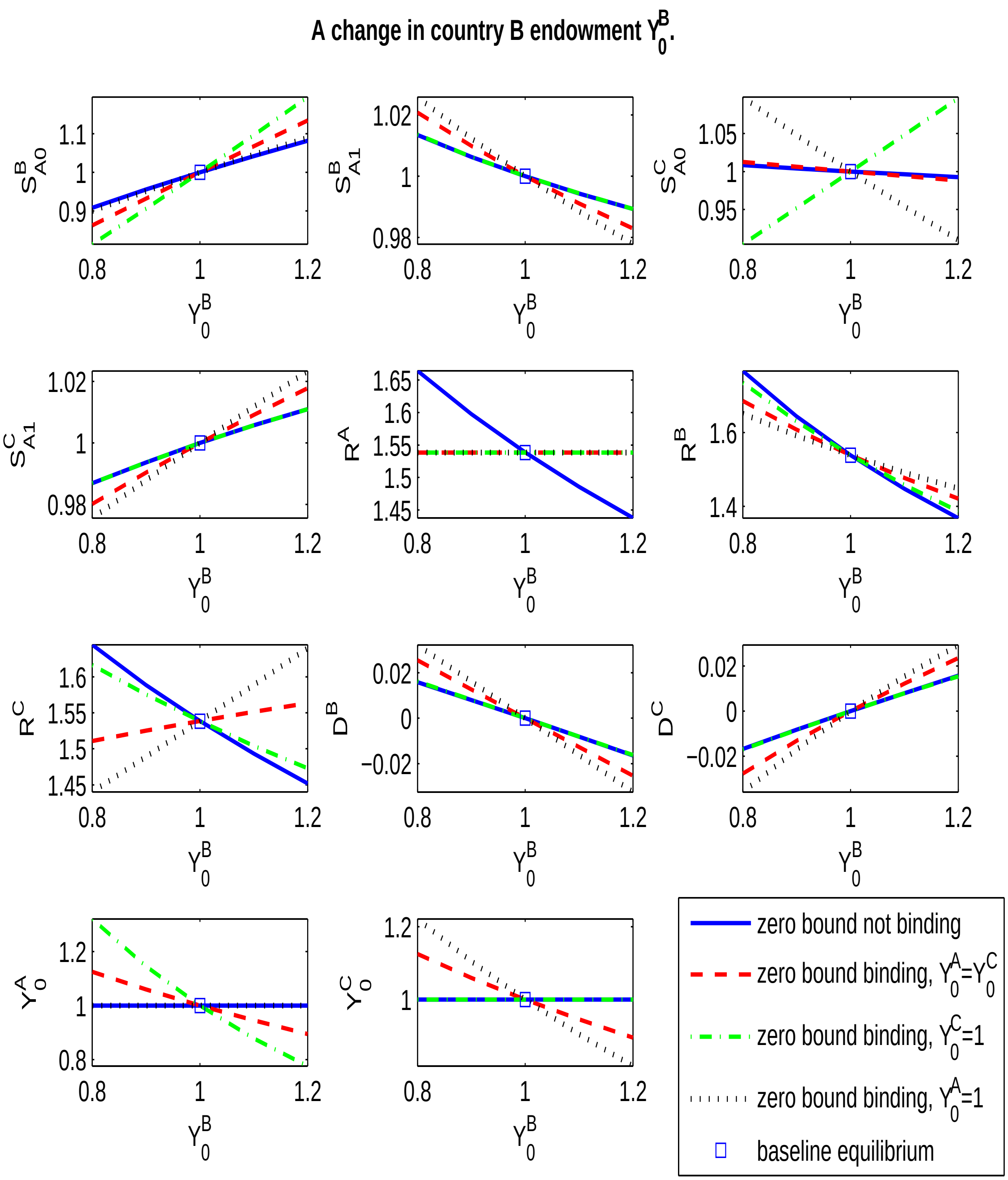

Figure 11: Comparative statics of symmetric three-country model with different period 0 endowment in country B, Y_0^B

rise more, is less sensitive to the endowment shock.

The results for the case in which the zero bound is binding in country A and country A and C share the shortfall in demand is presented in Figure 8. The difference in the simulation results of the two models when the zero bound is binding in country A and country A and C share the fall in demand can be accounted for similarly. Note that the interest rate in country C rises in the model with a leverage constraint. This is because the terms of trade in country C strengthens against country A in period 0 and weakens in period 1. Since the interest rate in country A is fixed, the UIP implies that the interest rate in C must rise in order to satisfy the assumption that $Y_0^A = Y_0^C$.

Figure 9 presents the results for the case in which the zero bound is binding in country A and country A alone suffers from the fall in demand. The responses to a productivity shock in the two models are similar, but the reactions in the terms of trade in period 0 and the interest rates are increased in the leveraged system.

Figure 10 presents the results for the case in which the zero bound is binding in country A and country C alone suffers from the loss in demand. The responses in the unleveraged model and in the leveraged model are almost identical. In particular, in the model without a leverage constraint, the debt in country A is unchanged in the face of a shock in endowment in country B. This is because when country B worsens its terms of trade against country A by any fraction, country A worsens its terms of trade against country C by an equal fraction to pass the burden of a fall in demand to country C.

Figure 11 compares the responses of a change in country B endowment in the three-country leveraged model when the zero bound is not binding in country A and when it is. It shows that country B's terms of trade in period 0 has to worsen by more when the zero bound is binding in country A. This is because, when the zero bound is binding there is a shortfall in demand, in either country A or country C or both, which means that country B has to worsen its terms of trade by more to ensure its resources are fully utilised.

5 The Global Effect of Deleveraging

The leveraged model in the previous section allows us to study a deleveraging shock in the US. Specifically, the deleveraging shock is a fall in the exogenous debt limit in the US. This means that the debt-constrained consumers in the US are forced to consume less in the initial period.

5.1 A Deleveraging Shock in the Two-country Model

In fact, we are studying an international version of Eggertsson and Krugman (2010). Eggertsson and Krugman (2010) is a closed endowment economy in which there are two types of consumers: patient consumers similar to the unconstrained consumption smoothers in

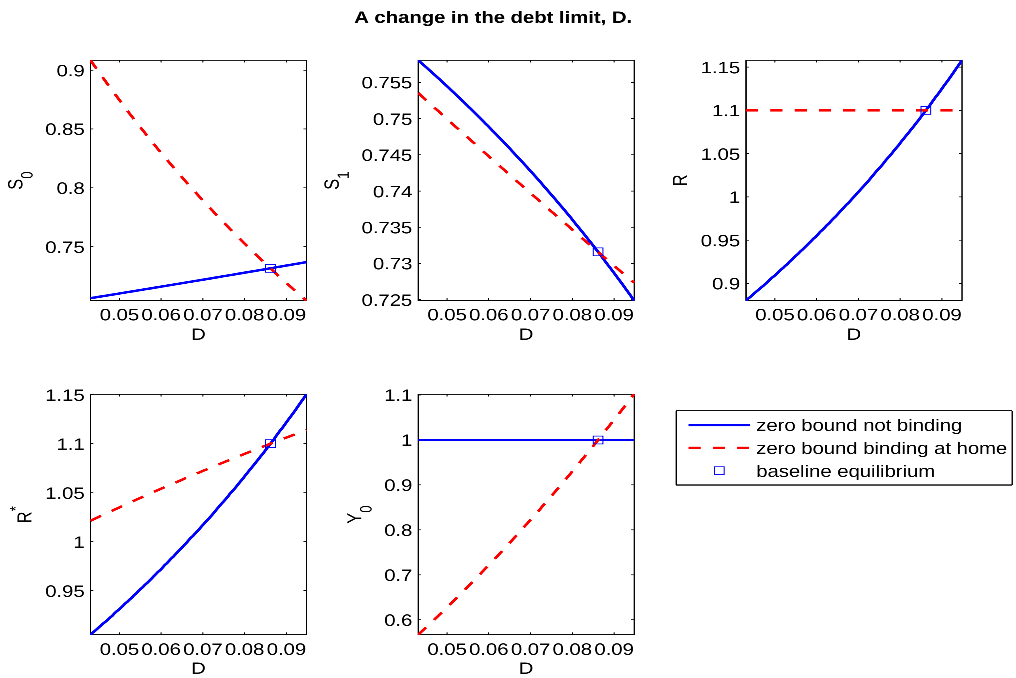

Figure 12: Comparative statics of the calibrated two-period model with different debt limits D

our model and impatient consumers similar to the debt-constrained consumers in our model. They consider a deleveraging shock, which is a fall in the exogenous debt limit to the debt-constrained consumers. They show that when this happens, debt-constrained consumers will cut consumption and the interest rates has to fall to induce the consumption smoothers to consume. However, if the deleveraging shock is so large that the natural interest rate is below the zero bound then there is a recession because the aggregate demand is less than the endowment.

Our model 'extends' Eggertsson and Krugman by turning the system to an open macroeconomic system. In our two-country system, we add a foreign economy whose policy objective is maintaining foreign demand and add an additional instrument, which are the terms of trade between these two countries. We assume that the foreign country – which we call China – controls the terms of trade to maintain its demand.

The simulation results for the deleveraging shock are reported in Figure 12. Consider a fall in the debt limit, D, in the model in which the zero bound does not bind. *Ceteris paribus*, the demand for US goods in period 0 falls, so the home terms of trade worsen to induce demand for home goods. The foreign interest rate falls to shift demand forwards. In period 1, home consumers pay a smaller debt interest, so its terms of trade improve to reduce the demand for home goods. Resources remain fully utilised in both countries in both periods. These results are consistent to Eggertsson and Krugman (2010).

In the model in which the zero bound is binding in the home country, in contrast, there is a big fall in the demand for home goods in the initial period. This happens because the foreign country worsens its terms of trade in respond to the fall in debt limit. This triggers the downward spiral in which demand for home goods and home income fall and the terms of trade in the foreign country deteriorate further. This means that adding a foreign economy in the way we do aggravates the shortfall in demand in the home economy compared with Eggertsson and Krugman because the foreign country 'steals' demand from the home country by manipulating the terms of trade, making the zero bound at home more binding.

5.2 A Deleveraging Shock in the Three-country Model

By adding one more country to this system, we are able to study how the terms of trade movements affect the allocation of the shortfall in global aggregate demand in the face of a deleveraging shock in the US. We study a fall in debt limit in the US in the three-country leveraged model.

Figure 13 present the results of this exercise. In the model in which the zero bound is not binding in country A, country B and C are symmetric. As the debt limit in country A falls, *ceteris paribus*, there is a fall in demand in country A goods in period 0. As consumers in country A cannot smooth consumption intertemporally, the terms of trade in country A must worsen against country B and C in order to keep resources fully utilised in country A. In period 1, since debt repayment in country A is reduced, *ceteris paribus*, there is excess demand in country A and insufficient demand in the rest of the world. Therefore the terms of trade in country A improve vis-a-vis country B and C. World interest rates fall to shift consumption to period 0 in country B and C so that markets clear in these countries. Debts in the rest of the world rise. Again, the fact that a fall in interest rates is necessary to keep resources fully utilised is consistent with Eggertsson and Krugman (2010).

Consider the model in which the zero bound is binding in country A and both country B and C manipulate the terms of trade such that all the fall in demand emerges in country A. Clearly, country B and C are still symmetric. When debt limit in country A falls, *ceteris paribus*, the demand in country A in period 0 decreases. By construction, the terms of trade are controlled by the rest of the world and the real interest rate in country A is fixed at the zero bound, so some goods in country A are not consumed and have to be disposed. The income of country A's consumers falls, and, *ceteris paribus*, they demand less goods from the rest of the world. The rest of the world weakens their terms of trade in order to keep resources fully utilised, further driving down the demand in country A. This triggers a vicious circle with reduction in both country A consumers' income and demand for country A's goods. Our simulation results show that the fall in country A demand is substantial, and for this reason the rest of the world worsens their terms of trade and cuts interest rates to reallocate demands to their countries in period

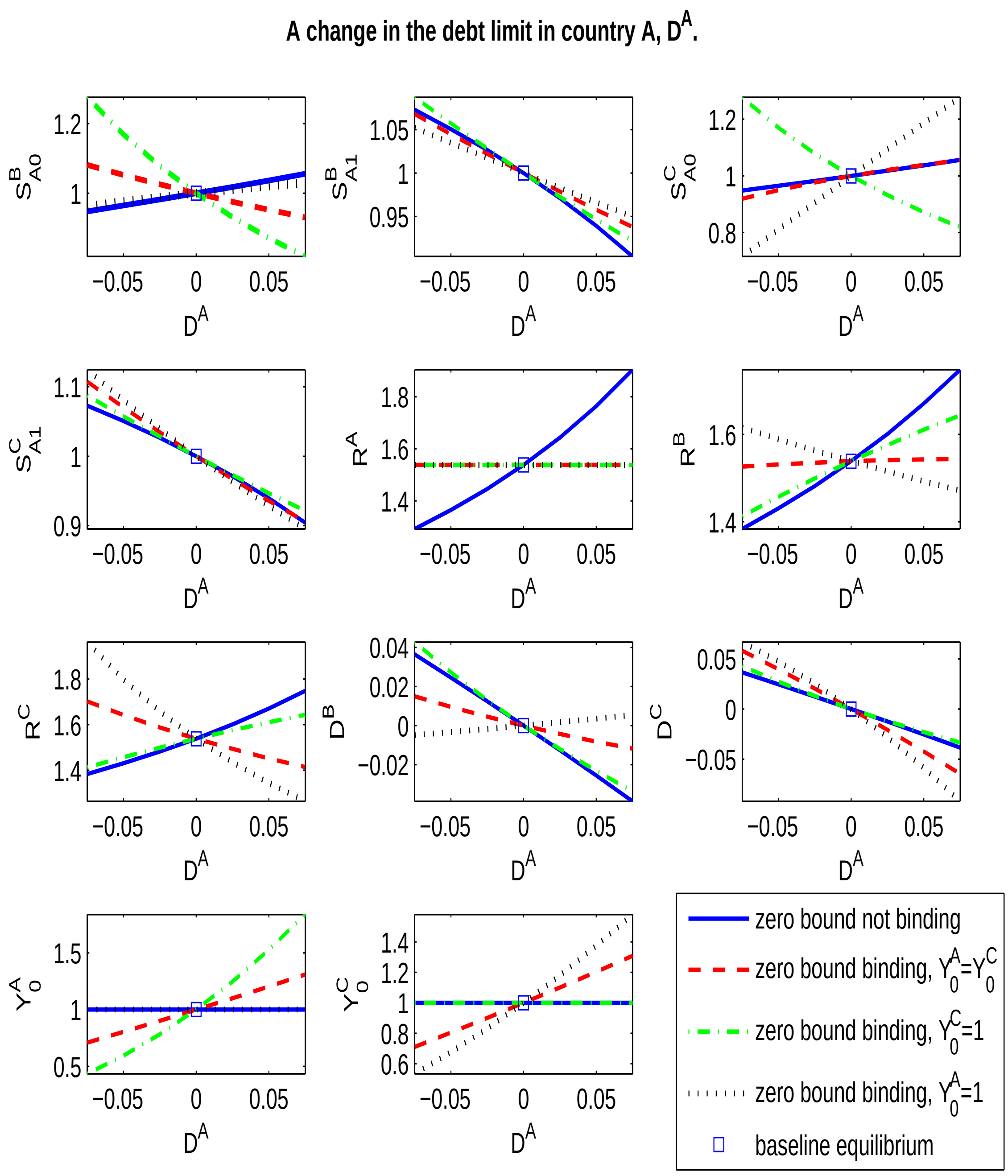

Figure 13: Comparative statics of symmetric three-country model with different debt limits in country A

0.

In the model in which the zero bound is binding in country A and both country A and B can manipulate the terms of trade such that all the fall in demand emerges in country C, after a fall in debt limit, *ceteris paribus*, the demand for country A's goods falls. As the terms of trade vis-a-vis country B are controlled by country B, country A, whose consumers do not smooth consumption intertemporally, must worsen its terms of trade against country C, *i.e.* S^C_{A0} must fall. Country C's interest rate falls to switch some of its consumption to the current period, but as markets have to clear in period 1, the fall in interest rate will not eliminate the loss in demand in country C. The resulting fall in income earned by country C consumers reduces their demand for world goods, leading to further deterioration in the terms of trade in country A and B. As a result, country C income and demand fall further. Our simulation results show that the fall in country C demand is so large that country B has to worsen its terms of trade to clear its goods market in period 0. Also, although the shortfall in demand country C is large, it is not as large as the fall in demand in country A in the system in which country C controls the terms of trade. This is because country C is not debt-constrained and can move consumption to the initial period in response to lower interest rate.

Finally, in the model in which the zero bound is binding in country A and country A and C share the loss in demand equally, the simulated lines of all the variables are sandwiched by the lines in the above two cases.

One can see from this set of simulations that when the zero bound is binding, a deleveraging shock will in general lead to a fall in the demand, but the extent to which the demand will fall depends on the country's ability to manipulate the terms of trade and transfer the shortfall in demand to the rest of the world.

6 Issues Relating to Global Macroeconomic Policy Cooperation

The above analysis suggests that there may be room for policy cooperation in the world in which the zero bound is binding in the US, in the face of a positive productivity shock in China or deleveraging in the US. China depreciates its currency to 'steal' demand from the rest of the world. US and Europe compete for less output loss in order to maximise the welfare in their respective countries in a Nash game.

However, the model does not have enough structure to show how the policymakers would compete against each other. This is because the only way to do this competition is to push the terms of trade in a country's favour, but both country cannot control the terms of trade by pushing the terms of trade in the opposite direction. To overcome this difficulty, we may need a more elaborate setup to model quantitative easing as a policy instrument controlled by the US policymaker.

Furthermore, in order to derive a utility-based loss function, we need to have disutility of labour in the utility function and an endogenous production system. Otherwise, suppose it does not require any effort to produce. Then there is no optimal level of output at which the marginal utility of consumption equals the marginal disutility of labour supply.

In the current model, devaluation hurts the welfare of the country when the zero bound is not binding. Consider the utility of country A, given in Equation (55). The consumption Euler equation and the assumption that the real interest rate is fixed at $1/\beta$ due to the zero bound imply that $C_0 = C_1$, which means that the welfare is maximised when aggregate consumption is the highest. Therefore, country A wants its terms of trade to strengthen, for any given value of endowment, to increase its purchasing power. This means competitive devaluation will not occur in this model when zero bound is not binding. This result is consistent with Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001) which show that when a microfounded welfare measure is used instead of an ad-hoc loss function, no country has the welfare incentive to 'retaliate' to foreign devaluation. A deterioration of the terms of trade abroad unambiguously increases domestic welfare because domestic consumption is higher and labour supply is lower.

When the zero bound is binding, however, a worsening of the terms of trade may increase welfare. Even although, as in the case in which the zero bound is not binding, a fall in purchasing power decreases welfare, the indirect effect of a rise in output, amplified by the Keynesian multiplier, may more than offset the direct effect when there is a shortfall of global aggregate demand.

Following the analysis of Oudiz and Sachs (1984), when the loss function contains quadratic terms in output gap and inflation, foreign devaluation reduces home welfare and causes the home country to devalue. This leads us to think that adding nominal rigidities in the Calvo (1983) style in our three-country model and using the microfounded quadratic loss function following Woodford (2003) may enable us to study the policy coordination problem along this line. (In fact, Obstfeld and Rogoff (1995) and Corsetti and Pesenti (1997) have nominal rigidities in their model, but they assume that prices are fixed for one period and flexible afterwards, which means that there is no inflation term in their welfare function.)

In this case, inflation will be an argument in the loss function. In the optimisation problem, the policy maker has to consider the trade-off between the marginal cost of inflation stabilisation and output stabilisation. This ensures the existence of a solution to this problem.

To summarise, in order to answer this question of policy coordination, the model will have to be amended so as to contain the production side of the economy, endogenous labour supply, a production subsidy, utility-based welfare measure and Calvo (1983) style nominal rigidities.

7 Conclusion

We presented a simple, microfounded, and intertemporal, model of global imbalances, and extended this to the case of three countries.

We followed Blanchard and Milesi Ferretti (2011) in using this model to reveal the effects of a zero bound to world interest rates, in both the two-country and three-country versions of the model. We do this in the context of an increase in production in China. There are two features of interest of the results.

First, we are able to solve, endogenously, for the extent of underutilisation of resources that is caused by the zero bound, when production increases in China. We can do this, even although no consumers are income-constrained in their consumption. The zero bound prevents aggregate demand from being as large as aggregate supply, which reduces consumers' real incomes. Even although all consumers follow an Euler equation, their 'marginal propensity to consume' out of current goods, where by this phrase we mean the marginal addition to the demand for home goods created by a marginal increase in home income, is, in effect, less than one. This is because the effects of any loss of income are spread across home and foreign consumption, and between present and future consumption. That ensures that there is a Keynesian multiplier of finite size. We are able to study the determinants of the size of this multiplier.

Second, we are able, in the three-country version, to analyse how the underutilisation of resources is distributed between the US and Europe. This depends on the outcome for the Eurodollar exchange-rate. We have no formal theory of how this is determined in the zero-bound world. But we suggest that it might be influenced by quantitative easing. The more the price of US is held down relative to the price of European goods, the larger the proportion of the underutilisation resources which happens in Europe.

Third, we are able to study how the existence of leverage-constrained consumers in one part of the world, the US, magnifies the global responses to the Chinese productivity shock. In such a world, interest rates must fall by a large amount following a shock to production in China. And if there is a zero bound, the size of the fall in demand which emerges is larger. This analysis applies the 'balance-sheet macroeconomics' of Eggertsson and Krugman (2010) to a study of the global economy.

The final part of the paper takes such a world in which the consumers in the US are leverage constrained, and discusses the global effects that emerge when these consumers deleverage. When there is no zero bound, it is obvious that deleveraging in one country will cause the world interest rate to fall. When there is a zero bound, deleveraging will cause an underutilisation of resources. In the three-country world of US, China, and Europe, the way in which the underutilisation of resources is shared between the terms of trade between the US and Europe depends on the behaviour of the Eurodollar exchange rates. As noted above we do not have a good theory of this.

In the final brief section of the paper we noted that this model is not yet set up in such a way as to make it possible to study the international coordination of macroeconomic policies. That is a task for future work.

Finally, what our study of deleveraging does not do is to show the way in which the effects of deleveraging can be cushioned by an endogenous public sector deficit. In reality, we know that, if there is deleveraging by the private sector in the US, an increase in the US fiscal deficit will cushion the effects of such deleveraging. In our model deleveraging has to be cushioned by the behaviour of other countries, and by a fall in the world interest rate. If there is a zero bound, we can see from the model that the effects of the leveraging might be significant. An endogenous public sector deficit would dampen these effects.

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A Appendix: A Two-Country Model without a Leverage Constraint

A.1 The Model

Consumers consume both home and foreign goods. We use a CES aggregator for simplicity.

$$
\begin{aligned}
C_t &= \left(\alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} && (103)\\
C_t^* &= \left((1-\alpha)^{\frac{1}{\eta}} \left(C_{Ht}^*\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{Ft}^*\right)^{\eta-1} \eta\right)^{\frac{\eta}{\eta-1}} && (104)
\end{aligned}
$$

Here, C_{Ht} and C_{Ft} denote the consumption of home and foreign goods by home consumers, and C_{Ht}^* and C_{Ft}^* denote the consumption of home and foreign goods by foreign consumers. Home and foreign consumers both have home bias in their consumption, which is captured by the parameter α, where $\alpha \geq 0.5$. When $\alpha = 0.5$, there is no home bias in consumption. We assume that the elasticity of substitution between home and foreign goods is larger than unity, that is $\eta > 1$.

Consumers in the home country choose their consumption of home and foreign goods to minimise aggregate expenditure. As a result, the demand for home and foreign goods, by home consumers, can be shown to be

$$
\begin{aligned}
C_{Ht} &= \alpha \left(\frac{P_t}{P_{Ht}}\right)^{\eta} C_t && (105)\\
C_{Ft} &= (1-\alpha) \left(\frac{P_t}{P_{Ft}}\right)^{\eta} C_t && (106)
\end{aligned}
$$

where P_{Ht} and P_{Ft} denote the price of home and foreign goods in the home country. Similarly for foreign consumers

$$
\begin{aligned}
C_{Ht}^* &= (1-\alpha) \left(\frac{P_t^*}{P_{Ht}^*}\right)^{\eta} C_t^* && (107)\\
C_{Ft}^* &= \alpha \left(\frac{P_t^*}{P_{Ft}^*}\right)^{\eta} C_t^* && (108)
\end{aligned}
$$

where P_{Ht}^* and P_{Ft}^* denote the price of home and foreign goods in the foreign country and the aggregate price levels in the home country and foreign country are

$$
\begin{aligned}
P_t &= \left(\alpha P_{Ht}^{1-\eta} + (1-\alpha) P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} && (109)\\
P_t^* &= \left((1-\alpha)\left(P_{Ht}^*\right)^{1-\eta} + \alpha \left(P_{Ft}^*\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} && (110)
\end{aligned}
$$

We assume that the law of one price holds, *i.e.* that after conversion at the ruling nominal exchange rate, e_t, each good sells at the same price in each country:

$$e_t = \frac{P^*_{Ht}}{P_{Ht}} = \frac{P^*_{Ft}}{P_{Ft}} \tag{111}$$

A rise in e_t is a depreciation of the foreign currency.

We let S_t denote the terms of trade (from the foreign country's perspective),

$$S_t \equiv \frac{P^*_{Ht}}{P^*_{Ft}} = \frac{P_{Ht}}{P_{Ft}} \tag{112}$$

Then,

$$\frac{P_t}{P_{Ht}} = \left(\frac{\alpha P_{Ht}^{1-\eta} + (1-\alpha) P_{Ft}^{1-\eta}}{P_{Ht}^{1-\eta}}\right)^{\frac{1}{1-\eta}} = \left(\alpha + (1-\alpha) S_t^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} \tag{113}$$

$$\frac{P_t}{P_{Ft}} = \left((1-\alpha) + \alpha S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{114}$$

$$\frac{P^*_t}{P^*_{Ht}} = \left((1-\alpha) + \alpha S_t^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} \tag{115}$$

$$\frac{P^*_t}{P^*_{Ft}} = \left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{116}$$

Substituting these equations into the consumption demands, we get:

$$C_{Ht} = \alpha \left(\alpha + (1-\alpha) S_t^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} C_t \tag{117}$$

$$C_{Ft} = (1-\alpha) \left((1-\alpha) + \alpha S_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}} C_t \tag{118}$$

$$C^*_{Ht} = (1-\alpha) \left((1-\alpha) + \alpha S_t^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} C^*_t \tag{119}$$

$$C^*_{Ft} = \alpha \left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}} C^*_t \tag{120}$$

We let Y_t and Y^*_t denote the exogenous endowments in the home and foreign country in period t respectively. As a result, the goods market clearing conditions are

$$Y_0 = C_{H0} + C^*_{H0}, \qquad Y_1 = C_{H1} + C^*_{H1} \tag{121}$$

$$Y^*_0 = C_{F0} + C^*_{F0}, \qquad Y^*_1 = C_{F1} + C^*_{F1} \tag{122}$$

There is a real debt market between the two countries. Consumers can borrow from the other country in period 0 and repay, with interest, in period 1. The budget constraint for the home country in period 0 is

$$P_0 D = P_0 C_0 - P_{H0} C_{H0} - P^*_{H0} C^*_{H0} e_0^{-1} \tag{123}$$

$$= P_0 C_0 - P_{H0}(C_{H0} + C^*_{H0}) \tag{124}$$

$$= P_0 C_0 - P_{H0} Y_0 \tag{125}$$

where D denotes real debt in the home country.

In period 1, the budget constraint for the home country is

$$\begin{aligned} 0 &= P_1 C_1 - P_{H1} C_{H1} - P^*_{H1} C^*_{H1} e_1^{-1} + P_1 D R && (126) \\ &= P_1 C_1 - P_{H1} (C_{H1} + C^*_{H1}) + P_1 D R && (127) \\ &= P_1 C_1 - P_{H1} Y_1 + P_1 D R && (128) \end{aligned}$$

where R is the gross real interest rate in the home country. We combine these two equations to eliminate debt.

$$C_1 = \frac{P_{H1}}{P_1} Y_1 + R \frac{P_{H0}}{P_0} Y_0 - R C_0 \qquad (129)$$

This equation above is the intertemporal budget constraint. It can be expressed in terms of the terms of trade

$$C_1 = \left(\alpha + (1-\alpha) S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} R Y_0 + \left(\alpha + (1-\alpha) S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1 - R C_0 \qquad (130)$$

For simplicity we assume that home consumers have perfect foresight and a logarithmic utility function of the following form:

$$U_0 = \ln C_0 + \beta \ln C_1 \qquad (131)$$

Taking their endowment as given, home consumers maximise utility subject to their budget constraint. This gives the consumption Euler equation for home consumers:

$$C_1 = \beta R C_0 \qquad (132)$$

We now substitute the Euler equation into the budget constraint to solve explicitly consumption demands in the two periods:

$$C_0 = \frac{1}{(1+\beta)} \left(\alpha + (1-\alpha) S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0 + \frac{1}{(1+\beta) R} \left(\alpha + (1-\alpha) S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1 \qquad (133)$$

and

$$C_1 = \frac{\beta R}{(1+\beta)} \left(\alpha + (1-\alpha) S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0 + \frac{\beta}{(1+\beta)} \left(\alpha + (1-\alpha) S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1 \qquad (134)$$

Home consumers' demands for home and foreign goods can be solved for using Equations (117) to (120).

The intertemporal choice problem for foreign consumers is similar to the problem for home consumers. The budget constraint for the foreign country in period 0 is

$$\begin{aligned} P^*_0 D^* &= P^*_0 C^*_0 - e_0 P_{F0} C_{F0} - P^*_{F0} C^*_{F0} && (135) \\ &= P^*_0 C^*_0 - P^*_{F0} Y^*_0 && (136) \end{aligned}$$

where D^* is the real debt by foreign consumers. The budget constraint for the foreign country in period 1 is

$$
\begin{aligned}
0 &= P_1^* C_1^* - e_1 P_{F1} C_{F1} - P_{F1}^* C_{F1}^* + R^* P_1^* D^* && (137) \\
&= P_1^* C_1^* - P_{F1}^* Y_1^* + R^* P_1^* D^* && (138)
\end{aligned}
$$

where R^* is the gross real interest rate in the foreign economy.

Combining the two constraints we obtain:

$$
C_1^* = \left(\alpha + (1-\alpha) S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}} R^* Y_0^* + \left(\alpha + (1-\alpha) S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}} Y_1^* - R^* C_0^* \quad (139)
$$

This is the intertemporal budget constraint for foreign consumers. Foreign consumers have the utility of the following form:

$$
U_0^* = \ln C_0^* + \beta \ln C_1^* \quad (140)
$$

Taking the endowments as given, foreign consumers maximise utility subject to the budget constraint. This gives the consumption Euler equation for foreign consumers:

$$
C_1^* = \beta R^* C_0^* \quad (141)
$$

Again, by substituting the consumption Euler equation into the budget constraint, we can solve explicitly for consumption demands in the two periods.

$$
\begin{aligned}
C_0^* &= \frac{1}{(1+\beta)} \left(\alpha + (1-\alpha) S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}} Y_0^* + \frac{1}{(1+\beta)R^*} \left(\alpha + (1-\alpha) S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}} Y_1^* && (142) \\
C_1^* &= \frac{\beta R^*}{(1+\beta)} \left(\alpha + (1-\alpha) S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}} Y_0^* + \frac{\beta}{(1+\beta)} \left(\alpha + (1-\alpha) S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}} Y_1^* && (143)
\end{aligned}
$$

Foreign consumers' demands for home and foreign goods can be solved for using Equations (117) to (120).

The debt market must clear in both periods. We show in this section that this implies the uncovered interest parity (UIP) holds.

We rearrange the home budget constraint in period 0:

$$
\begin{aligned}
P_0 D &= P_0 C_0 - P_{H0} C_{H0} - P_{H0}^* C_{H0}^* e_0^{-1} && (144) \\
&= P_{H0} C_{H0} + P_{F0} C_{F0} - P_{H0} C_{H0} - P_{H0} C_{H0}^* && (145) \\
&= P_{F0} C_{F0} - P_{H0} C_{H0}^* && (146)
\end{aligned}
$$

Similarly, the foreign budget constraint in period 0 can be rearranged as follows:

$$
\begin{aligned}
P_0^* D^* &= P_0^* C_0^* - e_0 P_{F0} C_{F0} - P_{F0}^* C_{F0}^* && (147) \\
\frac{P_0^*}{e_0} D^* &= P_{H0} C_{H0}^* - P_{F0} C_{F0} && (148)
\end{aligned}
$$

We add up the two budget constraints to obtain a relation between home and foreign debt.

$$P_0 D + \frac{P_0^*}{e_0} D^* = 0 \tag{149}$$

We rearrange the home budget constraint in period 1:

$$\begin{aligned}
0 &= P_1 C_1 - P_{H1} C_{H1} - P_{H1}^* C_{H1}^* e_1^{-1} + P_1 D R && (150)\\
&= P_{H1} C_{H1} + P_{F1} C_{F1} - P_{H1} C_{H1} - P_{H1} C_{H1}^* + P_1 D R && (151)\\
&= P_{F1} C_{F1} - P_{H1} C_{H1}^* + P_1 D R && (152)
\end{aligned}$$

Similarly, we rearrange the foreign budget constraint in period 1:

$$\begin{aligned}
0 &= P_1^* C_1^* - e_1 P_{F1} C_{F1} - P_{F1}^* C_{F1}^* + R^* P_1^* D^* && (153)\\
&= P_{H1} C_{H1}^* - P_{F1} C_{F1} + R^* \frac{P_1^*}{e_1} D^* && (154)
\end{aligned}$$

We add up the two budget constraints, and substitute in the debt market clearing condition in period 0, Equation (149) to obtain the uncovered interest parity.

$$\frac{R^*}{R} = \left(\frac{e_1 P_1}{P_1^*}\right) / \left(\frac{e_0 P_0}{P_0^*}\right) \tag{155}$$

where $e_t P_t / P_t^*$ is the real exchange rate in period t. It is easy to show that

$$\frac{e_t P_t}{P_t^*} = \frac{\left((1-\alpha) + \alpha S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}} \tag{156}$$

Therefore the uncovered interest parity can be expressed in terms of the terms of trade as

$$R^* = R \frac{\left(\alpha + (1-\alpha) S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left((1-\alpha) + \alpha S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}} \times \frac{\left((1-\alpha) + \alpha S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + (1-\alpha) S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}} \tag{157}$$

When $\alpha = 0.5$, home and foreign consumers have identical preferences, the real exchange rate is always equal to unity, so home and foreign interest rates are always equal.

A.2 The Full System

In equilibrium the home and foreign goods markets in both periods must clear and the debt market clears, which implies that uncovered interest parity holds . Therefore,

$$
\begin{aligned}
Y_0 &= \frac{\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-1}Y_0 \\
&+\frac{\alpha}{(1+\beta)R}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_1 \\
&+\frac{1-\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-1}S_0^{-\eta}Y_0^* \\
&+\frac{1-\alpha}{(1+\beta)R^*}S_0^{-\eta}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^* \qquad (158)\\
Y_0^* &= \frac{1-\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-1}S_0^{\eta}Y_0 \\
&+\frac{1-\alpha}{(1+\beta)R}S_0^{\eta}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_1 \\
&+\frac{\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-1}Y_0^* \\
&+\frac{\alpha}{(1+\beta)R^*}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^* \qquad (159)\\
Y_1 &= \frac{\alpha\beta R}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}Y_0 \\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-1}Y_1 \\
&+\frac{(1-\alpha)\beta R^*}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{\eta}{1-\eta}}S_1^{-\eta}Y_0^* \\
&+\frac{(1-\alpha)\beta}{(1+\beta)}S_1^{-\eta}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-1}Y_1^* \qquad (160)\\
Y_1^* &= \frac{(1-\alpha)\beta R}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}S_1^{\eta}Y_0 \\
&+\frac{(1-\alpha)\beta}{(1+\beta)}\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-1}S_1^{\eta}Y_1 \\
&+\frac{\alpha\beta R^*}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{\eta}{1-\eta}}Y_0^* \\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-1}Y_1^* \qquad (161)\\
R^* &= R\frac{S_1}{S_0}\times\frac{\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}\times\frac{\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}} \qquad (162)
\end{aligned}
$$

By Walras' Law, one of the five market clearing conditions is redundant. We have four equations to solve for the four relative prices (R, R^*, S_0, S_1).

In general, this system is complex and the analytical solution for the relative prices in levels cannot be computed. For $\alpha > 0.5$, we log-linearise the system around this symmetric steady state as follows: (where the 'hat' notation denotes the log deviation around the symmetric steady state).

$$\begin{bmatrix} 1-\alpha \\ \alpha-(1+\beta) \\ -(1-\alpha) \\ -\alpha \\ 0 \end{bmatrix} \hat{Y}_0^* = \begin{bmatrix} \Xi & 0 & \beta & 0 \\ -\Xi-\beta\Lambda & \beta\Lambda & \beta & 0 \\ 0 & -\Xi & 1 & 0 \\ -\Lambda & \Xi+\Lambda & 1 & 0 \\ -\Lambda & \Lambda & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{S}_0 \\ \hat{S}_1 \\ \hat{R} \\ \hat{R}^* \end{bmatrix} \tag{163}$$

where

$$\begin{aligned} \Xi &= (1+\beta)(1-\alpha)(1+2\alpha(\eta-1)) > 0, \\ \Lambda &= 2\alpha-1 > 0. \end{aligned}$$

The first-order approximated solution for this system is

$$\hat{R} = -\frac{(1-\alpha)(1+2\alpha(\eta-1))}{1+4\alpha(1-\alpha)(\eta-1)}\hat{Y}_0^* \tag{164}$$

$$\hat{R}^* = -\frac{\alpha(1+2(1-\alpha)(\eta-1))}{1+4\alpha(1-\alpha)(\eta-1)}\hat{Y}_0^* \tag{165}$$

$$\hat{S}_0 = \frac{1+\beta(2\alpha(\eta-1)+1)+4\alpha(1-\alpha)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+4\alpha(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{166}$$

$$\hat{S}_1 = -\frac{2\alpha(2\alpha-1)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+4\alpha(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{167}$$

A.3 A Zero Bound in the Two-country Model without a Leverage Constraint

This section assumes that the zero bound is binding in the home economy so that the real interest rate is not allowed to move, *i.e.* $\hat{R} = 0$. This means that we have fixed one of the relative prices in the system exogenously. In order to solve this system, we have to have an additional degree of freedom. We choose this to be the level of demand for home goods in period 0, $\hat{Y}_0$.

The foreign country controls the terms of trade to ensure that the goods markets in the foreign country clear in both period 0 and 1.

The (log-linearised) market clearing conditions are:

$$\hat{Y}_0 = \frac{1-\alpha}{1+\beta}\hat{Y}_0^* - (1-\alpha)(1+2\alpha(\eta-1))\hat{S}_0 + \frac{\alpha}{1+\beta}\hat{Y}_0 + \frac{\alpha\beta}{1+\beta}\hat{Y}_1 \tag{168}$$

$$\begin{aligned}\left(1-\frac{\alpha}{1+\beta}\right)\hat{Y}_0^* &= \frac{1-\alpha}{1+\beta}\hat{Y}_0 - \frac{\beta}{1+\beta}(2\alpha-1)\hat{S}_1 \\ &\quad + \frac{1}{1+\beta}[(1+\beta)(1-\alpha)(1+2\alpha(\eta-1)) + \beta(2\alpha-1)]\hat{S}_0 + \frac{(1-\alpha)\beta}{1+\beta}\hat{Y}_1\end{aligned} \tag{169}$$

$$\hat{Y}_1 = \frac{1-\alpha}{1+\beta}\hat{Y}_0^* + \frac{\alpha}{1+\beta}\hat{Y}_0 - (1-\alpha)(1+2\alpha(\eta-1))\hat{S}_1 + \frac{\alpha\beta}{1+\beta}\hat{Y}_1 \tag{170}$$

$$\begin{aligned}0 &= \frac{1-\alpha}{1+\beta}\hat{Y}_0 + \frac{\alpha}{1+\beta}\hat{Y}_0^* - \frac{1}{1+\beta}(2\alpha-1)\hat{S}_0 \\ &\quad + \frac{1}{1+\beta}[(1+\beta)(1-\alpha)(1+2\alpha(\eta-1)) + (2\alpha-1)]\hat{S}_1 + \frac{(1-\alpha)\beta}{1+\beta}\hat{Y}_1\end{aligned} \tag{171}$$

We further assume that the demand for home goods in period 1 equals the fixed endowment, *i.e.* $\hat{Y}_1 = 0$. Then, as we will show, some country A goods in period 0 are not demanded. We assume that these goods are disposed costlessly. From the two foreign goods market clearing conditions, we solve for the percentage changes in the terms of trade necessary to clear the foreign goods markets:

$$\hat{S}_0 = -\frac{1-\alpha}{\Xi}\hat{Y}_0 + \frac{(1+\beta-\alpha)\Xi + (1-\alpha)(1+\beta)\Lambda}{\Xi(\Xi+(1+\beta)\Lambda)}\hat{Y}_0^* \tag{172}$$

$$\hat{S}_1 = -\frac{1-\alpha}{\Xi}\hat{Y}_0 - \frac{\alpha\Xi - (1-\alpha)(1+\beta)\Lambda}{\Xi(\Xi+(1+\beta)\Lambda)}\hat{Y}_0^* \tag{173}$$

A fall in home income reduces the consumption of foreign goods by home consumers, so the terms of trade in the foreign country have to worsen by more to make foreign goods more attractive.

We substitute the terms of trade into the home market clearing condition in period 0, Equation (168), to compute the demand for home goods in that period:

$$\hat{Y}_0 = \frac{1}{1+\beta}\hat{Y}_0 - \frac{\beta}{1+\beta} \times \frac{\Xi}{\Xi+(1+\beta)\Lambda}\hat{Y}_0^* \tag{174}$$

Therefore, there is a solution for the demand for home goods in period 0, $\hat{Y}_0$:

$$\hat{Y}_0 = -\frac{\Xi}{\Xi+(1+\beta)\Lambda}\hat{Y}_0^* \tag{175}$$

$$= -\frac{(1-\alpha)}{\alpha}\frac{(1+2\alpha(\eta-1))}{(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \tag{176}$$

The terms of trade are:

$$
\begin{aligned}
\hat{S}_0 &= \frac{(1+\beta-\Lambda)\Xi + (1-\alpha)(1+\beta)\Lambda}{\Xi(\Xi+(1+\beta)\Lambda)}\hat{Y}_0^* \\
&= \frac{1+\beta(1+2\alpha(\eta-1)) + 4\alpha(1-\alpha)(\eta-1)}{\alpha(1+\beta)(1+2\alpha(\eta-1))(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \qquad (177) \\
\hat{S}_1 &= \frac{-\Lambda\Xi + (1-\alpha)(1+\beta)\Lambda}{\Xi(\Xi+(1+\beta)\Lambda)}\hat{Y}_0^* \\
&= -\frac{2(2\alpha-1)(\eta-1)}{(1+\beta)(1+2\alpha(\eta-1))(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \qquad (178)
\end{aligned}
$$

And the foreign interest rate is

$$\hat{R}^* = (2\alpha-1)(\hat{S}_1 - \hat{S}_0) = -\frac{2\alpha-1}{\alpha(1+2(1-\alpha)(\eta-1))}\hat{Y}_0^* \qquad (179)$$

Finally, we verify, using Equation (170), that the demand for home goods in period 1 is indeed unchanged so that the home goods market in period 1 clears:

$$
\begin{aligned}
& \frac{1}{1+\beta}\left((1-\alpha)\hat{Y}_0^* + \alpha\hat{Y}_0 - \Xi\hat{S}_1\right) \qquad (180) \\
=\ & \frac{1}{1+\beta}\left((1-\alpha) - \frac{\alpha\Xi}{\Xi+(1+\beta)\Lambda} - \frac{-\Lambda\Xi+(1-\alpha)(1+\beta)\Lambda}{\Xi+(1+\beta)\Lambda}\right)\hat{Y}_0^* \qquad (181) \\
=\ & 0 \qquad (182)
\end{aligned}
$$

Therefore, the home goods market in period 1 clears.

B Appendix: A Three-counry Model Without a Leverage Constraint

B.1 The Model

In what follows, we set up the three-country model and examine the response of the model to a Chinese productivity shock.

Suppose there are three countries labeled $J = \{A, B, C\}$, which we will sometimes denote as the US, China and Europe. Denote C_t^J the consumption aggregate in country J in period t. For simplicity, we use a CES aggregator with home bias in consumption.

$$
\begin{aligned}
C_t^A &= \left(\alpha^{\frac{1}{\eta}}(C_{At}^A)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Bt}^A)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Ct}^A)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \qquad (183) \\
C_t^B &= \left((\alpha^*)^{\frac{1}{\eta}}(C_{At}^B)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{Bt}^B)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Ct}^B)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \qquad (184) \\
C_t^C &= \left((\alpha^*)^{\frac{1}{\eta}}(C_{At}^C)^{\frac{\eta-1}{\eta}} + (\alpha^*)^{\frac{1}{\eta}}(C_{Bt}^C)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{Ct}^C)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \qquad (185)
\end{aligned}
$$

Throughout the model, the superscripts of the consumption indexes denote the country of the consumer and the subscripts denote the country of the endowment. For instance, C_B^A denotes consumption of good B by the consumers of country A. $t = \{0, 1\}$ is the time subscript. α is the consumption home bias, and $\alpha \geq 1/3$. Consumption of foreign goods are spread equally between the two foreign countries for simplicity and

$$\alpha^* \equiv \frac{1-\alpha}{2} \tag{186}$$

Country J's demand for country I's good in period t is given by

$$\begin{aligned}
C_{At}^A &= \alpha \left(\frac{P_t^A}{P_{At}^A}\right)^\eta C_t^A, \qquad & C_{Bt}^A &= \alpha^* \left(\frac{P_t^A}{P_{Bt}^A}\right)^\eta C_t^A, \qquad & C_{Ct}^A &= \alpha^* \left(\frac{P_t^A}{P_{Ct}^A}\right)^\eta C_t^A, \\
C_{At}^B &= \alpha^* \left(\frac{P_t^B}{P_{At}^B}\right)^\eta C_t^B, \qquad & C_{Bt}^B &= \alpha \left(\frac{P_t^B}{P_{Bt}^B}\right)^\eta C_t^B, \qquad & C_{Ct}^B &= \alpha^* \left(\frac{P_t^B}{P_{Ct}^B}\right)^\eta C_t^B, \\
C_{At}^C &= \alpha^* \left(\frac{P_t^C}{P_{At}^C}\right)^\eta C_t^C, \qquad & C_{Bt}^C &= \alpha^* \left(\frac{P_t^C}{P_{Bt}^C}\right)^\eta C_t^C, \qquad & C_{Ct}^C &= \alpha \left(\frac{P_t^C}{P_{Ct}^C}\right)^\eta C_t^C.
\end{aligned} \tag{187}$$

where P_{It}^J is country J's price of country I's good in period t and P_t^J is the aggregate price level in country J, defined by

$$P_t^A = \left(\alpha (P_{At}^A)^{1-\eta} + \alpha^* (P_{Bt}^A)^{1-\eta} + \alpha^* (P_{Ct}^A)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{188}$$

$$P_t^B = \left(\alpha^* (P_{At}^B)^{1-\eta} + \alpha (P_{Bt}^B)^{1-\eta} + \alpha^* (P_{Ct}^B)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{189}$$

$$P_t^C = \left(\alpha^* (P_{At}^C)^{1-\eta} + \alpha^* (P_{Bt}^C)^{1-\eta} + \alpha (P_{Ct}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{190}$$

We assume that the law of one price holds. We define three nominal exchange rates e_{BA}, e_{CA} and e_{CB} as follows:

$$e_{BAt} = \frac{P_{Jt}^B}{P_{Jt}^A}, \qquad \text{where } J = \{A, B, C\} \tag{191}$$

$$e_{CAt} = \frac{P_{Jt}^C}{P_{Jt}^A}, \qquad \text{where } J = \{A, B, C\} \tag{192}$$

$$e_{CBt} = \frac{P_{Jt}^C}{P_{Jt}^B}, \qquad \text{where } J = \{A, B, C\} \tag{193}$$

Clearly,

$$e_{CAt} = e_{BAt} \times e_{CBt} \tag{194}$$

We also define the terms of trade as follows:

$$S_{It}^J = \frac{P_{It}^J}{P_{Jt}^J} \tag{195}$$

Then,

$$S_{It}^J = \frac{P_{It}^J}{P_{Jt}^J} = \frac{P_{It}^I}{e_{IJt}} \times \frac{e_{IJt}}{P_{Jt}^I} = \frac{P_{It}^I}{P_{Jt}^I} = \frac{1}{S_{Jt}^I}, \qquad \text{for } I, J = \{A, B, C\} \tag{196}$$

Also,

$$S_{Bt}^{C}=\frac{P_{Bt}^{C}}{P_{Ct}^{C}}=\frac{P_{Bt}^{A}e_{CAt}}{P_{Ct}^{A}e_{CAt}}=\frac{P_{Bt}^{A}}{P_{Ct}^{A}}=\frac{P_{Bt}^{A}}{P_{At}^{A}}\times\frac{P_{At}^{A}}{P_{Ct}^{A}}=S_{Bt}^{A}\times\frac{1}{S_{Ct}^{A}}=S_{At}^{C}\times\frac{1}{S_{At}^{B}} \tag{197}$$

Then, the relative prices can be expressed in terms of the terms of trade. For country A,

$$\begin{aligned}
\frac{P_t^A}{P_{At}^A} &= \left(\alpha+\alpha^*(S_{At}^B)^{-(1-\eta)}+\alpha^*(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (198)\\
\frac{P_t^A}{P_{Bt}^A} &= \left(\alpha^*+\alpha(S_{At}^B)^{1-\eta}+\alpha^*(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (199)\\
\frac{P_t^A}{P_{Ct}^A} &= \left(\alpha^*+\alpha(S_{At}^C)^{1-\eta}+\alpha^*(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}} && (200)
\end{aligned}$$

For country B,

$$\begin{aligned}
\frac{P_t^B}{P_{At}^B} &= \left(\alpha^*+\alpha(S_{At}^B)^{-(1-\eta)}+\alpha^*(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (201)\\
\frac{P_t^B}{P_{Bt}^B} &= \left(\alpha+\alpha^*(S_{At}^B)^{1-\eta}+\alpha^*(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (202)\\
\frac{P_t^B}{P_{Ct}^B} &= \left(\alpha^*+\alpha^*(S_{At}^C)^{1-\eta}+\alpha(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}} && (203)
\end{aligned}$$

For country C,

$$\begin{aligned}
\frac{P_t^C}{P_{At}^C} &= \left(\alpha^*+\alpha^*(S_{At}^B)^{-(1-\eta)}+\alpha(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (204)\\
\frac{P_t^C}{P_{Bt}^C} &= \left(\alpha^*+\alpha^*(S_{At}^B)^{1-\eta}+\alpha(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} && (205)\\
\frac{P_t^C}{P_{Ct}^C} &= \left(\alpha+\alpha^*(S_{At}^C)^{1-\eta}+\alpha^*(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{1}{1-\eta}} && (206)
\end{aligned}$$

Substituting these equations into the consumption demands, we get

$$\begin{aligned}
C_{At}^A &= \alpha\left(\alpha+\alpha^*(S_{At}^B)^{-(1-\eta)}+\alpha^*(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^A\\
C_{Bt}^A &= \alpha^*\left(\alpha^*+\alpha(S_{At}^B)^{1-\eta}+\alpha^*(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^A\\
C_{Ct}^A &= \alpha^*\left(\alpha^*+\alpha(S_{At}^C)^{1-\eta}+\alpha^*(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}C_t^A\\
C_{At}^B &= \alpha^*\left(\alpha^*+\alpha(S_{At}^B)^{-(1-\eta)}+\alpha^*(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^B\\
C_{Bt}^B &= \alpha\left(\alpha+\alpha^*(S_{At}^B)^{1-\eta}+\alpha^*(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^B\\
C_{Ct}^B &= \alpha^*\left(\alpha^*+\alpha^*(S_{At}^C)^{1-\eta}+\alpha(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}C_t^B\\
C_{At}^C &= \alpha^*\left(\alpha^*+\alpha^*(S_{At}^B)^{-(1-\eta)}+\alpha(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^C\\
C_{Bt}^C &= \alpha^*\left(\alpha^*+\alpha^*(S_{At}^B)^{1-\eta}+\alpha(S_{At}^B)^{1-\eta}(S_{At}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}C_t^C\\
C_{Ct}^C &= \alpha\left(\alpha+\alpha^*(S_{At}^C)^{1-\eta}+\alpha^*(S_{At}^B)^{-(1-\eta)}(S_{At}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}C_t^C && (207)
\end{aligned}$$

We let Y_t^A, Y_t^B and Y_t^C denote the exogenous endowments in the three countries in period t. As a result, the goods market clearing conditions are:

$$Y_0^A = C_{A0}^A + C_{A0}^B + C_{A0}^C, \qquad Y_1^A = C_{A1}^A + C_{A1}^B + C_{A1}^C \tag{208}$$
$$Y_0^B = C_{B0}^A + C_{B0}^B + C_{B0}^C, \qquad Y_1^B = C_{B1}^A + C_{B1}^B + C_{B1}^C \tag{209}$$
$$Y_0^C = C_{C0}^A + C_{C0}^B + C_{C0}^C, \qquad Y_1^C = C_{C1}^A + C_{C1}^B + C_{C1}^C \tag{210}$$

We consider country A's budget constraint first:

$$\begin{aligned}
P_0^A D^A &= P_0^A C_0^A - P_{A0}^A C_{A0}^A - P_{A0}^B C_{A0}^B e_{BA0}^{-1} - P_{A0}^C C_{A0}^C e_{CA0}^{-1} && (211)\\
&= P_0^A C_0^A - P_{A0}^A (C_{A0}^A + C_{A0}^B + C_{A0}^C) && (212)\\
&= P_0^A C_0^A - P_{A0}^A Y_0^A && (213)
\end{aligned}$$

where D^A denotes real debt in country A.

In period 1, the budget constraint for country A is

$$\begin{aligned}
0 &= P_1^A C_1^A + P_1^A D^A R^A - P_{A1}^A C_{A1}^A - P_{A1}^B C_{A1}^B e_{BA1}^{-1} - P_{A1}^C C_{A1}^C e_{CA1}^{-1} && (214)\\
&= P_1^A C_1^A + P_1^A D^A R^A - P_{A1}^A Y_1^A && (215)
\end{aligned}$$

Combing the two period budget constraints, we obtain:

$$C_1^A = \frac{P_{A0}^A}{P_0^A} R^A Y_0^A + \frac{P_{A1}^A}{P_1^A} Y_1^A - R^A C_0^A \tag{216}$$

where R^A is the gross real interest rate in country A. This is the intertemporal budget constraint for country A. It can be expressed in terms of the terms of trade.

$$\begin{aligned}
C_1^A &= \left(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} R^A Y_0^A \\
&\quad + \left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1^A - R^A C_0^A && (217)
\end{aligned}$$

We assume that country A consumers have perfect foresight and a logarithmic utility function of the following form:

$$U_0^A = \ln C_0^A + \beta \ln C_1^A \tag{218}$$

Taking their endowment as given, country A consumers maximise utility subject to their budget constraint. This gives the consumption Euler equation for country A consumers:

$$C_1^A = \beta R^A C_0^A \tag{219}$$

We now substitute the Euler equation into the budget constraint to solve explicitly consumption demands in the two periods:

$$\begin{aligned}
C_0^A &= \frac{1}{(1+\beta)} \left(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0^A \\
&\quad + \frac{1}{(1+\beta)R^A} \left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1^A && (220)\\
C_1^A &= \frac{\beta R^A}{(1+\beta)} \left(\alpha + \alpha^*(S_{A0}^B)^{-(1-\eta)} + \alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_0^A \\
&\quad + \frac{\beta}{(1+\beta)} \left(\alpha + \alpha^*(S_{A1}^B)^{-(1-\eta)} + \alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} Y_1^A && (221)
\end{aligned}$$

The intertemporal choice for country B and C consumers are derived similarly. We obtain:

$$\begin{aligned} C_0^B &= \frac{1}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_0^B \\ &\quad+\frac{1}{(1+\beta)R^B}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_1^B \qquad (222) \\ C_1^B &= \frac{\beta R^B}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_0^B \\ &\quad+\frac{\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}Y_1^B \qquad (223) \\ C_0^C &= \frac{1}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_0^C \\ &\quad+\frac{1}{(1+\beta)R^C}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^C \qquad (224) \\ C_1^C &= \frac{\beta R^C}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_0^C \\ &\quad+\frac{\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^C \qquad (225) \end{aligned}$$

The demands for each country's goods can be found by Equations (207).

The debt market must clear in both periods. This implies the uncovered interest parity holds between any pairs of the three countries. We rearrange the budget constraints in period 0. For country A:

$$\begin{aligned} P_0^A D^A &= P_0^A C_0^A - P_{A0}^A Y_0^A \qquad (226) \\ &= P_{A0}^A C_{A0}^A + P_{B0}^A C_{B0}^A + P_{C0}^A C_{C0}^A - P_{A0}^A (C_{A0}^A + C_{A0}^B + C_{A0}^C) \qquad (227) \\ &= P_{B0}^A C_{B0}^A + P_{C0}^A C_{C0}^A - P_{A0}^A C_{A0}^B - P_{A0}^A C_{A0}^C \qquad (228) \end{aligned}$$

Similarly, the period 0 budget constraints for country B and C are

$$\begin{aligned} \frac{P_0^B}{e_{BA0}} D^B &= P_{A0}^A C_{A0}^B + P_{C0}^A C_{C0}^B - P_{B0}^A C_{B0}^A - P_{B0}^A C_{B0}^C \qquad (229) \\ \frac{P_0^C}{e_{CA0}} D^C &= P_{A0}^A C_{A0}^C + P_{B0}^A C_{B0}^C - P_{C0}^A C_{C0}^A - P_{C0}^A C_{C0}^B \qquad (230) \end{aligned}$$

Adding up the last three equations, we get the debt market clearing condition for period 0:

$$P_0^A D^A + \frac{P_0^B}{e_{BA0}} D^B + \frac{P_0^C}{e_{CA0}} D^C = 0 \qquad (231)$$

We also rearrange the budget constraints in period 1. For country A:

$$\begin{aligned} 0 &= P_1^A C_1^A - P_{A1}^A Y_1^A + R^A P_1^A D^A \qquad (232) \\ &= P_{A1}^A C_{A1}^A + P_{B1}^A C_{B1}^A + P_{C1}^A C_{C1}^A - P_{A1}^A (C_{A1}^A + C_{B1}^A + C_{C1}^A) + R^A P_1^A D^A \qquad (233) \\ &= P_{B1}^A C_{B1}^A + P_{C1}^A C_{C1}^A - P_{A1}^A C_{B1}^A - P_{A1}^A C_{C1}^A + R^A P_1^A D^A \qquad (234) \end{aligned}$$

Similarly, for country B and C:

$$
\begin{aligned}
0 &= P^A_{A1}C^B_{A1} + P^A_{C1}C^B_{C1} - P^A_{B1}C^A_{B1} - P^A_{B1}C^C_{B1} + \frac{R^B P^B_1}{e_{BA1}} D^B && (235) \\
0 &= P^A_{A1}C^C_{A1} + P^A_{B1}C^C_{B1} - P^A_{C1}C^A_{C1} - P^A_{C1}C^B_{C1} + \frac{R^C P^C_1}{e_{CA1}} D^C && (236)
\end{aligned}
$$

Adding up the last three equations, we obtain the market clearing condition in period 1:

$$
R^A P^A_1 D^A + \frac{R^B P^B_1}{e_{BA1}} D^B + \frac{R^C P^C_1}{e_{CA1}} D^C = 0 \tag{237}
$$

From the two debt market clearing conditions, it is clear that, for any values of D^B and D^C, the following two equations must hold:

$$
\begin{aligned}
D^A &= -\frac{P^B_0}{P^A_0 e_{BA0}} D^B - \frac{P^C_0}{P^A_0 e_{CA0}} D^C && (238) \\
0 &= \left(R^B \frac{P^B_1}{e_{BA1}} - R^A \frac{P^A_1 P^B_0}{P^A_0 e_{BA0}} \right) D^B + \left(R^C \frac{P^C_1}{e_{CA1}} - R^A \frac{P^A_1 P^C_0}{P^A_0 e_{BA0}} \right) D^C && (239)
\end{aligned}
$$

This implies

$$
\begin{aligned}
\frac{R^B}{R^A} &= \left(\frac{e_{BA1} P^A_1}{P^B_1} \right) / \left(\frac{e_{BA0} P^A_0}{P^B_0} \right) && (240) \\
\frac{R^C}{R^A} &= \left(\frac{e_{CA1} P^A_1}{P^C_1} \right) / \left(\frac{e_{CA0} P^A_0}{P^C_0} \right) && (241)
\end{aligned}
$$

where $e_{BAt}P^A/P^B$ and $e_{CAt}P^A/P^C$ are the real exchange rates for country B and country C. Hence, the uncovered interest parity holds in this system.

We may further express the real exchange rates in terms of the terms of trade:

$$
\begin{aligned}
\frac{e_{BAt} P^A_t}{P^B_t} &= \frac{P^A_t / P^A_{At}}{P^B_t / P^B_{At}} = \frac{\left(\alpha + \alpha^* (S^B_{At})^{-(1-\eta)} + \alpha^* (S^C_{At})^{-(1-\eta)} \right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^* (S^B_{At})^{1-\eta} + \alpha^* (S^B_{At})^{1-\eta} (S^C_{At})^{-(1-\eta)} \right)^{\frac{1}{1-\eta}}} S^B_{At} && (242) \\
\frac{e_{CAt} P^A_t}{P^C_t} &= \frac{P^A_t / P^A_{At}}{P^C_t / P^C_{At}} = \frac{\left(\alpha + \alpha^* (S^B_{At})^{-(1-\eta)} + \alpha^* (S^C_{At})^{-(1-\eta)} \right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^* (S^C_{At})^{1-\eta} + \alpha^* (S^B_{At})^{-(1-\eta)} (S^C_{At})^{1-\eta} \right)^{\frac{1}{1-\eta}}} S^C_{At} && (243)
\end{aligned}
$$

The uncovered interest parities can also be expressed in terms of the terms of trade as follows:

$$
\begin{aligned}
\frac{R^B}{R^A} &= \frac{S^B_{A1}}{S^B_{A0}} \frac{\left(\alpha + \alpha^*(S^B_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{1-\eta}(S^C_{A0})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^*(S^B_{A0})^{-(1-\eta)} + \alpha^*(S^C_{A0})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}} \\
&\quad \times \frac{\left(\alpha + \alpha^*(S^B_{A1})^{-(1-\eta)} + \alpha^*(S^C_{A1})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^*(S^B_{A1})^{1-\eta} + \alpha^*(S^B_{A1})^{1-\eta}(S^C_{A1})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}} \qquad (244) \\
\frac{R^C}{R^A} &= \frac{S^C_{A1}}{S^C_{A0}} \frac{\left(\alpha + \alpha^*(S^C_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{-(1-\eta)}(S^C_{A0})^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^*(S^B_{A0})^{-(1-\eta)} + \alpha^*(S^C_{A0})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}} \\
&\quad \times \frac{\left(\alpha + \alpha^*(S^B_{A1})^{-(1-\eta)} + \alpha^*(S^C_{A1})^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + \alpha^*(S^C_{A1})^{1-\eta} + \alpha^*(S^B_{A1})^{-(1-\eta)}(S^C_{A1})^{1-\eta}\right)^{\frac{1}{1-\eta}}} \qquad (245)
\end{aligned}
$$

The uncovered interest parities ensure that the debt markets clear.

B.2 The Full System

The full system contains the six goods market clearing conditions and two debt market clearing conditions. The debt market clearing conditions are the uncovered interest parities, Equations (57) and (58). The goods market clearing conditions are:

$$
\begin{aligned}
Y_0^A &= \frac{\alpha}{1+\beta}\left(\alpha + \alpha^*(S^B_{A0})^{-(1-\eta)} + \alpha^*(S^C_{A0})^{-(1-\eta)}\right)^{-1} Y_0^A \\
&\quad + \frac{\alpha}{(1+\beta)R^A}\left[\begin{array}{c} \left(\alpha + \alpha^*(S^B_{A0})^{-(1-\eta)} + \alpha^*(S^C_{A0})^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha + \alpha^*(S^B_{A1})^{-(1-\eta)} + \alpha^*(S^C_{A1})^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} \end{array}\right] Y_1^A \\
&\quad + \frac{\alpha^*}{(1+\beta)}(S^B_{A0})^{-\eta}\left(\alpha + \alpha^*(S^B_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{1-\eta}(S^C_{A0})^{-(1-\eta)}\right)^{-1} Y_0^B \\
&\quad + \frac{\alpha^*}{(1+\beta)R^B}(S^B_{A0})^{-\eta}\left[\begin{array}{c} \left(\alpha + \alpha^*(S^B_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{1-\eta}(S^C_{A0})^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha + \alpha^*(S^B_{A1})^{1-\eta} + \alpha^*(S^B_{A1})^{1-\eta}(S^C_{A1})^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}} \end{array}\right] Y_1^B \\
&\quad + \frac{\alpha^*}{(1+\beta)}(S^C_{A0})^{-\eta}\left(\alpha + \alpha^*(S^C_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{-(1-\eta)}(S^C_{A0})^{1-\eta}\right)^{-1} Y_0^C \\
&\quad + \frac{\alpha^*}{(1+\beta)R^C}(S^C_{A0})^{-\eta}\left[\begin{array}{c} \left(\alpha + \alpha^*(S^C_{A0})^{1-\eta} + \alpha^*(S^B_{A0})^{-(1-\eta)}(S^C_{A0})^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha + \alpha^*(S^C_{A1})^{1-\eta} + \alpha^*(S^B_{A1})^{-(1-\eta)}(S^C_{A1})^{1-\eta}\right)^{-\frac{1}{1-\eta}} \end{array}\right] Y_1^C \qquad (246)
\end{aligned}
$$

$$
\begin{aligned}
Y_0^B \;=\; & \frac{\alpha^*}{1+\beta}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-1}(S_{A0}^B)^{\eta}Y_0^A \\
& +\frac{\alpha^*}{(1+\beta)R^A}(S_{A0}^B)^{\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^A \\
& +\frac{\alpha}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^B \\
& +\frac{\alpha}{(1+\beta)R^B}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^B \\
& +\frac{\alpha^*}{(1+\beta)}(S_{A0}^B)^{\eta}(S_{A0}^C)^{-\eta}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-1}Y_0^C \\
& +\frac{\alpha^*}{(1+\beta)R^C}(S_{A0}^B)^{\eta}(S_{A0}^C)^{-\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^C
\end{aligned}
\tag{247}
$$

$$
\begin{aligned}
Y_0^C \;=\; & \frac{\alpha^*}{1+\beta}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-1}(S_{A0}^C)^{\eta}Y_0^A \\
& +\frac{\alpha^*}{(1+\beta)R^A}(S_{A0}^C)^{\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^A \\
& +\frac{\alpha^*}{(1+\beta)}(S_{A0}^B)^{-\eta}(S_{A0}^C)^{\eta}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^B \\
& +\frac{\alpha^*}{(1+\beta)R^B}(S_{A0}^B)^{-\eta}(S_{A0}^C)^{\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^B \\
& +\frac{\alpha}{(1+\beta)}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-1}Y_0^C \\
& +\frac{\alpha}{(1+\beta)R^C}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^C
\end{aligned}
\tag{248}
$$

$$
\begin{aligned}
Y_1^A \;=\;& \frac{\alpha\beta R^A}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right]Y_0^A\\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}Y_1^A\\
&+\frac{\alpha^*\beta R^B}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{-\eta}Y_0^B\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{-\eta}Y_1^B\\
&+\frac{\alpha^*\beta R^C}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^C)^{-\eta}Y_0^C\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}(S_{A1}^C)^{-\eta}Y_1^C
\end{aligned}
\tag{249}
$$

$$
\begin{aligned}
Y_1^B \;=\;& \frac{\alpha^*\beta R^A}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{\eta}Y_0^A\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{\eta}Y_1^A\\
&+\frac{\alpha\beta R^B}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right]Y_0^B\\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}Y_1^B\\
&+\frac{\alpha^*\beta R^C}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{\eta}(S_{A1}^C)^{-\eta}Y_0^C\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}(S_{A1}^B)^{\eta}(S_{A1}^C)^{-\eta}Y_1^C
\end{aligned}
\tag{250}
$$

$$
\begin{aligned}
Y_1^C \;=\;& \frac{\alpha^*\beta R^A}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^C)^\eta Y_0^A\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^C)^\eta Y_1^A\\
&+\frac{\alpha^*\beta R^B}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{-\eta}(S_{A1}^C)^\eta Y_0^B\\
&+\frac{\alpha^*\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{-\eta}(S_{A1}^C)^\eta Y_1^B\\
&+\frac{\alpha\beta R^C}{(1+\beta)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right]Y_0^C\\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}Y_1^C
\end{aligned}
\tag{251}
$$

By Walras' Law, one of the eight market clearing conditions is redundant. We have seven equations to solve for the seven relative prices $\{R^A, R^B, R^C, S_{A0}^B, S_{A0}^C, S_{A1}^B, S_{A1}^C\}$.

For $\alpha > 1/3$, we log-linearise the system around the symmetric steady state in which all endowments are unity, the terms of trade are unity and the real interest rates are $1/\beta$. We consider a rise in endowment in country B in period 0, Y_0^B.

The system can be written more simply in matrix form:

$$
\begin{bmatrix}\alpha^*\\ \alpha-(1+\beta)\\ \alpha^*\\ -\alpha^*\\ -\alpha\\ -\alpha^*\\ 0\\ 0\end{bmatrix}\hat{Y}_0^B=
\begin{bmatrix}
\Omega & \Omega & 0 & 0 & \beta & 0 & 0\\
-2\Omega-\beta\Psi & \Omega & \beta\Psi & 0 & \beta & 0 & 0\\
\Omega & -2\Omega-\beta\Psi & 0 & \beta\Psi & \beta & 0 & 0\\
0 & 0 & -\Omega & -\Omega & 1 & 0 & 0\\
-\Psi & 0 & 2\Omega+\Psi & -\Omega & 1 & 0 & 0\\
0 & -\Psi & -\Omega & 2\Omega+\Psi & 1 & 0 & 0\\
-\Psi & 0 & \Psi & 0 & 1 & -1 & 0\\
0 & -\Psi & 0 & \Psi & 1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}\hat{S}_{A0}^B\\ \hat{S}_{A0}^C\\ \hat{S}_{A1}^B\\ \hat{S}_{A1}^C\\ \hat{R}^A\\ \hat{R}^B\\ \hat{R}^C\end{bmatrix}
\tag{252}
$$

where

$$
\begin{aligned}
\Omega &= \alpha^*(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*)) > 0,\\
\Psi &= (1-3\alpha^*) > 0.
\end{aligned}
$$

The solution is

$$\hat{R}^A = R^C = -\frac{\Omega}{3\Omega + (1+\beta)\Psi}\hat{Y}_0^B \tag{253}$$

$$= -\frac{\alpha^*(1+(\eta-1)(1+\alpha-\alpha^*))}{1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{254}$$

$$\hat{R}^B = -\frac{\Omega + (1+\beta)\Psi}{3\Omega + (1+\beta)\Psi}\hat{Y}_0^B \tag{255}$$

$$= -\frac{(1-3\alpha^*) + \alpha^*(1+(\eta-1)(1+\alpha-\alpha^*))}{1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*)}\hat{Y}_0^B \tag{256}$$

$$\hat{S}_{A0}^B = \frac{(3\alpha^* + \beta)\Omega + \alpha^*(1+\beta)\Psi}{\Omega(3\Omega + (1+\beta)\Psi)}\hat{Y}_0^B \tag{257}$$

$$= \frac{(1+\beta) + (\beta + 3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{258}$$

$$\hat{S}_{A1}^B = -\frac{\Psi(\Omega - \alpha^*(1+\beta))}{\Omega(3\Omega + (1+\beta)\Psi)}\hat{Y}_0^B \tag{259}$$

$$= -\frac{(1-3\alpha^*)(\eta-1)(1+\alpha-\alpha^*)}{(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*))(1+3\alpha^*(\eta-1)(1+\alpha-\alpha^*))}\hat{Y}_0^B \tag{260}$$

$$\hat{S}_{A0}^C = \hat{S}_{A1}^C = 0 \tag{261}$$

And the debts are

$$D^A = D^C = \frac{\beta}{(1+\beta)}\frac{(\Omega - \alpha^*(1+\beta))}{\Omega(3\Omega + (1+\beta)\Psi)}\hat{Y}_0^B \tag{262}$$

$$D^B = -\frac{2\beta}{(1+\beta)}\frac{(\Omega - \alpha^*(1+\beta))}{\Omega(3\Omega + (1+\beta)\Psi)}\hat{Y}_0^B \tag{263}$$

The solutions of the three-country model are summarised in Table 1.

B.3 A Zero Bound in the Three-country model

We continue to consider a Chinese productivity shock in this section. But now we suppose that the zero bound is binding in the US.

Now, again suppose that the zero bound is binding in country A, or the US, so that the real interest rate is not allowed to move, *i.e.* $\hat{R}^A = 0$, and that the shock is not large enough to make the other two interest rates bind. We study what happens when the endowment in country B (China) in period 0 rises. The market clearing conditions are:

$$\begin{aligned}
\hat{Y}_0^A &= \frac{\alpha}{1+\beta}\hat{Y}_0^A + \frac{\alpha^*}{1+\beta}\hat{Y}_0^B + \frac{\alpha^*}{1+\beta}\hat{Y}_0^C + \frac{\alpha\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^B \\
&\quad + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^C - \frac{\Omega}{1+\beta}\hat{S}_{A0}^B - \frac{\Omega}{1+\beta}\hat{S}_{A0}^C \qquad (264) \\
\hat{Y}_0^B &= \frac{\alpha^*}{1+\beta}\hat{Y}_0^A + \frac{\alpha}{1+\beta}\hat{Y}_0^B + \frac{\alpha^*}{1+\beta}\hat{Y}_0^C + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha\beta}{1+\beta}\hat{Y}_1^B \\
&\quad + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^C + \frac{2\Omega+\beta\Psi}{1+\beta}\hat{S}_{A0}^B - \frac{\Omega}{1+\beta}\hat{S}_{A0}^C - \frac{\beta\Psi}{1+\beta}\hat{S}_{A1}^B \qquad (265) \\
\hat{Y}_0^C &= \frac{\alpha^*}{1+\beta}\hat{Y}_0^A + \frac{\alpha^*}{1+\beta}\hat{Y}_0^B + \frac{\alpha}{1+\beta}\hat{Y}_0^C + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^B \\
&\quad + \frac{\alpha\beta}{1+\beta}\hat{Y}_1^C - \frac{\Omega}{1+\beta}\hat{S}_{A0}^B + \frac{2\Omega+\beta\Psi}{1+\beta}\hat{S}_{A0}^C - \frac{\beta\Psi}{1+\beta}\hat{S}_{A1}^C \qquad (266) \\
\hat{Y}_1^A &= \frac{\alpha}{1+\beta}\hat{Y}_0^A + \frac{\alpha^*}{1+\beta}\hat{Y}_0^B + \frac{\alpha^*}{1+\beta}\hat{Y}_0^C + \frac{\alpha\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^B \\
&\quad + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^C - \frac{\Omega}{1+\beta}\hat{S}_{A1}^B - \frac{\Omega}{1+\beta}\hat{S}_{A1}^C \qquad (267) \\
\hat{Y}_1^B &= \frac{\alpha^*}{1+\beta}\hat{Y}_0^A + \frac{\alpha}{1+\beta}\hat{Y}_0^B + \frac{\alpha^*}{1+\beta}\hat{Y}_0^C + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha}{1+\beta}\hat{Y}_0^B \\
&\quad + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^C \frac{\Psi}{1+\beta}\hat{S}_{A0}^B + \frac{2\Omega+\Psi}{1+\beta}\hat{S}_{A1}^B - \frac{\Omega}{1+\beta}\hat{S}_{A1}^C \qquad (268) \\
\hat{Y}_1^C &= \frac{\alpha^*}{1+\beta}\hat{Y}_0^A + \frac{\alpha^*}{1+\beta}\hat{Y}_0^B + \frac{\alpha}{1+\beta}\hat{Y}_0^C + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^A + \frac{\alpha^*\beta}{1+\beta}\hat{Y}_1^B \\
&\quad + \frac{\alpha\beta}{1+\beta}\hat{Y}_1^C - \frac{\Psi}{1+\beta}\hat{S}_{A0}^C - \frac{\Omega}{1+\beta}\hat{S}_{A1}^B + \frac{2\Omega+\Psi}{1+\beta}\hat{S}_{A1}^C \qquad (269)
\end{aligned}$$

We assume that China controls the terms of trade against the US, that is $\hat{S}_{A0}^B$ and $\hat{S}_{A1}^B$ to ensure that the goods markets in China clear in each of the two periods. From the two market clearing conditions for country B, we compute the change in terms of trade that is necessary to achieve this:

$$\begin{aligned}
\hat{S}_{A0}^B &= -\frac{\alpha^*}{2\Omega}(\hat{Y}_0^A + \hat{Y}_0^C) - \frac{\alpha^*\beta}{2\Omega}(\hat{Y}_1^A + \hat{Y}_1^C) + \frac{2\Omega+\Psi}{2(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C \\
&\quad + \frac{\beta\Psi}{2(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C + \frac{\beta\Omega+\alpha^*(2\Omega+(1+\beta)\Psi)}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (270) \\
\hat{S}_{A1}^B &= -\frac{\alpha^*}{2\Omega}(\hat{Y}_0^A + \hat{Y}_0^C) - \frac{\alpha^*\beta}{2\Omega}(\hat{Y}_1^A + \hat{Y}_1^C) + \frac{\Psi}{2(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C \\
&\quad + \frac{2\Omega+\beta\Psi}{2(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C + \frac{-\Omega+\alpha^*(2\Omega+(1+\beta)\Psi)}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (271)
\end{aligned}$$

We substitute these terms of trade into the period 0 market clearing conditions in country A and country C:

$$
\begin{aligned}
\hat{Y}_0^A &= \frac{2\alpha+\alpha^*}{2(1+\beta)}\hat{Y}_0^A + \frac{3\alpha^*}{2(1+\beta)}\hat{Y}_0^C + \frac{(2\alpha+\alpha^*)\beta}{2(1+\beta)}\hat{Y}_1^A + \frac{3\alpha^*\beta}{2(1+\beta)}\hat{Y}_1^C \\
&\quad -\frac{\Omega[2(2\Omega+(1+\beta)\Psi)+2\Omega+\Psi]}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C - \frac{\beta\Psi\Omega}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C \\
&\quad -\frac{\beta\Omega}{(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (272) \\
\hat{Y}_0^C &= \frac{3\alpha^*}{2(1+\beta)}\hat{Y}_0^A + \frac{2\alpha+\alpha^*}{2(1+\beta)}\hat{Y}_0^C + \frac{3\alpha^*\beta}{2(1+\beta)}\hat{Y}_1^A + \frac{(2\alpha+\alpha^*)\beta}{2(1+\beta)}\hat{Y}_1^C \\
&\quad +\frac{2(2\Omega+(1+\beta)\Psi)(2\Omega+\beta\Psi)-\Omega(2\Omega+\Psi)}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C - \frac{\beta\Psi[\Omega+2(2\Omega+(1+\beta)\Psi)]}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C \\
&\quad -\frac{\beta\Omega}{(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (273) \\
\hat{Y}_1^A &= \frac{2\alpha+\alpha^*}{2(1+\beta)}\hat{Y}_0^A + \frac{3\alpha^*}{2(1+\beta)}\hat{Y}_0^C + \frac{(2\alpha+\alpha^*)\beta}{2(1+\beta)}\hat{Y}_1^A + \frac{3\alpha^*\beta}{2(1+\beta)}\hat{Y}_1^C \\
&\quad -\frac{\Omega\Psi}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C - \frac{\Omega[2(2\Omega+(1+\beta)\Psi)+2\Omega+\beta\Psi]}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C \\
&\quad +\frac{\Omega}{(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (274) \\
\hat{Y}_1^C &= \frac{3\alpha^*}{2(1+\beta)}\hat{Y}_0^A + \frac{2\alpha+\alpha^*}{2(1+\beta)}\hat{Y}_0^C + \frac{3\alpha^*\beta}{2(1+\beta)}\hat{Y}_1^A + \frac{(2\alpha+\alpha^*)\beta}{2(1+\beta)}\hat{Y}_1^C \\
&\quad -\frac{\Psi[2(2\Omega+(1+\beta)\Psi)+\Omega]}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A0}^C + \frac{2(2\Omega+\Psi)(2\Omega+(1+\beta)\Psi)-\Omega(2\Omega+\beta\Psi)}{2(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{S}_{A1}^C \\
&\quad +\frac{\Omega}{(1+\beta)(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \qquad (275)
\end{aligned}
$$

B.3.1 Case 1: Country A and Country C have the same shortfall in demand

In this subsection, we assume that the demands for these two countries' goods fall by equal proportion such that $\hat{Y}_0^A = \hat{Y}_0^C$. We will also impose the assumption that the demands for these two countries' goods in period 1 equals the fixed endowment, *i.e.* $\hat{Y}_1^A = \hat{Y}_1^C = 0$. This implies the terms of trade between these two countries must be

$$
\begin{aligned}
\hat{S}_{A0}^C &= \frac{2(3\Omega+\Psi)}{3\Omega\Psi(3\Omega+(1+\beta)\Psi)}[(2\Omega+(1+\beta)\Psi)\hat{Y}_0^A+\Omega\hat{Y}_0^B] \qquad (276) \\
\hat{S}_{A1}^C &= \frac{2}{3\Omega(3\Omega+(1+\beta)\Psi)}[(2\Omega+(1+\beta)\Psi)\hat{Y}_0^A+\Omega\hat{Y}_0^B] \qquad (277)
\end{aligned}
$$

We substitute these terms of trade into the market clearing conditions of country A

and C:

$$\hat{Y}_0^A = -\frac{2\Omega}{(1+\beta)\Psi}\hat{Y}_0^A - \frac{\Omega}{(1+\beta)\Psi}\hat{Y}_0^B \tag{278}$$

$$\hat{Y}_0^C = \frac{2(\Omega+(1+\beta)\Psi)}{(1+\beta)\Psi}\hat{Y}_0^C + \frac{\Omega}{(1+\beta)\Psi}\hat{Y}_0^B \tag{279}$$

This enables us to compute the common output in the two countries:

$$\hat{Y}_0^A = \hat{Y}_0^C = -\frac{\Omega}{2\Omega+(1+\beta)\Psi}\hat{Y}_0^B \tag{280}$$

The terms of trade between country A and C are unchanged:

$$\hat{S}_{A0}^C = \hat{S}_{A1}^C = 0 \tag{281}$$

The terms of trade between country A and B are

$$\hat{S}_{A0}^B = \frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \tag{282}$$

$$\hat{S}_{A1}^B = -\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \tag{283}$$

B.3.2 Case 2: Country C controls bilateral terms of trade

In this subsection, we assume country C controls $\hat{S}_{A0}^C$ and $\hat{S}_{A1}^C$ to keep the demand for its goods in each of the two periods unchanged, that is $\hat{Y}_0^C = \hat{Y}_1^C = 0$. From the market clearing conditions of country C, we compute the terms of trade between these two countries necessary to achieve this:

$$\hat{S}_{A0}^C = -\frac{\alpha^*}{\Omega}\hat{Y}_0^A - \frac{\alpha^*\beta}{\Omega}\hat{Y}_1^A + \frac{\beta\Omega}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \tag{284}$$

$$\hat{S}_{A1}^C = -\frac{\alpha^*}{\Omega}\hat{Y}_0^A - \frac{\alpha^*\beta}{\Omega}\hat{Y}_1^A - \frac{\Omega}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \tag{285}$$

We now further assume that the demand for country A's goods in period 1 equals the exogenous supply, *i.e.* $\hat{Y}_1^A = 0$. Substituting these two terms of trade into the country A market clearing condition in period 0, we obtain

$$\hat{Y}_0^A = \frac{1}{1+\beta}\hat{Y}_0^A - \frac{\beta}{(1+\beta)}\frac{\Omega}{(\Omega+(1+\beta)\Psi)}\hat{Y}_0^B \tag{286}$$

The marginal propensity to consumption is $1/(1+\beta)$. The demand for country A goods in period 0 in this case is

$$\hat{Y}_0^A = -\frac{\Omega}{\Omega+(1+\beta)\Psi}\hat{Y}_0^B \tag{287}$$

The terms of trade between country A and country C are

$$\hat{S}^B_{A0} = \frac{(2\Omega + (1+\beta)\Psi)[(3\alpha^* + \beta)\Omega + \alpha^*(1+\beta)\Psi]}{\Omega(\Omega + (1+\beta)\Psi)(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{288}$$

$$\hat{S}^B_{A1} = -\frac{(2\Omega + (1+\beta)\Psi)(\Omega - \alpha^*(1+\beta))\Psi}{\Omega(\Omega + (1+\beta)\Psi)(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{289}$$

$$\hat{S}^C_{A0} = \frac{(3\alpha^* + \beta)\Omega + \alpha^*(1+\beta)\Psi}{(\Omega + (1+\beta)\Psi)(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{290}$$

$$\hat{S}^C_{A1} = -\frac{\Psi(\Omega - \alpha^*(1+\beta))}{(\Omega + (1+\beta)\Psi)(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{291}$$

One can check from the period 1 market clearing condition for country A to verify that the goods market in period 1 clears.

B.3.3 Case 3: Country A controls bilateral terms of trade

In this section, we assume instead country A controls $\hat{S}^C_{A0}$ and $\hat{S}^C_{A1}$ to keep the demands for its goods in each of the two periods unchanged, that is $\hat{Y}^A_0 = \hat{Y}^C_0 = 0$. This case is asymmetrical to the previous case because only country A is at the zero bound.

We compute from the market clearing conditions of country A the terms of trade between these two countries necessary to keep the demand for country A's goods constant:

$$\hat{S}^C_{A0} = \frac{\alpha^*}{\Omega}\hat{Y}^C_0 + \frac{\alpha^*\beta}{\Omega}\hat{Y}^C_1 - \frac{\beta}{(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{292}$$

$$\hat{S}^C_{A1} = \frac{\alpha^*}{\Omega}\hat{Y}^C_0 + \frac{\alpha^*\beta}{\Omega}\hat{Y}^C_1 + \frac{1}{(3\Omega + (1+\beta)\Psi)}\hat{Y}^B_0 \tag{293}$$

As before, we impose the assumption that the demand for country C goods in period 1 equals the fixed endowment. Substituting the above expressions for the terms of trade into country C's market clearing condition in period 0, we obtain

$$\hat{Y}^C_0 = \frac{1}{1+\beta}\hat{Y}^C_0 - \frac{\beta}{1+\beta}\hat{Y}^B_0 \tag{294}$$

Since this market must clear in equilibrium, this implies

$$\hat{Y}^C_0 = -\hat{Y}^B_0 \tag{295}$$

The terms of trade and interest rates are:

$$\hat{S}^B_{A0} = \frac{(3\alpha+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0 \tag{296}$$

$$\hat{S}^B_{A1} = -\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0 \tag{297}$$

$$\hat{S}^C_{A0} = -\frac{(3\alpha+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0 \tag{298}$$

$$\hat{S}^C_{A1} = \frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0 \tag{299}$$

$$\hat{R}^B = -\frac{(1+\beta)\Psi}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0 \tag{300}$$

$$\hat{R}^C = \frac{(1+\beta)\Psi}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0 \tag{301}$$

C Appendix: Including a Leverage Constraint in the Model

C.1 The Full System of a Two-country Model with a Leverage Constraint

The full system of the two-country model with a leverage constraint contains the goods market clearing conditions in both the home and foreign countries, and for both periods, as well as the debt market clearing condition, Equation (18). The system is presented as follows:

$$
\begin{aligned}
Y_0 &= \alpha\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-1}Y_0+\alpha\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}D \\
&+\frac{1-\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-1}S_0^{-\eta}Y_0^* \\
&+\frac{1-\alpha}{(1+\beta)R^*}S_0^{-\eta}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^* \qquad (302)\\
Y_0^* &= (1-\alpha)\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{-1}S_0^{\eta}Y_0+(1-\alpha)\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}S_0^{\eta}D \\
&+\frac{\alpha}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-1}Y_0^* \\
&+\frac{\alpha}{(1+\beta)R^*}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-\frac{1}{1-\eta}}Y_1^* \qquad (303)\\
Y_1 &= \alpha\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-1}Y_1-\alpha\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}RD \\
&+\frac{(1-\alpha)\beta R^*}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{\eta}{1-\eta}}S_1^{-\eta}Y_0^* \\
&+\frac{(1-\alpha)\beta}{(1+\beta)}S_1^{-\eta}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-1}Y_1^* \qquad (304)\\
Y_1^* &= (1-\alpha)\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{-1}S_1^{\eta}Y_1-(1-\alpha)\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}S_1^{\eta}RD \\
&+\frac{\alpha\beta R^*}{(1+\beta)}\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{-\frac{1}{1-\eta}}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{\eta}{1-\eta}}Y_0^* \\
&+\frac{\alpha\beta}{(1+\beta)}\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{-1}Y_1^* \qquad (305)
\end{aligned}
$$

$$
R^* = R\frac{S_1}{S_0}\times\frac{\left(\alpha+(1-\alpha)S_0^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha+(1-\alpha)S_0^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}\times\frac{\left(\alpha+(1-\alpha)S_1^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\alpha+(1-\alpha)S_1^{1-\eta}\right)^{\frac{1}{1-\eta}}} \qquad (306)
$$

C.2 A Zero Bound in the Two-country System with a Leverage Constraint

In this section, suppose the home interest rate is at the zero bound, so it cannot fall further. There is no analytical solution for this system in general. Therefore, in the following, we study the special case in which the debt limit in the home country tends to zero and an analytical solution exists.

Suppose the endowments are $Y_0 = Y_0^* = Y_1 = Y_1^* = 1$, then the terms of trade are unity and the foreign interest rate R^* is $1/\beta$. The home interest rate is implied by the uncovered interest parity. Further suppose that the foreign economy controls the terms of trade, S_0 and S_1, to make sure that the foreign goods markets clear.

We proceed by log-linearising the market clearing conditions around the symmetric steady state.

$$
\begin{aligned}
\hat{Y}_0 &= \alpha \hat{Y}_0 + \frac{1-\alpha}{1+\beta}\hat{Y}_0^* - \frac{1}{1+\beta}[\Xi - (1-\alpha)\alpha\beta]\hat{S}_0 - \frac{\alpha\beta(1-\alpha)}{1+\beta}\hat{S}_1 && (307)\\
\left(1 - \frac{\alpha}{1+\beta}\right)\hat{Y}_0^* &= (1-\alpha)\hat{Y}_0 - \frac{\alpha^2\beta}{1+\beta}\hat{S}_1 + \frac{1}{1+\beta}(\Xi + \beta\alpha^2)\hat{S}_0 && (308)\\
\hat{Y}_1 &= \alpha \hat{Y}_1 + \frac{1-\alpha}{1+\beta}\hat{Y}_0^* - \frac{\alpha(1-\alpha)}{1+\beta}\hat{S}_0 - \frac{1}{1+\beta}[\Xi - \alpha(1-\alpha)]\hat{S}_1 && (309)\\
0 &= (1-\alpha)\hat{Y}_1 + \frac{\alpha}{1+\beta}\hat{Y}_0^* - \frac{\alpha^2}{1+\beta}\hat{S}_0 + \frac{1}{1+\beta}(\alpha^2 + \Xi)\hat{S}_1 && (310)
\end{aligned}
$$

We further assume that the demand for home goods in period 1 equals the fixed endowment, *i.e.* $\hat{Y}_1 = 0$. From the two foreign goods markets clearing conditions, we solve out the percentage changes in the terms of trade necessary to clear the foreign goods markets:

$$
\begin{aligned}
\hat{S}_0 &= -\frac{(1-\alpha)(1+\beta)(\alpha^2+\Xi)}{\Xi(\Xi + \alpha^2(1+\beta))}\hat{Y}_0 + \frac{\alpha^2(1-\alpha)(1+\beta) + (1+\beta-\alpha)\Xi}{\Xi(\Xi + \alpha^2(1+\beta))}\hat{Y}_0^* && (311)\\
\hat{S}_1 &= -\frac{\alpha^2(1-\alpha)(1+\beta)}{\Xi(\Xi + \alpha^2(1+\beta))}\hat{Y}_0 - \frac{\alpha(\Xi - \alpha(1-\alpha)(1+\beta))}{\Xi(\Xi + \alpha^2(1+\beta))}\hat{Y}_0^* && (312)
\end{aligned}
$$

We substitute the terms of trade into the home market clearing condition in period 0 to compute the demand for home goods in that period:

$$
\hat{Y}_0 = \left(1 - \frac{\alpha(1-\alpha)\beta}{\Xi + \alpha^2(1+\beta)}\right)\hat{Y}_0 - \frac{\beta(\Xi - \alpha(1-\alpha)(1+\beta))}{(1+\beta)(\Xi + \alpha^2(1+\beta))}\hat{Y}_0^* \qquad (313)
$$

The demand for home goods is :

$$
\begin{aligned}
\hat{Y}_0 &= -\frac{\Xi - \alpha(1-\alpha)(1+\beta)}{\alpha(1-\alpha)(1+\beta)}\hat{Y}_0^* && (314)\\
&= -\left(\frac{1-\alpha}{\alpha} + 2(\eta - 1)\right)\hat{Y}_0^* && (315)
\end{aligned}
$$

The relative prices in this system with zero bound and debt limit are

$$
\hat{S}_0 = \frac{1}{\alpha}\hat{Y}_0^*, \quad \hat{S}_1 = 0, \quad \hat{R} = 0, \quad \hat{R}^* = -\frac{2\alpha - 1}{\alpha}\hat{Y}_0^* \qquad (316)
$$

It is easy to check that the home market clearing condition in period 1, Equation (309), holds.

C.3 A Three-Country Leveraged Model

The model contains two debt market clearing conditions, which are the uncovered interest parities, Equations (57) and (58), and six goods market clearing conditions as follows:

$$
\begin{aligned}
Y_0^A = {} & \alpha\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^A \\
& +\alpha\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}D^A \\
& +\frac{\alpha^*}{(1+\beta^B)}(S_{A0}^B)^{-\eta}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^B \\
& +\frac{\alpha^*}{(1+\beta^B)R^B}(S_{A0}^B)^{-\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^B \\
& +\frac{\alpha^*}{(1+\beta^C)}(S_{A0}^C)^{-\eta}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-1}Y_0^C \\
& +\frac{\alpha^*}{(1+\beta^C)R^C}(S_{A0}^C)^{-\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^C
\end{aligned} \tag{317}
$$

$$
\begin{aligned}
Y_0^B = {} & \alpha^*\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-1}(S_{A0}^B)^{\eta}Y_0^A \\
& +\alpha^*\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}(S_{A0}^B)^{\eta}D^A \\
& +\frac{\alpha}{(1+\beta^B)}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^B \\
& +\frac{\alpha}{(1+\beta^B)R^B}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^B \\
& +\frac{\alpha^*}{(1+\beta^C)}(S_{A0}^B)^{\eta}(S_{A0}^C)^{-\eta}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-1}Y_0^C \\
& +\frac{\alpha^*}{(1+\beta^C)R^C}(S_{A0}^B)^{\eta}(S_{A0}^C)^{-\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^C
\end{aligned} \tag{318}
$$

$$
\begin{aligned}
Y_0^C = {} & \alpha^*\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{-1}(S_{A0}^C)^{\eta}Y_0^A \\
& +\alpha^*\left(\alpha+\alpha^*(S_{A0}^B)^{-(1-\eta)}+\alpha^*(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}(S_{A0}^C)^{\eta}D^A \\
& +\frac{\alpha^*}{(1+\beta^B)}(S_{A0}^B)^{-\eta}(S_{A0}^C)^{\eta}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-1}Y_0^B \\
& +\frac{\alpha^*}{(1+\beta^B)R^B}(S_{A0}^B)^{-\eta}(S_{A0}^C)^{\eta}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^B \\
& +\frac{\alpha}{(1+\beta^C)}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-1}Y_0^C \\
& +\frac{\alpha}{(1+\beta^C)R^C}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\end{array}\right]Y_1^C
\end{aligned} \tag{319}
$$

$$\begin{aligned}
Y_1^A &= \alpha\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}Y_1^A \\
&\quad -\alpha\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}R^AD^A \\
&\quad +\frac{\alpha^*\beta^BR^B}{(1+\beta^B)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{-\eta}Y_0^B \\
&\quad +\frac{\alpha^*\beta^B}{(1+\beta^B)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{-\eta}Y_1^B \\
&\quad +\frac{\alpha^*\beta^CR^C}{(1+\beta^C)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^C)^{-\eta}Y_0^C \\
&\quad +\frac{\alpha^*\beta^C}{(1+\beta^C)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}(S_{A1}^C)^{-\eta}Y_1^C \qquad (320)
\end{aligned}$$

$$\begin{aligned}
Y_1^B &= \alpha^*\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{\eta}Y_1^A \\
&\quad -\alpha^*\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}(S_{A1}^B)^{\eta}R^AD^A \\
&\quad +\frac{\alpha\beta^BR^B}{(1+\beta^B)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right]Y_0^B \\
&\quad +\frac{\alpha\beta^B}{(1+\beta^B)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}Y_1^B \\
&\quad +\frac{\alpha^*\beta^CR^C}{(1+\beta^C)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{\eta}(S_{A1}^C)^{-\eta}Y_0^C \\
&\quad +\frac{\alpha^*\beta^C}{(1+\beta^C)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}(S_{A1}^B)^{\eta}(S_{A1}^C)^{-\eta}Y_1^C \qquad (321)
\end{aligned}$$

$$\begin{aligned}
Y_1^C &= \alpha^*\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^C)^{\eta}Y_1^A \\
&\quad -\alpha^*\left(\alpha+\alpha^*(S_{A1}^B)^{-(1-\eta)}+\alpha^*(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}(S_{A1}^C)^{\eta}R^AD^A \\
&\quad +\frac{\alpha^*\beta^BR^B}{(1+\beta^B)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^B)^{1-\eta}+\alpha^*(S_{A0}^B)^{1-\eta}(S_{A0}^C)^{-(1-\eta)}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}}\end{array}\right](S_{A1}^B)^{-\eta}(S_{A1}^C)^{\eta}Y_0^B \\
&\quad +\frac{\alpha^*\beta^B}{(1+\beta^B)}\left(\alpha+\alpha^*(S_{A1}^B)^{1-\eta}+\alpha^*(S_{A1}^B)^{1-\eta}(S_{A1}^C)^{-(1-\eta)}\right)^{-1}(S_{A1}^B)^{-\eta}(S_{A1}^C)^{\eta}Y_1^B \\
&\quad +\frac{\alpha\beta^CR^C}{(1+\beta^C)}\left[\begin{array}{c}\left(\alpha+\alpha^*(S_{A0}^C)^{1-\eta}+\alpha^*(S_{A0}^B)^{-(1-\eta)}(S_{A0}^C)^{1-\eta}\right)^{-\frac{1}{1-\eta}}\\ \times\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\end{array}\right]Y_0^C \\
&\quad +\frac{\alpha\beta^C}{(1+\beta^C)}\left(\alpha+\alpha^*(S_{A1}^C)^{1-\eta}+\alpha^*(S_{A1}^B)^{-(1-\eta)}(S_{A1}^C)^{1-\eta}\right)^{-1}Y_1^C \qquad (322)
\end{aligned}$$

Table 1: Solutions of the three-country model after a shock in the endowment of country B in period 0

Case:	No zero bound	Zero bound Binding in Country A: Country A & C share shortfall in demand equally	Zero bound Binding in Country A: Country C controls S^C_{A0}, S^C_{A1}	Zero bound Binding in Country A: Country A controls S^C_{A0}, S^C_{A1}
$\hat{Y}^A_0$	0	$-\frac{\Omega}{2\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	$-\frac{\Omega}{\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	0
$\hat{Y}^C_0$	0	$-\frac{\Omega}{2\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	0	$-\hat{Y}^B_0$
$\hat{Y}^A_1, \hat{Y}^B_1, \hat{Y}^C_1$	0	0	0	0
$\hat{R}^A$	$-\frac{\Omega}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	0	0	0
$\hat{R}^B$	$-\frac{\Omega+(1+\beta)\Psi}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	$-\frac{(1+\beta)\Psi}{2\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	$-\frac{(2\Omega+(1+\beta)\Psi)(1+\beta)\Psi}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{(1+\beta)\Psi}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0$
$\hat{R}^C$	$-\frac{\Omega}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0$	0	$-\frac{(1+\beta)\Psi\Omega}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{(1+\beta)\Psi}{3\Omega+(1+\beta)\Psi}\hat{Y}^B_0$
$\hat{S}^B_{A0}$	$\frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{(2\Omega+(1+\beta)\Psi)((3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi)}{\Omega(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$
$\hat{S}^B_{A1}$	$-\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(2\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{(2\Omega+(1+\beta)\Psi)(\Omega-\alpha^*(1+\beta))\Psi}{\Omega(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$
$\hat{S}^C_{A0}$	0	0	$\frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{(3\alpha^*+\beta)\Omega+\alpha^*(1+\beta)\Psi}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$
$\hat{S}^C_{A1}$	0	0	$-\frac{\Psi(\Omega-\alpha^*(1+\beta))}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\Psi(\Omega-\alpha^*(1+\beta))}{\Omega(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$
D^A	$\frac{\beta}{(1+\beta)}\frac{(\Omega-\alpha^*(1+\beta))}{(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\beta}{(1+\beta)}\frac{(\Omega-\alpha^*(1+\beta))}{(2\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\beta}{(1+\beta)}\frac{(\Omega-\alpha^*(1+\beta))}{(\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	0
D^B	$-\frac{\beta}{(1+\beta)}\frac{2(\Omega-\alpha^*(1+\beta))}{(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{\beta}{(1+\beta)}\frac{2(\Omega-\alpha^*(1+\beta))}{(2\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{\beta}{(1+\beta)}\frac{(3\Omega+2(1+\beta)\Psi)(\Omega-\alpha^*(1+\beta))}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$-\frac{\beta}{(1+\beta)}\frac{3(\Omega-\alpha^*(1+\beta))}{(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$
D^C	$\frac{\beta}{(1+\beta)}\frac{(\Omega-\alpha^*(1+\beta))}{(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\beta}{(1+\beta)}\frac{(\Omega-\alpha^*(1+\beta))}{(2\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\beta}{(1+\beta)}\frac{\Psi(1+\beta)(\Omega-\alpha^*(1+\beta))}{(\Omega+(1+\beta)\Psi)(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$	$\frac{\beta}{(1+\beta)}\frac{3(\Omega-\alpha^*(1+\beta))}{(3\Omega+(1+\beta)\Psi)}\hat{Y}^B_0$

Note: $\Omega = \alpha^*(1+\beta)(1+(\eta-1)(1+\alpha-\alpha^*)) > 0, \;\; \Psi = (1-3\alpha^*) > 0.$

Table 2: Calibration of the two-country model with leverage and the model without a leverage constraint

	Definition	Symbol	Values in model with leverage	Values in model without a leverage constraint
Parameters	Elasticity of substitution between home and foreign goods	η	2	2
	Consumption home bias	α	0.7	0.7
	Discount factor in the US	β^{US}	–	0.825
	Discount factor in China	β	1.857	1.857
	US output in period 0	Y_0	1	1
	U.S output in period 1	Y_1	1.1	1.1
	Chinese output in period 0	Y_0^*	0.4	0.4
	Chinese output in period 1	Y_1^*	0.6	0.6
	U.S debt limit	D	0.087	0.087*
Equilibrium values	U.S real interest rate	R	1.1	1.102
	Chinese real interest rate	R^*	1.1	1.102
	Period 0 terms of trade	S_0	0.732	0.732
	Period 1 terms of trade	S_1	0.732	0.731

* D in the model without leverage is endogenous.

Table 3: Calibration of the symmetric leveraged three-country model

	Definition	Symbol	Values
Parameters	Elasticity of substitution between home and foreign goods	η	2
	Consumption home bias	α	0.7
	Consumption share of each foreign country	α^*	0.15
	Discount factor in China	β^B	0.65
	Discount factor in Europe	β^C	0.65
	US output in period 0	Y_0^A	1
	US output in period 1	Y_1^A	1
	Chinese output in period 0	Y_0^B	1
	Chinese output in period 1	Y_1^B	1
	Europe output in period 0	Y_0^C	1
	Europe output in period 1	Y_1^C	1
	US debt limit	D^A	0
Equilibrium values	US real interest rate	R^A	1.5385
	Chinese real interest rate	R^B	1.5385
	Europe real interest rate	R^C	1.5385
	Period 0 US-China terms of trade	S_{A0}^B	1
	Period 1 US-China terms of trade	S_{A1}^B	1
	Period 0 US-Europe terms of trade	S_{A0}^C	1
	Period 1 US-Europe terms of trade	S_{A1}^C	1
	Period 0 consumption in US	C_0^A	1
	Period 1 consumption in US	C_1^A	1
	Period 0 consumption in China	C_0^B	1
	Period 1 consumption in China	C_1^B	1
	Period 0 consumption in Europe	C_0^C	1
	Period 1 consumption in Europe	C_1^C	1
	Debt in China	D^B	0
	Debt in Europe	D^C	0